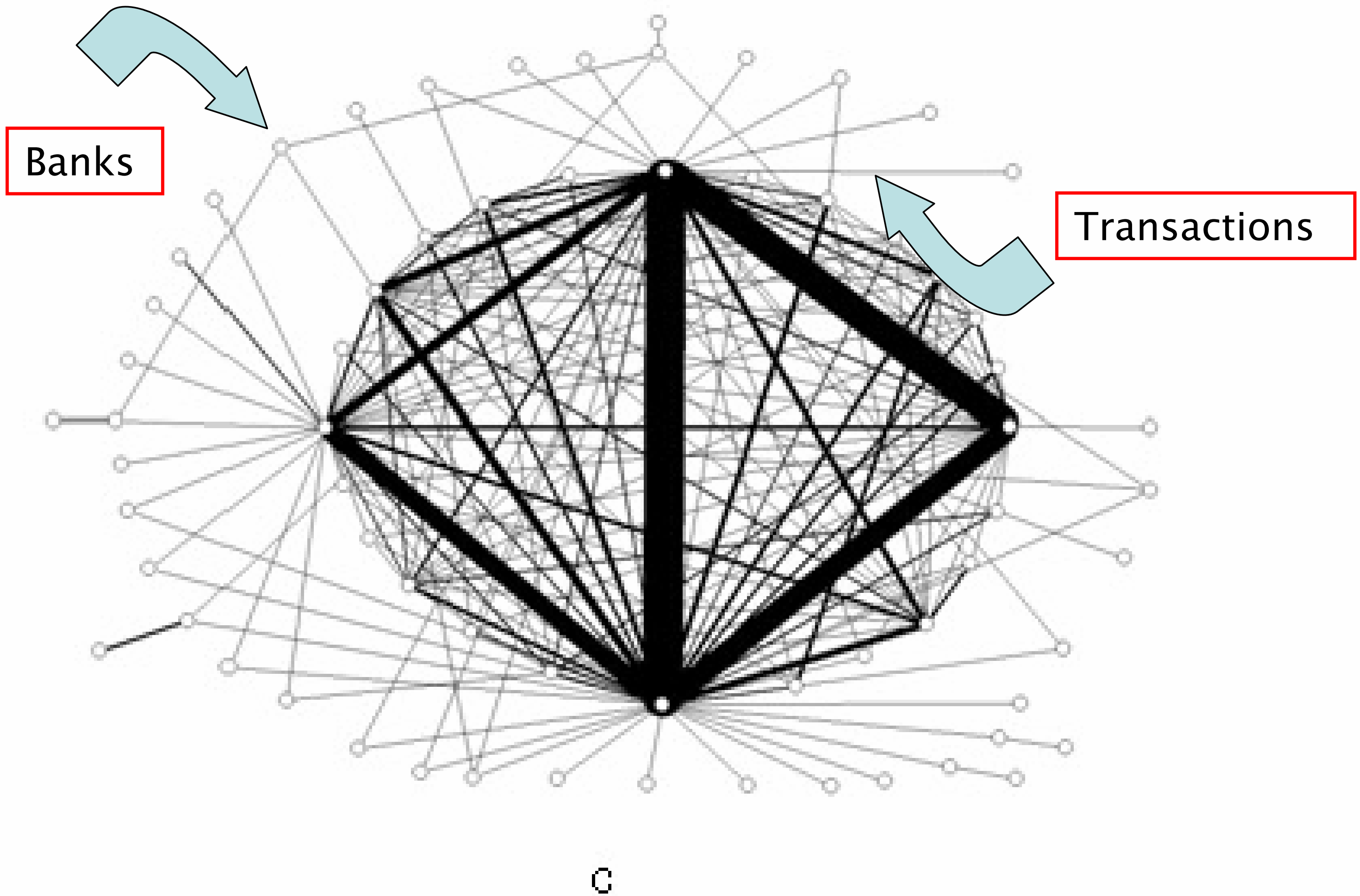
