# Magnetic-field dependence of the critical dynamics of a superconducting $YBa_2Cu_3O_{7-\delta}$ detwinned single crystal

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The in-plane longitudinal resistivities $\rho_{aa}$ and $\rho_{bb}$ of a single-crystal of $YBa_2Cu_3O_{7-\delta}$ have been measured as a function of magnetic field $H$ (less than 6.75 T) and temperature $T$ near $T_c$. We found that the $\rho_{aa}(T,H)$ data in the mixed state collapse into a single curve as a function of $\rho_{bb}(T,H)$. The extracted fluctuation conductivity exhibits dynamic scaling over a wide range of $T$ and $H$: assuming that the static universality class is the three-dimensional $XY$ model yields the observed dynamic critical exponent $z=1.5\pm 0.1$ $(H>0)$, apparently consistent with the dynamic universality class of superfluid $^4$He. In zero field, the data are inconclusive, and a reliable determination of $z$ is not possible. [S0163-1829(97)03026-9]

In contrast to conventional superconductors, high- $T_c$ superconductors have coherence lengths that are short enough to introduce appreciable fluctuation effects and scaling behavior which requires a theoretical treatment going beyond Gaussian corrections to mean-field theory. The fluctuation effects have been observed in several measurements, e.g., penetration depth,[1] magnetic susceptibility,[2–4] resistivity,[3–6] and specific heat.[7,8] In a strongly type-II superconductor, the coupling of the order parameters to gauge fields is weak, leading to an extended intermediate temperature interval over which the static critical behavior is that of a neutral complex scalar order parameter, i.e., the $XY$ model.[9] Observations of fluctuation effects are consistent with fully three-dimensional (3D) critical behavior in this universality class.

Observations of the dynamic critical exponent $z$ specifying the dynamic universality class are less abundant and are ambiguous to date. We emphasize that the identification of the intermediate asymptotic 3D-$XY$ static behavior is independent of the identification of the dynamic universality class. An exponent $z\cong 2$ is suggested for resistivity measurements in the presence of a magnetic field,[3,5] but the fluctuation conductivity data do not collapse convincingly onto a single curve. Other workers have reported a value $z\cong 3/2$ for the zero-field conductivity.[10,11] However, less than a decade of scaling behavior is observed, and the sensitivity for small changes in $T_c$ used in the fitting is difficult to assess.

The value of the dynamic critical exponent expected theoretically is equally unclear. Neither the true asymptotic critical regime, with fluctuating gauge fields, nor the intermediate asymptotic regime where the static universality class is 3D-$XY$ has been systematically studied to date. A heuristic argument, due to Halperin,[12] suggests that the number of Cooper pairs is not conserved because of plasma fluctuations, so that both the phase and magnitude of the order parameter obey relaxational dynamics in the universality class of model $A$, leading to $z\cong 2$ in three dimensions. This expectation contrasts sharply with the result for $^4$He: the charge density is conserved even though the phase fluctuates, and model $E$ dynamics are appropriate with $z\cong 3/2$.

We report here on the in-plane longitudinal resistivities ($\rho_{aa}$ and $\rho_{bb}$) for both $a$ and $b$ directions and on the critical fluctuations associated with the respective conductivities $\sigma_{aa}$ and $\sigma_{bb}$ near the critical temperature of a detwinned single crystal of $YBa_2Cu_3O_{7-\delta}$. We have used critical scaling theory to probe the dynamical universality class of the single crystal; this crystal has a higher $T_c$ and a lower resistivity than the samples used in other reported resistivity measurements.[3,4,6] Our high-quality sample is intended to preclude the role of defects in a magnetic field. (The effect of disorder on static critical behavior has been verified to be irrelevant in the case of zero field.[1,13,14])

