Dynamic Hysteresis and Spontaneous Symmetry Breaking in Magnets

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When a many-body interacting system, such as magnet, is driven by a time-varying external field it exhibits new and intriguing physics. In particular, when a magnet is driven by a periodically varying field $h(t) = h_o sin(wt)$ it exhibits dynamic hysteresis, that is, hysteresis that is not present in the adiabatic limit ($dh/dt \to 0$) with thermal fluctuations present. Under certain conditions-typically low T (temperature), high w, and small h_o-a dynamic transition to a spontaneously broken symmetry phase occurs, where the time average of the magnetization of the system becomes nonzero. We present both theoretical and experimental studies of dynamic hysteresis and transitions in magnets.

I. INTRODUCTION

In general, when a time-varying external perturbation is applied to a cooperatively interacting many-body system, the system will not be able to respond instantaneously to the perturbation. In this paper we study the consequences of the delayed response of a magnet driven by a periodically varying external field $h(t) = h_o sin(wt)$. In this case the magnetization $m(t)$ will have a delayed response to the external field due to the relaxational delay of the domains of the magnet [1].

The essential physics arrives from two completing time scales: the relaxation time for the magnetic system and the period of the external field [2]. The relaxation time of the magnet not only depends on the temperature T, but also the magnitude of the external driving field h_o, since a strong h_o will dramatically reduce the response time of magnetic domains flipping. In the basic scenario where the relaxation time of the magnet is much much smaller than the period of the external field, the magnetization will follow the external field (in phase). This is the adiabatic limit, in which the external field is varying so slowly ($dh/dt \to 0$) that the magnetization is able to respond with no measurable delay. However, there is a caveat in the case of a dirty, or disordered, magnet the magnetization will not be in phase with $h(t)$ since the domains of the magnet will be pinned by varying amounts by the disorder [3]. In order to avoid the additional complication of disorder, which is still an open topic, we will look at only pure magnetic systems. In this paper we are interested in two scenarios: when the relaxation time of the magnet is comparable to the period of the external field, and when the relaxation time of the magnet is much greater than the period of the external field.

When the relaxation time of a magnet is comparable to period of the external field, hysteresis occurs due to the delayed response of the magnetization to the external field $h(t)$ [2]. Since there is no hysteresis in pure magnets in the adiabatic limit [4], the hysteresis resulting from a periodic external driving field is called *dynamic hysteresis*. In order to quantitatively measure dynamic hysteresis, we find the hysteresis loop area A (which represents the energy loss of the magnetization-demagnitization cycle) defined as:

$$A = \oint m dh \tag{1}$$

At this stage the average magnetization over a period of the external field is zero, and thus the hysteresis loop is *symmetric* about the origin. *A priori* one would expect that the average magnetization would be zero since $h(t)$ is periodically oscillating between $-h_o$ and h_o. However, this is not always the case, when the relaxation time of a magnet is much greater than the period of the external field the magnetization of the system does not have enough time to relax, or follow the sign, of the external driving field $h(t)$. As a result of this the average magnetization acquires a nonzero value and the hysteresis loop becomes *asymmetric* about the origin. A phase transition occurs into a *spontaneous broken symmetry phase* [1]. We define the order parameter for the transition as follows:

$$Q = \frac{\omega}{2\pi} \oint m dt \tag{2}$$

Where Q is proportional to the average magnetization over one period of the external driving field. Thus Q is zero when $< m(t) >= 0$ and the hysteresis loop is symmetric about the origin, and nonzero when $< m(t) >\neq 0$ and the hysteresis loop is asymmetric about the origin.

Understanding dynamic hysteresis and transitions not only involves new and interesting physics, but also has many technological and engineering applications, especially related to recording [1]. In section II and III we begin by overviewing some of theoretical studies that have been done on this problem (Monte Carlo simulations of the Ising model, mean field theory, etc.) In section IV we overview some of the experimental work that have been done, in particular in ultrathin flims (2D systems). In section V we finish off with concluding remarks, discuss open questions that remain, and what the future holds for this field.