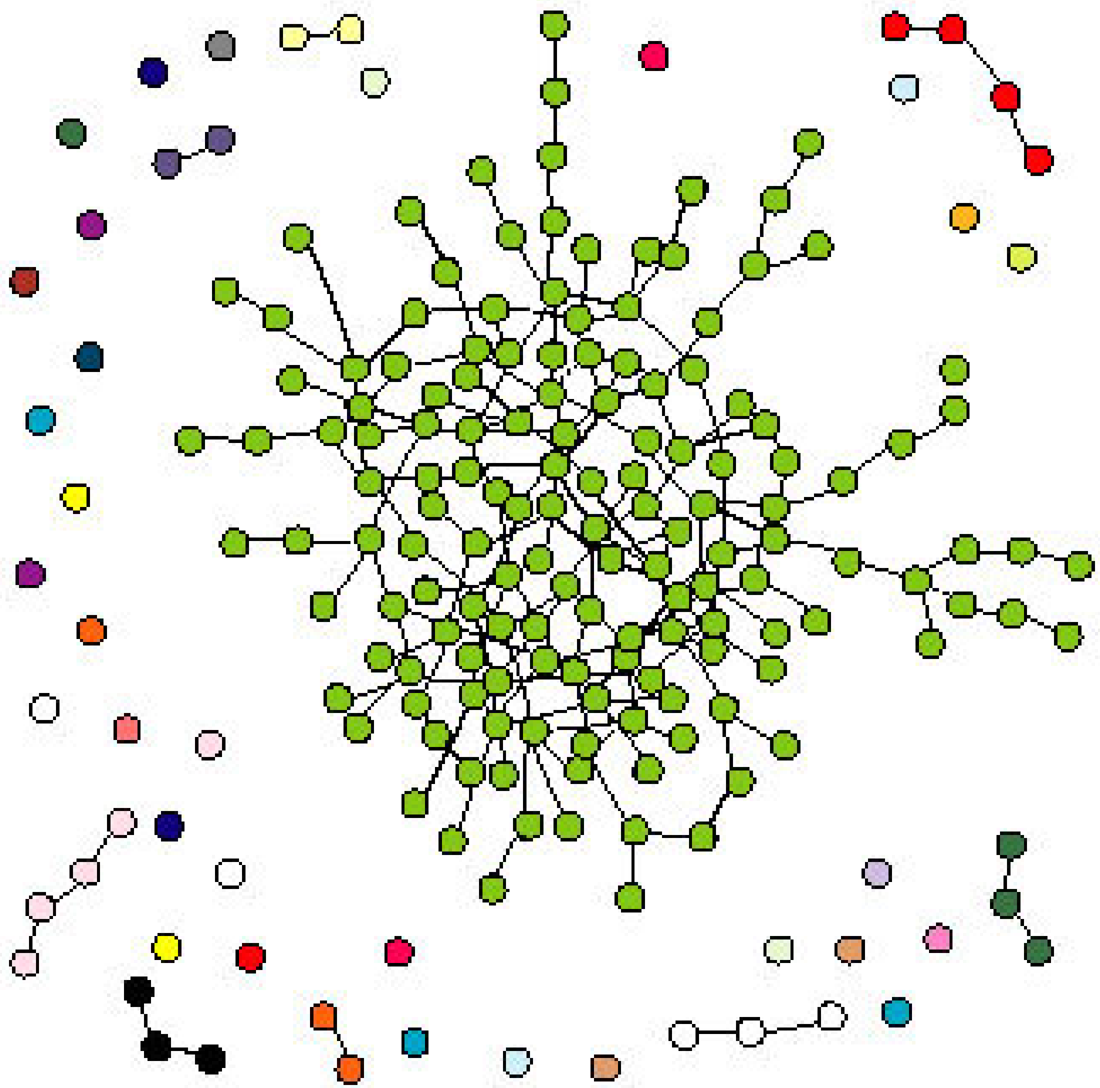
Collective Effects