The single crystal of $YBa_2Cu_3O_{7-\delta}$ was grown in an yttria-stabilized zirconia crucible by using a flux method described in detail elsewhere.[15] A diamagnetic transition temperature of 92.8 K and a transition width (10–90 %) of 0.25 K were measured with a Quantum Design 1-T superconducting quantum interference device magnetometer. The resistivities $\rho_{aa}$ and $\rho_{bb}$ measured on the same sample were, respectively, about 60 and 30 $\mu\Omega$ cm at $T=100$ K. The zero-resistance transition temperature was 93.4 K for both $a$- and $b$-axis conduction. The resistive transition width $\Delta T_c$ (10–90 %) was 0.15 K. The high-$T_c$, small-$\Delta T_c$, and low values of resistivity indicated that the sample was optimally oxygenated and of high quality.

The same sample was used for the resistivity measurements along both the $a$ and $b$ axes. It had a rectangular shape ($0.7\times 0.4$ mm$^2\times 40$ $\mu$m), and was detwinned at 450 °C to produce the desired orientations. The details of the detwinning process are described elsewhere.[16] The contacts for attaching gold wires were made by evaporating gold onto the crystal and heating it at 500 °C for 6 h in flowing $O_2$. Gold diffuses less than 0.2 $\mu$m into the sample during this process.[17] All of the contact resistances were less than 1 $\Omega$. The $T_c$ of the crystal was unchanged by detwinning and annealing it in flowing $O_2$ to relieve possible strains from the detwinning. The longitudinal resistivity was measured with a four-terminal geometry at a frequency of 37.7 Hz and a current of 1 mA flowing parallel to the longer edge (initially parallel to the $b$ axis). After the resistivity measurement along the $b$ axis was completed, the orientation of the Cu-O chains in the sample was changed by applying uniaxial pressure on the shorter edge at 450 °C. The sample was then

 

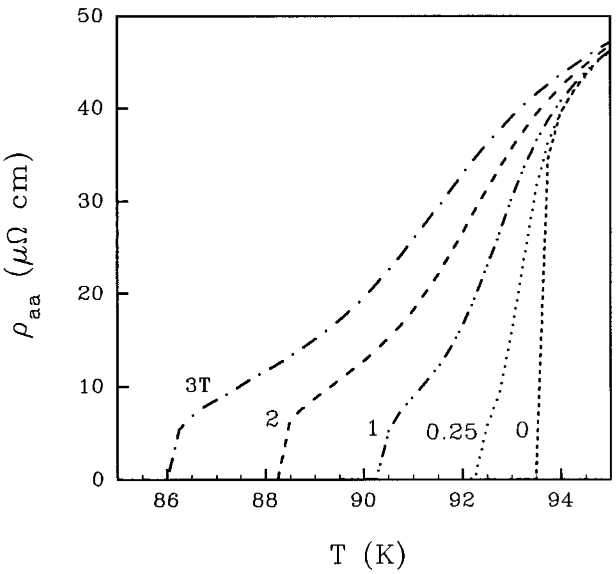


FIG. 1. Resistivity $\rho_{aa}(T,H)$ along the $a$ axis vs $T$ for a single crystal of $YBa_2Cu_3O_{7-\delta}$ in a magnetic field of several sizes with $H\|c$ axis. At zero field, the zero-resistance transition temperature is 93.4 K. The resistive transition width (10–90 %) from $\rho_{bb}$ is 0.15 K.

annealed in the same way, and the resistivity was measured with current flowing along the $a$ axis. The contacts for the gold wires remained unaltered throughout all the measurements.

