Interpretation of the temperature dependence of the electromagnetic penetration depth in $YBa_2Cu_3O_{7-\delta}$

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The low-temperature behavior of the *a-b* plane penetration depth, λ_{ab}, is a probe of the pairing state in $YBa_2Cu_3O_{7-\delta}$. A group-theoretic analysis shows that in orthorhombic or tetragonal crystals *all* singlet pairing states other than "*s* wave" would lead to $\Delta\lambda_{ab}(T)\equiv\lambda_{ab}(T)-\lambda_{ab}(0)\sim T$. In contrast, for an isotropic system, there are combinations of singlet pairing states and field directions that would give rise to $\Delta\lambda(T)\sim T^3$. We reanalyze the surface impedance data of Fiory *et al.* [Phys. Rev. Lett. **61**, 1419 (1988)], and show that these data exhibit neither a BCS temperature dependence nor a linear temperature dependence at low temperature, but instead follow $\Delta\lambda_{ab}(T)\sim T^2$. This behavior is probably not intrinsic, and possible explanations are discussed.

I. INTRODUCTION

Only at low temperature, T, is the temperature dependence of the electromagnetic penetration depth tensor $\lambda(T)$ a potential probe of the pairing state in superconductors.[1] This probe has a compelling virtue: it is a direct measure of the superfluid density and is thus unaffected by the possible presence of low-lying excitations not associated with nodes in the superconducting gap.

In a perfect orthorhombic single crystal, the penetration depth tensor has three principal components, which we will call λ_a, λ_b, and λ_c, where the subscript indicates the direction in which the screening currents flow. In this paper we consider the measurement of the penetration depth in situations where the screening currents flow predominantly in the *a-b* plane. This is a situation of current experimental interest, for which there exists a large body of experimental data.[2-7] In an isotropic system, unconventional singlet pairing states, for example, *d*-wave states, can give rise to a temperature dependence for λ_a which differs from that of λ_b, i.e.,

$$\frac{\Delta\lambda_a(T)}{\lambda_a(0)}\approx A\left[\frac{T}{T_c}\right]+O\left[\left[\frac{T}{T_c}\right]^2\right], \tag{1.1}$$

whereas

$$\frac{\Delta\lambda_b(T)}{\lambda_b(0)}\approx C\left[\frac{T}{T_c}\right]^3+O\left[\left[\frac{T}{T_c}\right]^4\right]. \tag{1.2}$$

Here we define $\Delta\lambda_a\equiv\lambda_a(T)-\lambda_a(0)$, etc., and A and C are constants of order unity. Throughout this paper we shall use the term "unconventional" to refer to a superconducting pairing state whose gap function does not have the full crystal point group symmetry.[1]

The purpose of this paper is to point out that because of the oyrthorhombic (or near tetragonal) symmetry of $YBa_2Cu_3O_{7-\delta}$, *any* conventional singlet pairing state would give rise to a linear temperature dependence in *both* λ_a and λ_b at low temperatures. This observation should be useful in ruling out candidates for the pairing state from experimental data, because the linear dependence, if present, would be much easier to distinguish from the *s*-wave BCS prediction than would a cubic temperature dependence. Below, we explain the origin of this result, and then discuss its implications for the existing experimental data for the *a-b* plane average of the penetration depth, λ_{ab}.[2-7] The data which are least noisy at low temperature are those obtained by measuring the inductive contribution to the surface impedance.[2] We have reanalyzed these data and find that for two films of thickness 500 and 2000 Å, $\lambda_{ab}(0)$ is 1425±25 Å for *both* films. The low temperature dependence follows $\Delta\lambda_{ab}(T)\sim T^2$, contrary to the predictions based on both *s*-wave and unconventional singlet pairing states. Certain caveats apply, of course, to these predictions, and these are briefly discussed too.