http://www.butleronline.id.au/3d%20Tulip%20Fields%20near%20Lisse%20Netherlands%20xxx.JPG



Soromaki et al (2006)

c



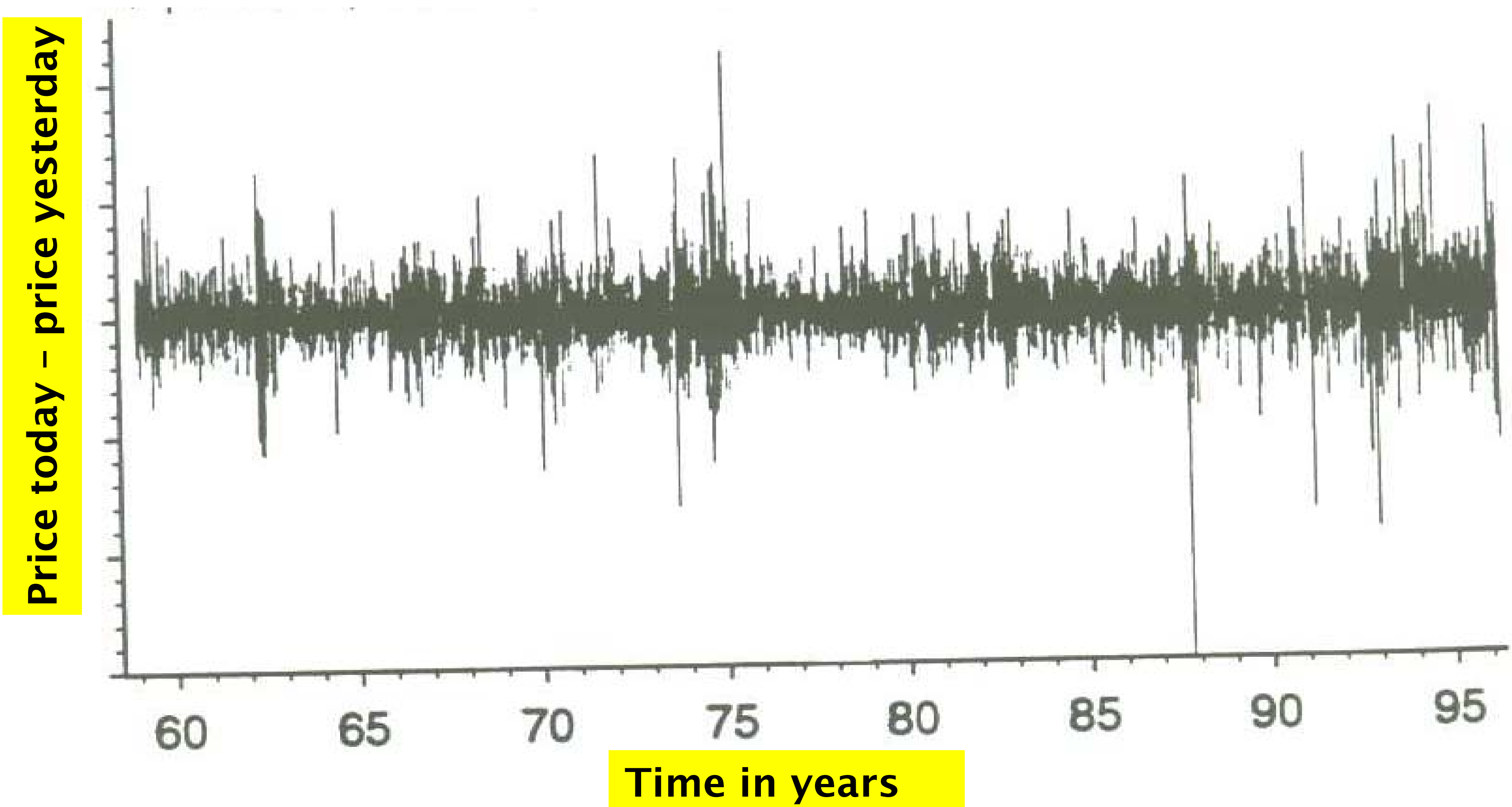
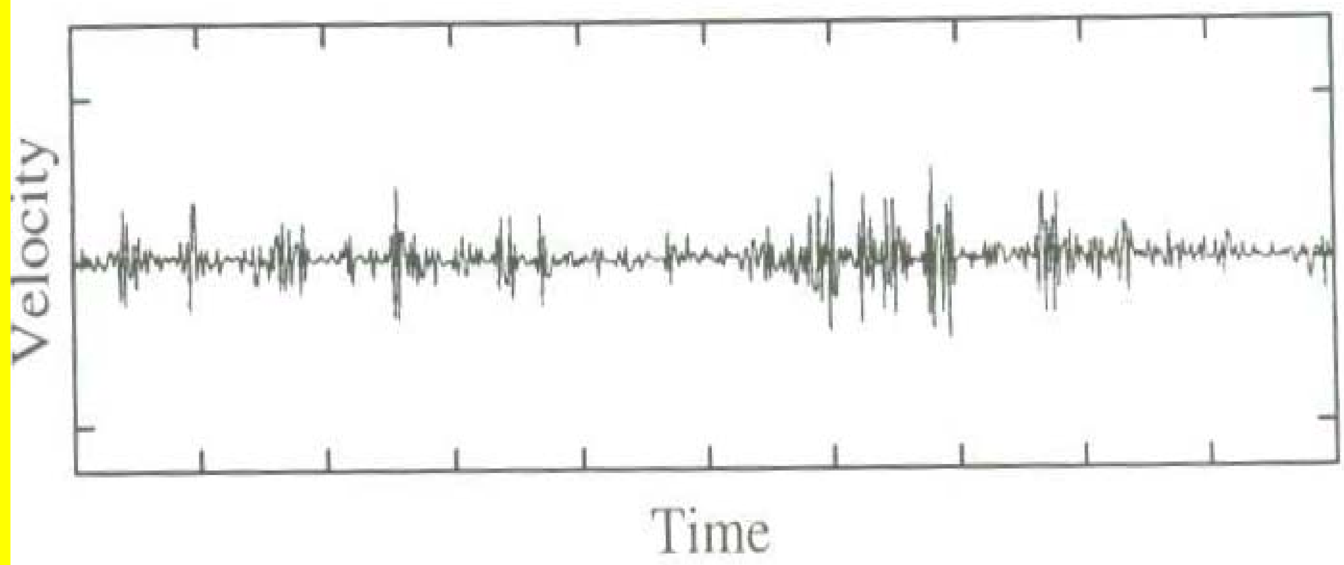