All of the data presented in this paper were taken at temperature and magnetic-field values above the melting line and therefore should refer to Ohmic behavior.[18] Figure 1 shows the resistivity $\rho_{aa}(T,H)$ along the $a$ axis vs $T$ for several values of the applied magnetic field with $H$ parallel to the $c$ axis. At zero field, the zero-resistance transition temperature was the same along both $a$ and $b$ axis, $T_c = 93.4$ K. Note that the temperature resolution was 10 mK. The transition to the resistive state was abrupt. In the mixed state the anisotropy of the longitudinal resistivities showed strong field and temperature dependences.[19]

A plot of $\log_{10}(\rho_{aa}(T,H))$ vs $\log_{10}(\rho_{bb}(T,H))$for $85 \leqslant T \leqslant 93$ K and $0 \leqslant H \leqslant 6.75$ T is shown in Fig. 2. The solid line is a fit to the data in the linear region on a log-log scale. The resistivity was measured at $\Delta T = 1$ K and $\Delta H = 0.25$ T intervals. All of the $\rho_{aa}$ data collapse onto a single curve as a function of $\rho_{bb}$. At low $H$or $T$, the data collapse onto a line with a slope of 1.05 and an intercept of 1.33, indicating that

$$\rho_{aa}(\text{in } \mu\Omega\text{ cm}) = 1.33[\rho_{bb}(\text{in } \mu\Omega\text{ cm})]^{1.05}. \qquad (1)$$

As $H$ or $T$ is increased, the curve of the collapsed data exhibits a positive curvature, and approaches the normal-state behavior; $\rho_{aa}/\rho_{bb}$ is approximately 2 (Refs. 19 and 20).

The fluctuation conductivities $\sigma^*_{aa}$ and $\sigma^*_{bb}$ were obtained by a background subtraction of a linear-with-$T$ resistivity from $\rho_{aa}$ and $\rho_{bb}$above 150 K. They are linearly proportional to each other, as shown in Fig. 3:

$$\sigma^*_{aa} \cong 0.75\sigma^*_{bb}. \qquad (2)$$

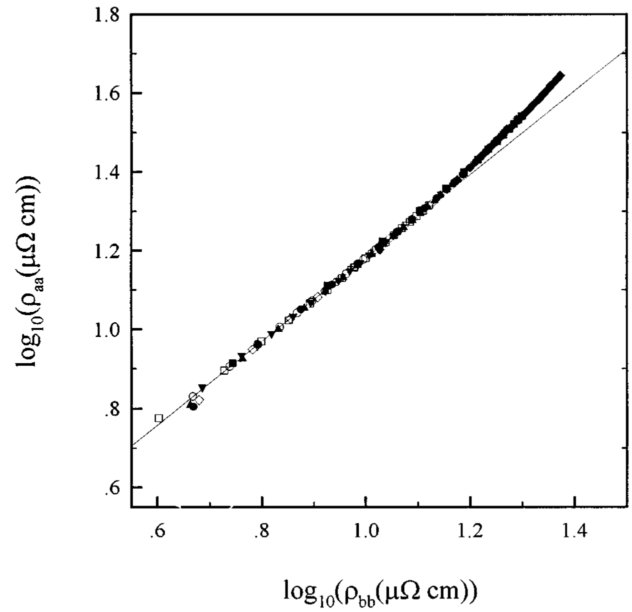


FIG. 2. $\log_{10}(\rho_{aa}(T,H))$ vs $\log_{10}(\rho_{bb}(T,H))$for $85 \leqslant T \leqslant 93$ K and $0 < H \leqslant 6.75$ T. The solid line is a fit in the linear region on a log-log scale; $\rho_{aa} = 1.33\rho_{bb}^{1.05}$.The resistivity is measured at temperature intervals of 1 K from 85 to 93 K. The temperatures and corresponding data-point symbols are 85 (○), 86 (▼), 87 (□), 88 (▲), 89 (◇), 90 (●), 91 (▽), 92 (■), and 93 (◆).