II. THEORY

In order to establish rigorous constraints on the pairing state from the temperature dependence of the electromagnetic penetration depth, it is necessary to determine the asymptotic behavior at low temperature. The penetration depth is determined by the superfluid density, so that at low temperature, if the gap on the Fermi surface is nonzero everywhere, $\Delta\lambda_{ab}(T)$ exhibits activated behavior. In *s*-wave BCS theory for a spherical Fermi surface,[8]

$$\frac{\Delta\lambda(T)}{\lambda(0)}\sim 3.33\left[\frac{T}{T_c}\right]^{1/2}\exp(-1.76T_c/T) \tag{2.1}$$

for small T. A similar formula would be valid for a nodeless triplet state, such as the Balian-Werthamer (BW) state. Conversely, if the excitation gap vanishes on points or lines of the Fermi surface, then the penetration depth will exhibit a power-law behavior at small T. In general, powers of T, T^2, T^3, or T^4 are possible depending on the type of nodes and the orientation of the applied field.[1,9] Thus, a measurement of the low temperature penetration depth enables one not only to determine whether or not the gap function has nodes, but also to investigate whether or not any such nodes correspond to points or lines on the Fermi surface.

It is not possible to determine unambiguously the pairing state from the temperature dependence away from the asymptotic low-temperature regime, since there are many effects which may influence the detailed form of the temperature dependence, including strong-coupling corrections, dirt, the precise shape of the Fermi surface, and gap anisotropy. In particular one cannot conclude that the pairing is s-wave by fitting the overall behavior to either the clean weak coupling isotropic s-wave BCS result of Ref. 8 or to the empirical Gorter-Casimir formula

$$\frac{\lambda(T)}{\lambda(0)}=[1-(T/T_c)^4]^{-1/2} \tag{2.2}$$

since other pairing states may lead to a similar looking temperature dependence away from the asymptotic low-temperature regime.[9]

At low temperature, the quantities $\Delta\lambda_a(T)$ and $\Delta\lambda_b(T)$ are proportional to the a and b axis diagonal components of the normal fluid density tensor,[9] defined in the absence of Fermi liquid corrections by

$$\rho_{ij}^n=\frac{m}{\hbar}\int\frac{d^2k}{(2\pi)^3|\mathbf{v}|}v_iv_j\int d\varepsilon_k\frac{\beta}{2}\text{sech}^2\left[\frac{\beta E_k}{2}\right] , \tag{2.3}$$

where the integral is over the Fermi surface, $\mathbf{v}$ is the Fermi velocity, $\beta=1/(k_BT)$, ε_k is the normal state band energy measured with respect to the chemical potential, and E_k is the quasiparticle energy, given by

$$E_k^2=\varepsilon_k^2+|\Delta_k|^2 \tag{2.4}$$

for singlet superconductors and by

$$E_k^2=\varepsilon_k^2+|\mathbf{d}_k|^2\pm|\mathbf{d}_k\times\mathbf{d}_k^*| \tag{2.5}$$

for triplet superconductors, where Δ_k and $\mathbf{d}_k$ are the respective gap functions. It is straightforward to see from Eq. (2.3) that if the excitation gap vanishes on lines on the Fermi surface, then some components of ρ_{ij}^n will approach zero linearly with T at small temperatures.

To be more quantitative, we evaluate ρ_{ij}^n assuming a cylindrical Fermi surface, appropriate for $YBa_2Cu_3O_{7-\delta}$, for various possible dispositions of line nodes on the Fermi surface. First, assuming a line of nodes in the plane $k_z=0$ we obtain

$$\rho^n\propto\text{diag}(I_0^x\eta,I_0^y\eta,I_2^z\eta^3) , \tag{2.6}$$

where I_0^x, etc., are nonzero constants of order unity, $\eta\equiv k_BT/\Delta_{\max}$, and $\Delta_{\max}$ is the maximum value of the zero temperature energy gap over the Fermi surface. The notation "diag" refers to the diagonal components of the tensor in question. Alternatively, if there are four-line nodes parallel to the c axis and separated by $\pi/2$ around the cylindrical Fermi surface, for example, along the lines $k_x=0$ and $k_y=0$, we obtain

$$\rho_{ij}^n\propto\text{diag}(I_0^x\eta+I_2^x\eta^3,I_0^y\eta+I_2^y\eta^3,I_0^z\eta) . \tag{2.7}$$

These simple results show that for both of these possible dispositions of line nodes we obtain $\Delta\lambda_a(T)\propto T$ and $\Delta\lambda_b(T)\propto T$ as $T\rightarrow 0$. These results are not at all sensitive to the detailed shape of the Fermi surface, so long as it has cylindrical or spherical topology.