FIGURE E1-1. Top: IBM stock from 1959 to 1996, in units of $10, plotted on logarithmic scale. Bottom: the corresponding relative daily price changes, in units of 1%.

Mandelbrot

WIND SPEED FLUCTUATIONS

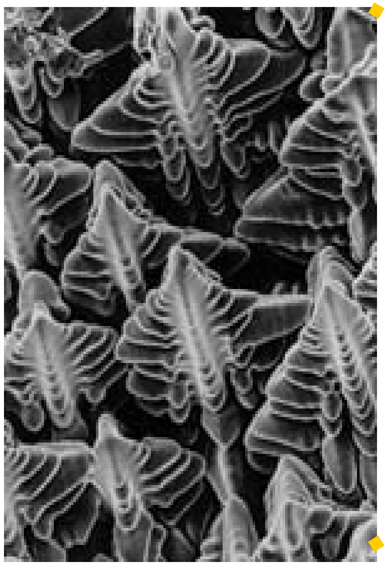
Speed now –speed a moment ago



Time

Gagne (1980)

Metals



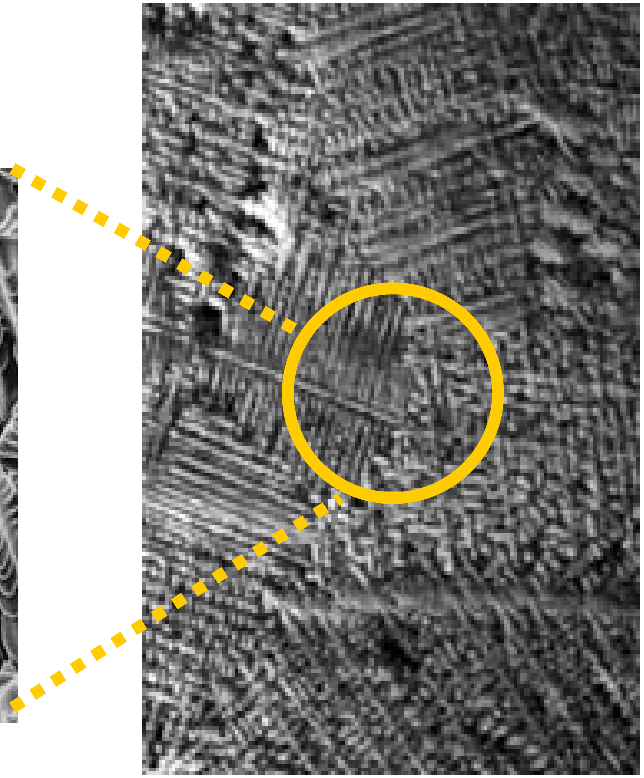
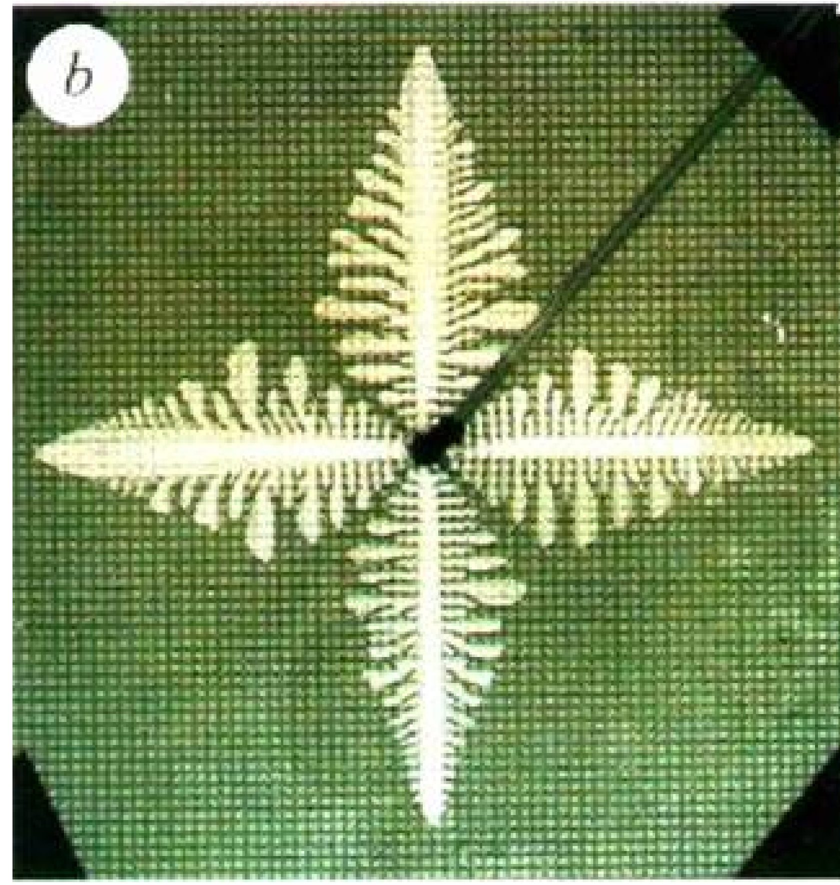