In the narrow region of temperature $85 \leqslant T \leqslant 93$ K, the background conductivities are approximately linear with $T$, and increase less rapidly with decreasing $T$ than the fluctuation conductivity. The resistivity is expressed as $1/(\sigma^* + \sigma_{BG})$,where $\sigma^*$ and $\sigma_{BG}$ are the fluctuation and the background conductivity, respectively. By using Eq. (2) we can rewrite $\rho_{aa}$ in terms of $\sigma^*_{bb}$ and the background conductivity along the $b$ axis $\sigma_{BG\|b}$:

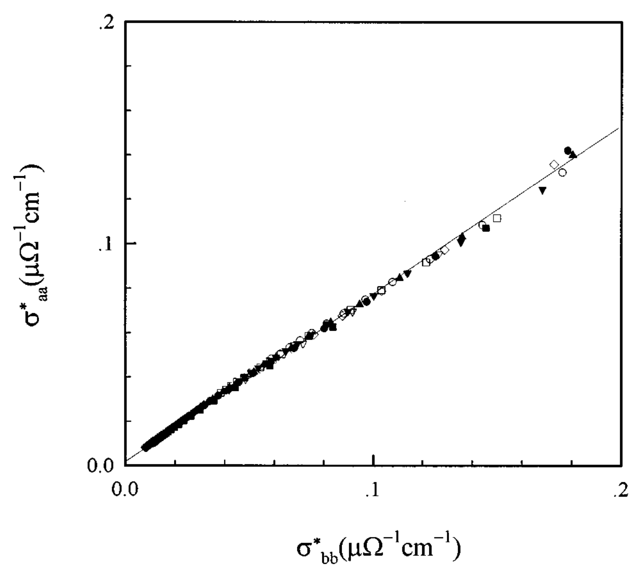


FIG. 3. Fluctuation conductivity $\sigma^*_{aa}(T,H)$ vs $\sigma^*_{bb}(T,H)$. The solid line is the fit: $\sigma^*_{aa} = 0.75\sigma^*_{bb}$.The resistivity is measured at temperature intervals of 1 K from 85 to 93 K; the corresponding data-point symbols are the same as in Fig. 2.

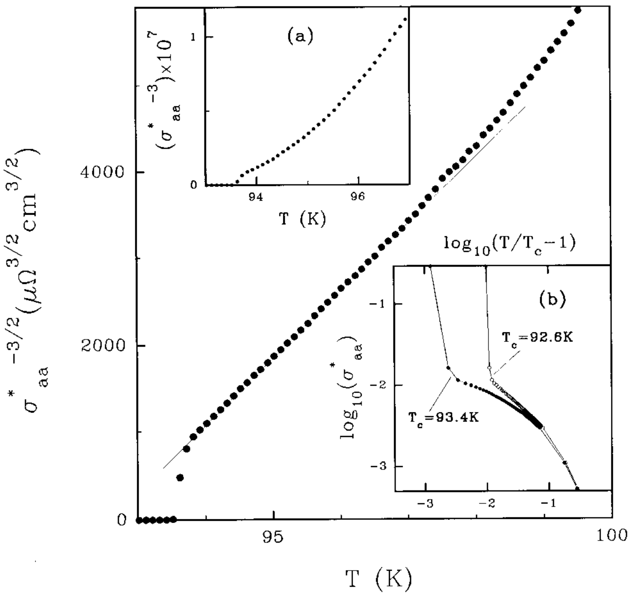


FIG. 4. $\sigma_{aa}^{*\,-3/2}$ vs $T$ at zero magnetic field. The solid line is a guide to the eye. Linear behavior would correspond to $z \cong 2$. Inset (a): $\sigma_{aa}^{*\,-3}$vs $T$ at zero magnetic field with $z=1.5$; the units on the vertical axis are $\mu\Omega^3$ cm$^3$. Inset (b): $\log_{10}\sigma_{aa}^*$ vs $\log_{10}(T/T_c-1)$ at zero magnetic field with $T_c$=93.4 and 92.6 K. $\sigma_{aa}^*$ is expressed in $\mu\Omega$ cm before taking the logarithm for plotting on the vertical axis.