For superconductors with orthorhombic or tetragonal point groups, a group theoretic analysis shows that *all possible* singlet pairing states other than s-wave pairing have line nodes corresponding to one or more of the cases described above.[10] It follows that for all unconventional singlet states we would expect to have $\Delta\lambda_a(T)\propto T$ and $\Delta\lambda_b(T)\propto T$. If the experimental data do not show this behavior we would have to conclude that the pairing state in $YBa_2Cu_3O_{7-\delta}$ cannot be unconventional singlet pairing state.

In fact, a slightly more general conclusion can be drawn. In all measurements to date, λ_a and λ_b are not measured separately, either because of crystal twinning, or because the sample is a film or an oriented powder. What is measured may be assumed to be an average over the ab plane of the penetration depth, λ_{ab}. Consider a pairing state with just two-line nodes parallel to the c axis, corresponding, for example, to the intersection of the plane $k_x=0$ with the Fermi surface. This case does not arise if the pairing state is a singlet, for the reasons discussed in the preceding paragraph, but such a deployment of line nodes can occur in some of the candidate triplet pairing states. In this case we obtain

$$\rho_{ij}^n\propto\text{diag}(I_2^x\eta^3,I_0^y\eta,I_0^z\eta) .$$

Then, performing the average, we again predict that $\Delta\lambda_{ab}(T)\propto T$ as $T\rightarrow 0$. Thus if a linear temperature dependence is not observed experimentally, then this would suffice to rule out *all pairing states with line nodes* in orthorhombic or tetragonal crystals. This includes a great many of the triplet states in addition to the unconventional singlet states. The only states permitted would be either those with no nodes (including s-wave) or those which have point nodes.

It is important to point out that a number of rather stringent *caveats* apply to our results. It is well known that both scattering processes and Fermi liquid renormalizations are capable of generating a T^2 term in $\Delta\lambda$ even if there are line nodes in the gap.[9,11,12] For example, it is thought that these effects apply to the unconventional superconductor UPt_3 which has a penetration depth varying as T^2 at low T. It is also possible that the Fermi surface does not have the topology of a cylinder or sphere[13] in which case the pairing state may have "d-wave" symmetry, but not give rise to nodes in the energy gap.

III. EXPERIMENT

We now discuss the implications of the above results for recent experiments on $YBa_2Cu_3O_{7-\delta}$. As mentioned earlier, it is imperative to examine the data at low temperatures only. The surface impedance data in Ref. 2 exhibit the least scatter at low temperatures of all the data published. All the other data sets we have examined (Refs. 3–7) appear similar, but have either more scatter or fewer data points at low temperatures, making it harder to draw definitive conclusions. The data of Sridhar *et al.*[14] do not go below about $0.7T_c$, so that they cannot be used to study the low-temperature behavior of $\lambda_{ab}(T)$. The data of Anlage *et al.*[15] were analyzed in a way which explicitly assumed the Gorter-Casimir formula for $\lambda_{ab}(T)$, and therefore do not provide an independent test of the temperature dependence. For these reasons we focus on the data of Ref. 2 below.

Fiory *et al.* measure the surface impedance of two epitaxial thin films, of thickness 500 and 2000 Å. They extract the penetration depth λ_{ab} from the inductive component, L, using the formula

$$L=\frac{4\pi}{c^2}\frac{\lambda_{ab}^2}{d}\ , \qquad (3.1)$$

where d is the film thickness, c is the speed of light in vacuum, and cgs units are used. They fit their results to BCS theory and find that $\lambda_{ab}(0)$ is 1500 Å for the thinner film and 2100 Å for the thicker film. They attribute the discrepancy between these two results to Josephson coupling between grains in the 2000-Å film.