Image: David and Boatner (1997)

Scale of one millionth of a meter



Air bubble

Scale of one meter

dimensional analysis here that considerably simplifies the algebra. For convenience we use $\delta^2 = \epsilon$, and write Eq. (in 1-D as

$$\left[\partial_t - \partial_x^2 \left(1 + \partial_x^2\right)^2\right] \psi = \delta^2 \partial_x^2 (\psi^3 - \psi). \qquad (30)$$

The basic premise of the multiple scales analysis is that while the pattern itself varies on the scale of its wavelength $(2\pi/k_0)$, its amplitude varies on much larger length and time scales. It is then appropriate to introduce *slowly* varying arguments

$$X = \delta x \quad , \quad T = \delta^2 t \qquad (31)$$

for the envelope function $A(X,T)$. This scaling was previously applied by Gunaratne *et al.* [14] to the Swift-Hohenberg equation with success, and as the PFC equation is essentially a conserved analog of the Swift-Hohenberg equation we anticipate that the same scaling holds here.

Derivatives scale as follows

$$\begin{aligned}
\partial_x &\rightarrow \partial_x + \delta\partial_X \\
\partial_x^2 &\rightarrow \partial_x^2 + 2\delta\partial_X\partial_x + \delta^2\partial_X^2 \\
\partial_t &\rightarrow \delta^2\partial_T,
\end{aligned} \qquad (32)$$

whereas the operator

$$\partial_x^2 \left(1 + \partial_x^2\right)^2 \rightarrow \sum_{j=0}^{6} \delta^j \mathcal{L}_j \qquad (33)$$

such that

$$\begin{aligned}
\mathcal{L}_0 &= \partial_x^2 \left(1 + \partial_x^2\right)^2 \\
\mathcal{L}_1 &= 4\partial_X\partial_x^3 \left(1 + \partial_x^2\right) + 2\partial_X\partial_x \left(1 + \partial_x^2\right)^2 \\
\mathcal{L}_2 &= 4\partial_X^2\partial_x^4 + 10\partial_X^2\partial_x^2 \left(1 + \partial_x^2\right) + \partial_X^2 \left(1 + \partial_x^2\right)^2 \\
\mathcal{L}_3 &= 12\partial_X^3\partial_x^3 + 8\partial_X^3\partial_x \left(1 + \partial_x^2\right) \\
\mathcal{L}_4 &= 13\partial_X^4\partial_x^2 + 2\partial_X^4 \left(1 + \partial_x^2\right) \\
\mathcal{L}_5 &= 6\partial_X^5\partial_x \\
\mathcal{L}_6 &= \partial_X^6.
\end{aligned} \qquad (34)$$

We now expand ψ in a perturbation series in δ to get

$$\psi = \psi_0 + \delta\psi_1 + \delta^2\psi_2 + \delta^3\psi_3 + \ldots. \qquad (35)$$

Using Eq. (32) and the above series, the δ expansion of the nonlinear term in Eq. (30) can be written as

$$\begin{aligned}
\partial_x^2(\psi^3 - \psi) &= \partial_x^2(\psi_0^3 - \psi_0) \\
&+\delta[\partial_x^2(3\psi_0^2\psi_1 - \psi_1) + 2\partial_X\partial_x(\psi_0^3 - \psi_0)] \\
&+3\psi_0\psi_2^2 + 6\psi_0\psi_1\psi_3 + 3\psi_0^2\psi_4 - \psi_4) \\
&+2\partial_X\partial_x(\psi_1^2 + 6\psi_0\psi_1\psi_2 + 3\psi_0^2\psi_3 - \psi_3) \\
&+\partial_X^2(3\psi_0\psi_1^2 + 3\psi_0^2\psi_2 - \psi_2)] \\
&+\mathcal{O}(\delta^5).
\end{aligned} \qquad (36)$$