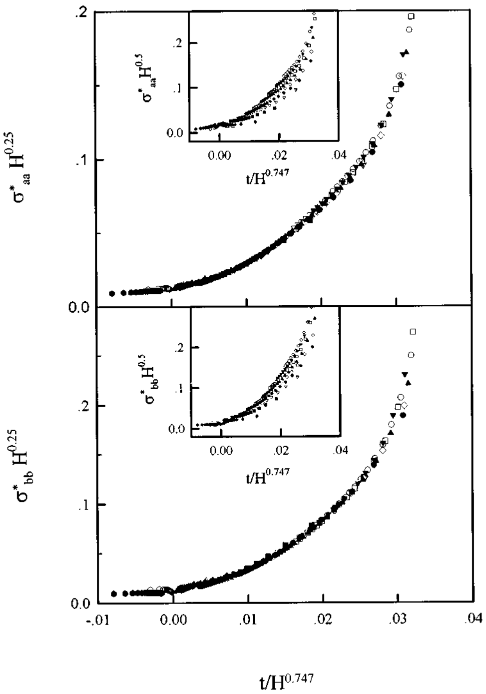


**FIG. 5. Fluctuation conductivities scaled with $z=1.5$. The resistivity is measured at temperature intervals of 1 K from 85 to 93 K; the corresponding data-point symbols are the same as in Fig. 2, and data appear for three additional temperatures: 93.25 K (△), 93.5 K (○), and 94 K (●). In plotting the data, $\sigma_{aa}$ and $\sigma_{bb}$ are expressed in $\mu\Omega^{-1}$ cm$^{-1}$, and $H$ is expressed in oersteds. Upper panel: $\sigma_{aa}^*H^{0.25}$ vs $t/H^{0.747}$. Inset: The fluctuation conductivity $\sigma_{aa}^*$ scaled with $z=2$. Lower panel: $\sigma_{bb}^*H^{0.25}$ vs $t/H^{0.747}$. Inset: The fluctuation conductivity $\sigma_{bb}^*$ scaled with $z=2$.**

$$\begin{aligned}\rho_{aa} &= 1/[\sigma_{aa}^* + \sigma_{\mathrm{BG}\|a}] \\ &\cong 1.33\rho_{bb}/[1-(\sigma_{\mathrm{BG}\|b} - 4/3\sigma_{\mathrm{BG}\|a})\rho_{bb}] \\ &\cong 1.33\rho_{bb}/[1-(0.039-3.038 \\ &\quad \times 10^{-4}\ \mathrm{T/K})\rho_{bb}/(\mu\Omega\ \mathrm{cm})], \end{aligned} \tag{3}$$

where $\sigma_{\mathrm{BG}\|a} \cong -1.548\times10^{-4}$ T$(/\mu\Omega$ cm K$)+0.032(/\mu\Omega$ cm$)$ and $\sigma_{\mathrm{BG}\|b} \cong -5.102\times10^{-4}$ T$(/\mu\Omega$ cm K$)+0.082(/\mu\Omega$ cm$)$. Therefore $\rho_{aa}$ is a function of $\rho_{bb}$ and the data collapse onto a single function. The anisotropy in fluctuation conductivities gives rise to the factor of 1.33 in Eq. (1). The fluctuation and background conductivities along the $b$ axis in the denominator of Eq. (3) are associated with the exponent 1.05 in Eq. (1).

The exponent in Eq. (1) provides a constraint for the value of the exponent $\gamma$ in the universal scaling law, $\rho_{xy} \sim \rho_{xx}^{\gamma}$, where $\rho_{xy}$ is the Hall resistivity and $\rho_{xx}$ is the longitudinal one.[21–24] From Onsager's relation, the exponents $\alpha$ and $\beta$ in $\rho_{ab} \sim \rho_{aa}^{\alpha}$ and $\rho_{ba} \sim \rho_{bb}^{\beta}$, should be such that $\rho_{aa}^{\alpha} \sim \rho_{bb}^{\beta}$ with $\beta/\alpha = 1.05$ in the region where $\rho_{aa} = 1.33\rho_{bb}^{1.05}$.