The formula for the surface inductance used by Fiory *et al.*, Eq. (3.1), is only valid when $d \ll \lambda_{ab}$. For the 2000-Å film, this condition is not satisfied. The complete expression, valid for all values of d/λ_{ab} is[15]

$$L=\frac{4\pi}{c^2}\lambda_{ab}\coth\left[\frac{d}{\lambda_{ab}}\right]. \qquad (3.2)$$

This expression follows from the solution of Maxwell's equations for electromagnetic waves incident on a free-standing slab of superconductor. When $d \ll \lambda$, we recover the expression used by Fiory *et al.* Both formulas ignore any possible effects of the substrate, which may be significant when $d<\lambda$. When the data are reanalyzed using this expression we obtain the results shown in Fig. 1. The discrepancy found in Ref. 2 between the zero temperature values of λ_{ab} has disappeared; the data from both films are consistent with a value $\lambda_{ab}(0)=1425\pm25$ Å.

In order to address the question of the precise form of the low temperature behavior we have plotted the data on an expanded scale, as shown in Fig. 2. From the figure it is apparent that the data exhibit upward curvature at even the lowest temperatures. Attempting to bound any coefficient of a linear temperature dependence in $\Delta\lambda_{ab}$, we write

$$\frac{\Delta\lambda_{ab}(T)}{\lambda_{ab}(0)}=A\left[\frac{T}{T_c}\right]+O\left[\left[\frac{T}{T_c}\right]^2\right]. \qquad (3.3)$$

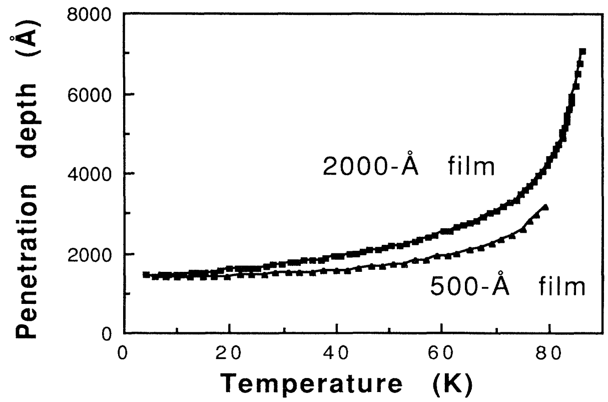

FIG. 1. λ_{ab} vs T for $YBa_2Cu_3O_{7-\delta}$ films of thickness 500 and 2000 Å. The data are from Ref. 2, reanalyzed as discussed in the text.

We estimate that A lies between 0 and approximately 0.13 for the thinner film and between 0 and 0.21 for the thicker film. For comparison the estimate of A, from evaluating Eq. 2.6 or 2.7, assuming an unconventional singlet pairing state and a cylindrical Fermi surface, but excluding corrections due to inelastic scattering and Fermi liquid effects, gives a value for A lying between $2.2k_BT_c/2\Delta_{\max}$ and $2.8k_BT_c/2\Delta_{\max}$, depending upon precisely which pairing state is being considered. Estimates of $2\Delta_{\max}/k_BT_c$ range form 3.5 to 8, implying that $0.28<A<0.9$. The curvature of the data, and the small or possibly zero value of A compared to the above expectations suggests that unconventional singlet pairing does not occur in these samples.