Substituting Eq. (35) in Eq. (30), and using the scaled operators in Eqns. (32-34), we can write equations satisfied by the ψ_m at each $\mathcal{O}(\delta^m)$. At $\mathcal{O}(1)$ we obtain,

$$\begin{aligned}
\mathcal{L}_0\psi_0 &= 0 \\
\Rightarrow \psi_0 &= \bar{\psi} + A_{01}(X,T)e^{ix} + \text{c.c.}
\end{aligned} \qquad (37)$$

where A_{mn} is the complex amplitude of mode n at $\mathcal{O}(\delta^m)$. At $\mathcal{O}(\delta)$ we get

$$\begin{aligned}
\mathcal{L}_0\psi_1 + \mathcal{L}_1\psi_0 &= 0 \\
\Rightarrow \psi_1 &= A_{11}(X,T)e^{ix} + \text{c.c.}
\end{aligned} \qquad (38)$$

where (and hereon) we neglect the constant term in view of its inclusion in Eq. (37). At the next order we have

$$\mathcal{L}_0\psi_2 = \partial_T\psi_0 - \mathcal{L}_1\psi_1 - \mathcal{L}_2\psi_0 - \partial_x^2(\psi_0^3 - \psi_0). \qquad (39)$$

For $\psi_2(x,t)$ to remain bounded we have to guarantee that the right hand side of Eq. (39) does not have a projection in the null space of $\mathcal{L}_0$, which yields a solvability condition [17, 18] (also known as the Fredholm alternative). Applying the alternative imposes the following condition on the amplitude at $\mathcal{O}(\delta^2)$:

$$\partial_T A_{01} = 4\partial_X^2 A_{01} + (1 - 3\bar{\psi}^2)A_{01} - 3A_{01}|A_{01}|^2. \qquad (40)$$

Thus,

$$\psi_2 = A_{21}e^{ix} + A_{22}e^{2ix} + A_{23}e^{3ix} + \text{c.c.} \qquad (41)$$

where $A_{22} = A_{01}^2\bar{\psi}/3$, and $A_{23} = A_{01}^3/64$.

At subsequent orders, the following equations are obtained for ψ_m:

$$\begin{aligned}
\mathcal{O}(\delta^3) : \mathcal{L}_0\psi_3 &= \partial_T\psi_1 - \mathcal{L}_1\psi_2 - \mathcal{L}_2\psi_1 - \mathcal{L}_3\psi_0 \\
&-[\partial_x^2(3\psi_0^2\psi_1 - \psi_1) + 2\partial_X\partial_x(\psi_0^3 - \psi_0)] \\
\mathcal{O}(\delta^4) : \mathcal{L}_0\psi_4 &= \partial_T\psi_2 - \mathcal{L}_1\psi_3 - \mathcal{L}_2\psi_2 - \mathcal{L}_3\psi_1 - \mathcal{L}_4\psi_0 \\
&-[\partial_x^2(3\psi_0\psi_1^2 + 3\psi_0^2\psi_2 - \psi_2) \\
&+2\partial_X\partial_x(3\psi_0^2\psi_1 - \psi_1) + \partial_X^2(\psi_0^3 - \psi_0)] \\
\mathcal{O}(\delta^5) : \mathcal{L}_0\psi_5 &= \partial_T\psi_3 - \mathcal{L}_1\psi_4 - \mathcal{L}_2\psi_3 - \mathcal{L}_3\psi_2 - \mathcal{L}_4\psi_1 \\
&-\mathcal{L}_5\psi_0 - [\partial_x^2(\psi_1^2 + 6\psi_0\psi_1\psi_2 + 3\psi_0^2\psi_3 \\
&-\psi_3) + 2\partial_X\partial_x(3\psi_0\psi_1^2 + 3\psi_0^2\psi_2 - \psi_2) \\
&+\partial_X^2(3\psi_0^2\psi_1 - \psi_1)] \\
\mathcal{O}(\delta^6) : \mathcal{L}_0\psi_6 &= \partial_T\psi_4 - \mathcal{L}_1\psi_5 - \mathcal{L}_2\psi_4 - \mathcal{L}_3\psi_3 - \mathcal{L}_4\psi_2 \\
&-\mathcal{L}_5\psi_1 - \mathcal{L}_6\psi_0 - [\partial_x^2(3\psi_1^2\psi_2 + 3\psi_0\psi_2^2 \\
&+6\psi_0\psi_1\psi_3 + 3\psi_0^2\psi_4 - \psi_4) + 2\partial_X\partial_x(\psi_1^2 \\
&+6\psi_0\psi_1\psi_2 + 3\psi_0^2\psi_3 - \psi_3) + \partial_X^2(3\psi_0\psi_1^2 \\
&+3\psi_0^2\psi_2 - \psi_2)],
\end{aligned} \qquad (42)$$