We turn now to a discussion of the critical behavior. Figure 4 shows that in zero magnetic field $\rho_{aa}^{*\,-3/2}$ is linear in $T$ in a temperature interval from 0.3 to about 4 K above $T_c$. Note that there is no need to fit a value for $T_c$ in this plot. The extent of the linear interval matches that over which the penetration depth varies in a manner consistent with the 3D-*XY* universality class.[1] Using the scaling result, $\sigma^* \sim t^{-\nu(2+z-d)}$,where $\nu=0.669$ (Ref. 25), dimension $d=3$, $t \equiv 1-T/T_c$, and $z$ is the dynamic critical exponent, we find that $z$ is consistent with a value of about 2.0. For comparison, using $z=1.5$ in plotting $\sigma_{aa}^{*\,-1/\nu(2+z-d)}$ vs $T$ does not produce a plot with a linear dependence on $T$, as shown in the inset (a) of Fig. 4. Inset (b) exhibits the log-log plot of the data, with a value of $T_c$=93.4 K; it is not possible to identify a substantial interval of reduced temperature $t$ where a simple power law can be reliably determined. In particular, there is no good evidence for an exponent of $z \cong 3/2$; with a value of $T_c$=92.6 K extrapolated from the main figure, $z \cong 2$. Note that our data closely resemble those reported previously.[10,11]

Very close to $T_c$, $\sigma_{aa}^*$ varies rapidly: possible explanations are a crossover to another universality class, which is not adequately resolved by existing data (including ours, of course), or nonintrinsic behavior, presumably caused by disorder or smearing of the transition. While we cannot rule out any of these possibilities at this stage, one might naively argue that the fact that the $z=2$ behavior extrapolates to a $T_c$=92.6 K lower than the actual $T_c$ mitigates against the disorder interpretation; but further study is clearly required.

In the presence of a magnetic field, Fig. 5 shows that the fluctuation conductivities $\sigma_{aa}^*$and $\sigma_{bb}^*$ (for $85 \leqslant T \leqslant 95$ K) collapse onto a function of $t/H^{1/2\upsilon}$, consistent with the 3D-*XY* scaling $\sigma^* \sim H^{-(2+z-d)/2}F(t/H^{0.747})$, where $d=3$, $F$ is a scaling function, and we used again the theoretical value $\nu=0.669$. The best scaling could be achieved by using $T_c$ as its value in $H=0$, i.e., 93.4 K, as in Refs. 2–5, and 7. This agrees with the resistivity data of Howson, Overend, and

Lawrie[5] that do not scale with the lowest-Landau-level model. Overend *et al.*[26] reported that the specific-heat data scale with the 3D-$XY$ model in magnetic fields up to 8 T and the lowest-Landau model does not work below 6 T. These findings are consistent with theoretical arguments which predict that below fields of order 10 T, the scaling should be governed by the zero-field critical point rather than the lowest-Landau-level scaling.[27] The insets demonstrate scaling with the dynamic critical exponent of $z=2$. The best scaling of the fluctuation conductivities $\sigma_{aa}^{*}$ and $\sigma_{bb}^{*}$ indicates that the dynamic critical exponent $z=1.5\pm0.1$, which is close to the value appropriate for the dynamic universality class of superfluid $^4$He.

The collapse of $\rho_{aa}(T,H)$ in the mixed state onto a single curve as a function of $\rho_{bb}(T,H)$ is a result of the fluctuations. The degree of nonlinearity of the function is a measure of the strength of fluctuations. The fluctuation conductivities $\sigma_{aa}^{*}$ and $\sigma_{bb}^{*}$ show a critical scaling behavior, consistent with the 3D-$XY$ model. When $H$ is nonzero, the universality class of $YBa_2Cu_3O_{7-\delta}$ is apparently or coincidentally the same as that of superfluid $^4$He. For zero field, there is arguably a case to be made for the $z\cong2$. The physical origin of this possible field dependence and the unexpected value for $z$ when $H>0$ are unknown to us, but are presently being explored theoretically. It would be valuable to examine the behavior near $T_c$ with better resolution of temperature and magnetic field.

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