Now we address the question of whether or not the data are consistent with s-wave BCS theory. Figure 3 shows the data plotted against T^2. The data fall on a straight line for $0<T<55$ K, suggesting that they are consistent with the expression

$$\frac{\Delta\lambda_{ab}(T)}{\lambda_{ab}(0)}=B\left[\frac{T}{T_c}\right]^2+O\left[\left[\frac{T}{T_c}\right]^3\right], \qquad (3.4)$$

where $B\approx0.63$ for the thinner film and $B\approx1.6$ for the

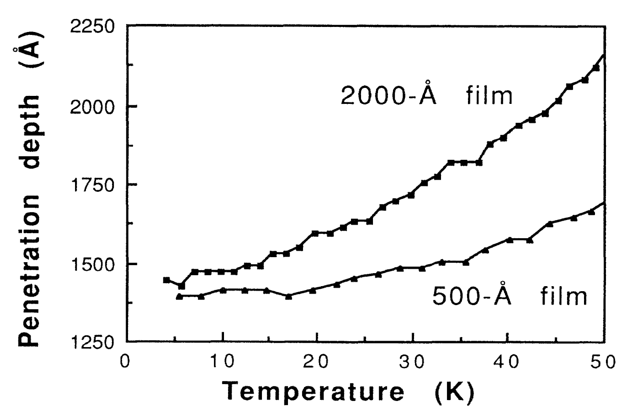

FIG. 2. The data of Fig. 1, shown on an expanded scale to emphasize the low-temperature region.

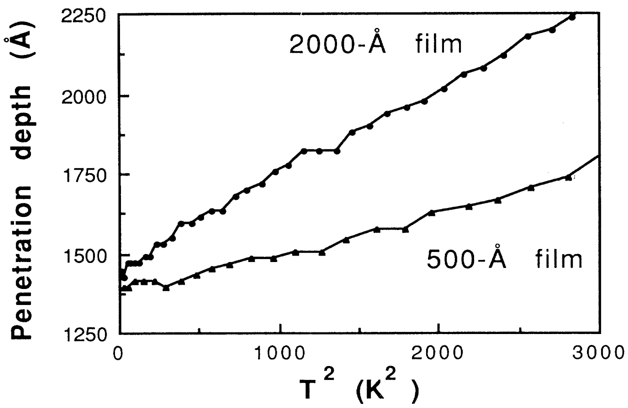

FIG. 3. The data of Fig. 1 plotted against T^2 in the low-temperature region.

thicker film. Therefore, it seems that these data do not follow the BCS prediction. Fiory *et al.* noted agreement with the BCS in the overall temperature dependence, but did not carefully examine the low-temperature behavior. It should be noted that the temperature dependence is essentially unchanged, whether the penetration depth is obtained from the thin-film formula of Eq. (3.1) or from the more general equation (3.2).

The fact that the coefficient of T^2 is sample dependent implies that nonintrinsic effects make a contribution to this term. Let us briefly consider possible causes for this behavior. First, as we have already mentioned above, unconventional pairing states may give rise to a T^2 behavior when either scattering processes or Fermi liquid corrections are taken into account. In the case of nonmagnetic impurity scattering, the coefficient of T^2 would depend upon the concentration of impurities, and so would be sample dependent.

Second, it may also be possible to explain the T^2 behavior within conventional BCS "*s*-wave" pairing. Hebard *et al.*[16] have shown that in the high-field regime (fields greater than about 1 T), the dominant contribution to the surface impedance comes not from the condensate, but from pinned vortices. They demonstrate that in this case, the penetration depth at a fixed external field is indeed proportional to T^2 at low temperature.[17] Might the ambient fields of 5 mG in Ref. 2 account for the T^2 term measured there? Using the expression in Ref. 16 we find that any such term is completely negligible, in agreement with the assertion in Re. 2 that vortex pinning is negligible. Another possibility is noted in Ref. 9; if there is a sufficient concentration of magnetic impurities present, then a T^2 term can be present even in an *s*-wave superconductor. However for this to occur the pair-breaking scattering rate at low temperature must be comparable with the gap function. A third possibility would be weak pair-breaking scattering whose magnitude is temperature dependent. Such scattering would contribute a temperature dependence to the superfluid density over and above that predicted by BCS theory.