$$\begin{aligned}
&+6i\partial_X(A_{01}^2 A_{01}^*) \\
\partial_T A_{21} &= -13\partial_X^4 A_{01} - 12i\partial_X^3 A_{11} + 4\partial_X^2 A_{21} \\
&-(1 - 3\bar{\psi}^2)(2i\partial_X A_{11} + \partial_X^2 A_{01}) \\
&+(1 - 3\bar{\psi}^2)A_{21} - 3A_{11}^2 A_{01}^* - 6|A_{01}|^2 A_{21} \\
&-6\bar{\psi}A_{22}A_{01}^* - 3A_{23}{A_{01}^*}^2 - 6A_{01}|A_{11}|^2 \\
&-3A_{01}^2 A_{21}^* + 6i\partial_X(A_{01}^2 A_{11}^* + 2|A_{01}|^2 A_{11}) \\
&+3\partial_X^2(A_{01}^2 A_{01}^*) \\
\partial_T A_{31} &= 6i\partial_X^5 A_{01} - 13\partial_X^4 A_{11} - 12i\partial_X^3 A_{21} + 4\partial_X^2 A_{31} \\
&-(1 - 3\bar{\psi}^2)(2i\partial_X A_{21} + \partial_X^2 A_{11}) \\
&+(1 - 3\bar{\psi}^2)A_{31} - 6A_{11}A_{21}A_{01}^* - 6|A_{01}|^2 A_{31} \\
&-6A_{01}A_{21}A_{11}^* - 6A_{23}A_{01}^*A_{11}^* - 6A_{01}A_{11}A_{21}^* \\
&-3A_{01}^2 A_{31}^* - 6\bar{\psi}A_{32}A_{01}^* - 3A_{33}{A_{01}^*}^2 \\
&-6\bar{\psi}A_{22}A_{11}^* + 6i\partial_X(2A_{01}|A_{11}|^2 + 2|A_{01}|^2 A_{21} \\
&+A_{01}^*A_{11}^2 + A_{01}^2 A_{21}^*) + 3\partial_X^2(A_{01}^2 A_{11}^* \\
&+2|A_{01}|^2 A_{21}^*) + \text{h.o.t.} \\
\partial_T A_{41} &= \partial_X^6 A_{01} + 6i\partial_X^5 A_{11} - 13\partial_X^4 A_{21} - 12i\partial_X^3 A_{31} \\
&+4\partial_X^2 A_{41} - (1 - 3\bar{\psi}^2)(2i\partial_X A_{31} + \partial_X^2 A_{21}) \\
&+(1 - 3\bar{\psi}^2)A_{41} - 3A_{21}^2 A_{01}^* - 6A_{11}A_{31}A_{01}^* \\
&-6A_{01}A_{41}A_{01}^* - 6A_{11}A_{21}A_{11}^* - 6A_{01}A_{31}A_{11}^* \\
&-3A_{11}^2 A_{21}^* - 6A_{01}|A_{21}|^2 - 6A_{01}A_{11}A_{31}^* \\
&-3A_{01}^2 A_{41}^* - 6\bar{\psi}A_{42}A_{01}^* - 3A_{43}{A_{01}^*}^2 \\
&-6\bar{\psi}A_{32}A_{11}^* - 6A_{33}A_{01}^*A_{11}^* - 3A_{23}{A_{11}^*}^2 \\
&-6\bar{\psi}A_{33}A_{22}^* - 6A_{01}|A_{23}|^2 + 6i\partial_X(2A_{11}A_{21}A_{01}^* \\
&+2|A_{01}|^2 A_{31} + A_{11}|A_{11}|^2 + 2A_{01}A_{21}A_{11}^* \\
&+2A_{01}A_{11}A_{21}^* + A_{01}^2 A_{31}^*) + 3\partial_X^2(2A_{01}|A_{11}|^2 \\
&+A_{01}^*A_{11}^2 + 2|A_{01}|^2 A_{21} + A_{01}^2 A_{21}^*) + \text{h.o.t.}
\end{aligned} \qquad (43)$$