There have been other reports[3,7,18–20] that $\Delta\lambda_{ab}$ is proportional to T^2, although there is no consensus about the value of B. Indeed, dc measurements are consistent with a value $0 \le B < 0.4$, whereas ac techniques yield values in the vicinity of $B \approx 1.0$. On the other hand, there are also claims[2–6] that $\lambda_{ab}(T)$ follows the BCS prediction over the range $0 < T < T_c$, although for most of these measurements, it is difficult to draw any first conclusions in the low-temperature region.

IV. CONCLUSION

To summarize, we have shown that the point-group symmetry of the high temperature superconductors implies that *any* unconventional singlet-pairing state leads to $\Delta\lambda_{ab} \propto T$, unless scattering or Fermi-liquid corrections are important. The data do not seem to be consistent with this linear temperature dependence. Kinetic measurements seem to give a quadratic temperature dependence, whose origin is unclear.

Finally, it is interesting to mention the constraints on unconventional pairing states obtained from other experiments.[1] Evidence against triplet pairing has recently been provided by the measurements of the anisotropy of the Knight shift.[21] A reliable and detailed study of the penetration depth at low temperatures would eliminate the uncertainties in the identification of the pairing state to which we have referred, and is clearly warranted.

Note added in proof: After completion of this work, we received copies of unpublished work from V. F. Grantmakher *et al.* and S. M. Anlage *et al.* which specifically examine the low-temperature behavior of $\lambda_{ab}(T)$ and report power laws with an exponent of two in the former case and an exponent with a value between 1.3 and 3.3 in the latter case.

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[1] For a detailed introducton to the definition and classification of the different pairing states in high-temperature superconductors, and their possible experimental consequences, see J. F. Annet, N. D. Goldenfeld, and S. R. Renn, in *Physical Properties of High Temperature Superconductors II*, edited by D. M. Ginsberg (World Scientific, New Jersey, 1990).
[2] A. T. Fiory, A. F. Hebard, P. M. Mankiewich, and R. E. Howard, Phys. Rev. Lett. **61**, 1419 (1988).
[3] D. R. Harshmann *et al.*, Phys. Rev. B **39**, 851 (1989).
[4] L. Krusin-Elbaum *et al.*, Phys. Rev. Lett. **62**, 217 (1989).
[5] S. Mitra *et al.*, Phys. Rev. B **40**, 2674 (1989).
[6] Y. J. Uemura *et al.*, Physica C **162-164**, 857 (1989).
[7] T. M. Riseman *et al.* (unpublished).
[8] B. Muhlschlegel, Z. Phys. **155**, 313 (1959).

[9]F. Gross *et al.*, Z. Phys. B **64**, 175 (1986).
[10]M. Sigrist and T. M. Rice, Z. Phys. B **68**, 9 (1987); J. F. Annett, Adv. Phys. **39**, 83 (1990).
[11]C. H. Choi and P. Muzikar, Phys. Rev. B **37**, 5947 (1988).
[12]C. M. Varma, K. Miyake and S. Schmitt-Rink, Phys. Rev. Lett. **57**, 626 (1986).
[13]J. R. Schrieffer, X. G. Wen, and S. C. Zhang, Phys. Rev. B **39**, 1166 (1989).
[14]S. Sridhar, D. H. Ho, and W. Kennedy, Phys. Rev. Lett. **63**, 1873 (1989).
[15]S. M. Anlage *et al.*, Appl. Phys. Lett. **54**, 2710 (1989).
[16]A. F. Hebard, P. L. Gammel, C. E. Rice, and A. F. J. Levi, Phys. Rev. B **40**, 5243 (1989).
[17]P. L. Gammel, A. F. Hebard, C. E. Rice, and A. F. J. Levi, Physica C **162-164**, 1565 (1989).
[18]J. R. Cooper *et al.*, Physica C **153-155**, 1491 (1988).
[19]J. P. Carini *et al.*, Phys. Rev. B **37**, 9726 (1988).
[20]L. Drabeck *et al.*, Phys Rev. B **39**, 785 (1989).
[21]S. E. Barrett *et al.*, Phys. Rev. B **41**, 6283 (1990).