Here, "h.o.t." refers to higher order terms that are functions of A_{01} and its derivatives. The amplitude function for the pattern (e^{ix}) can be written as

$$A(X,T) = A_{01}(X,T) + \delta A_{11}(X,T) + \delta^2 A_{21}(X,T) + \ldots. \qquad (44)$$

Using Eqns. (40), (43), and (44), and scaling back to original variables, i.e. $X \rightarrow \delta^{-1}x$ and $T \rightarrow \delta^{-2}t$, the amplitude equation to $\mathcal{O}(\delta^4)$ can be written as

$$\begin{aligned}
\partial_t A &= 4\partial_x^2 A - 12i\partial_x^3 A - 13\partial_x^4 A + 6i\partial_x^5 A + \partial_x^6 A \\
&-\delta^2(1 - 3\bar{\psi}^2)\left(2i\partial_x + \partial_x^2\right) A + \delta^2[(1 - 3\bar{\psi}^2)A \\
&-3A|A|^2 + 3\left(2i\partial_x + \partial_x^2\right)\left(A|A|^2\right)] - \delta^4\left(\frac{3}{64}A|A|^4\right. \\
&\left.+2\bar{\psi}^2 A|A|^2\right) + \mathcal{O}(\delta^6)
\end{aligned} \qquad (45)$$

or more compactly, after replacing $\delta^2 \rightarrow \epsilon$, to $\mathcal{O}(\epsilon^2)$

$$\partial_t A = -\left(1 - \mathcal{L}_{1D}\right)\mathcal{L}_{1D}{}^2 A - \epsilon(1 - 3\bar{\psi}^2)\mathcal{L}_{1D} A$$


SUPER
COMPUTER

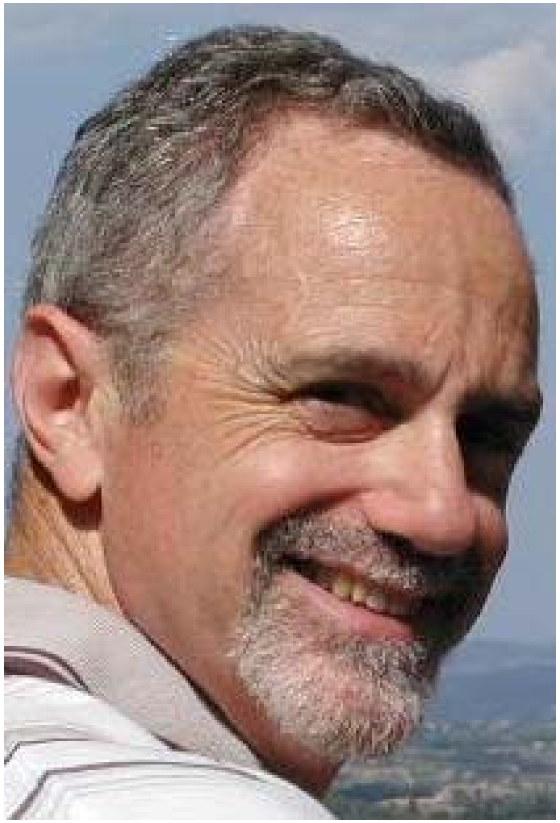

SUPER
COMPUTER
OUT
RULE
Wizard o'Art

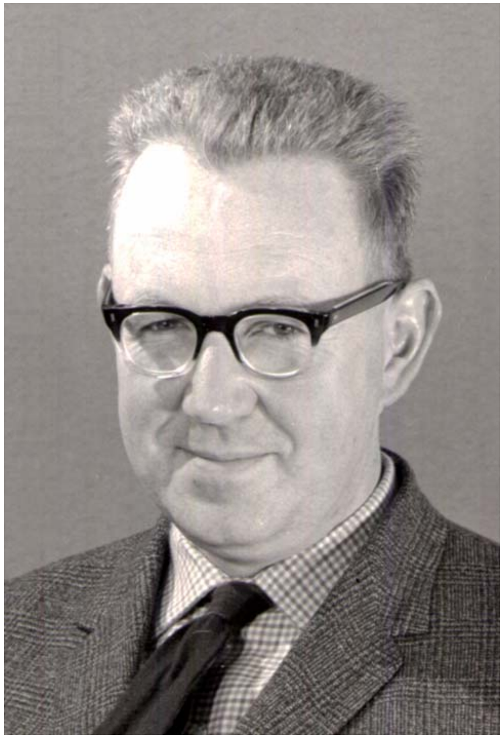
Sampling water and microbes at Yellowstone National Park

NG, Bruce Fouke

Surveying Minerva Terrace

NG, Hector Garcia Martin, John Veysey





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Collective Effects