II. THEORY-DYNAMIC HYSTERESIS

Computer simulations have played a large part in the investigation of dynamic hysteresis and transitions. The simplest model one can simulate that captures all of the physics is the ferromagnetic Ising model. The Ising model is a model of a magnet represented as a lattice of spins (the magnetic domains). Each spin can take on two values: up or down (1 or -1). In addition, each spin is coupled to its neighbor with a ferromagnetic (negative) coupling constant. The Hamiltonian for the system at a time t is [5]:

$$H = -\sum_{<ij>} J s_i s_j - h(t) \sum_i s_i \tag{3}$$

where $s_i = \pm 1$, $h(t) = h_o sin(wt)$, and $< ij >$ indicates nearest neighbor interactions. The magnetization is then $M = (1/N) \sum_i s_i$, and the period of the external driving field is $\tau = \frac{2\pi}{w}$. The results of two dimensional finite temperature Monte Carlo simulations in an N by N lattice are given in Fig. 1 [1]. Since $h(t)$ is periodic (with a period τ) we also expect the magnetization to be periodic, as we see in Fig. 1.

In Fig. 1 (from [1]) we see four distinct possibilities, in the top plot the high temperature results in a fast relaxation for the system, and thus we see that the magnetization closely follows the external field. Thus there is no hysteresis and no transition to a spontaneously symmetry broken state. In the upper middle plot the temperature has been reduced to the point where the relaxation time is comparable to the period so we see that there a phase delay between $m(t)$ and $h(t)$, this delay leads to energy dissipation and hysteresis, thus $A \neq 0$. In lower middle plot we finally see a spontaneous symmetry broken state, which was brought about by a further reduction in temperature and a strong reduction in h_o. In this scenario the relaxation time of the system is much longer than the period of $h(t)$. The frequency is just too fast, the spin flips in the system take too long compared to the period of $h(t)$, so the spins get stuck in one state [6]. So we have both $A \neq 0$ and $Q \neq 0$. We notice that in the lower middle plot the temperature falls below $T_c = 2.269...$ for the 2D Ising model, below which the relaxation time of the magnet is significantly increased since nearest neighbor interactions begin to dominate thermal fluctuations.

The A in the lower middle plot is actually smaller than the A in upper middle plot; while the area of the hysteresis loop increases as the phase delay between $m(t)$ and $h(t)$ increases, once the transition is make to the symmetry broken state the area of the hysteresis loop gets smaller and eventually goes to zero as in bottom plot. Once the symmetry is broken and the period of the external field continues to get smaller and smaller compared to the relaxation time of the system, the magnetization is able to respond less the less to the external field, thus reducing the area of the hysteresis loop (which measures the amount the magnetization has changed during the course of a single period in $h(t)$). At some point the external field will vary so rapidly compared to the relaxation time of the system that the system is unaffected as we see in the bottom plot of Fig. 1 [1].

So far it is clear that $A = A(h_o, w, T)$, through Monte Carlo simulations of the Ising model A actually obeys the following scaling form [2]:

$$A \sim h_o^\alpha T^{-\beta} g(w/h_o^\gamma T^\delta) \tag{4}$$

where

$$g(x) \sim x^\epsilon e^{-x^2}/\sigma \tag{5}$$

The constants: $\alpha, \beta, \gamma, \delta, \epsilon$, and σ depend on the dimension of the system. In Fig. 2 (from [2]) a collapse A, giving $g(x)$, is presented in $D = 1, 2, 3, 4$. From the functional form of $g(x)$ we indeed see that A peaks out and goes to zero for large w (or equivalently small h_o or T).

In addition to studying this problem through Monte Carlo simulations, we look at mean field theory. In mean field theory we can write down the equation of motion of the system [7]:

$$\frac{dm}{dt} = -m + tanh\Big(\frac{m + h_o sin(wt)}{T}\Big) \tag{6}$$

Where the microscopic relaxation time, which would come on the left, has been set to unity. One can solve the above differential equation numerically. However, there are some serious setbacks to this mean field analysis since mean field theory ignores nontrivial fluctuations that are present in finite dimensions. As a result of ignoring these fluctuations a non-zero area for the dynamic hysteresis loop, and even a dynamic transition is possible in the adiabatic limit ($dh/dt \to 0$) [1]. The area of the hysteresis loop in mean field theory scales as Eq. (4) where $g(x)$ is now [2]:

$$g(x) \sim x/(1 + cx^2) \tag{7}$$

So in mean field theory $g(x)$ is a Lorentzian, a collapse of this form is given in Fig. 2(e).

III. THEORY-DYNAMIC TRANSITIONS

Although there are setbacks to the mean field analysis of dynamic transitions, we can obtain a schematic picture of the phase diagram between the $Q = 0$ state and $Q \neq 0$ state. In Fig. 3 (from [1]) we present the phase diagram for the dynamic transition as determined by mean field theory for a fixed value of w [8]. The phase boundary includes a tri-critical point (TCP), separating a line of continuous transitions from a line of first order transitions. The first order transitions occurs for the segment of the phase boundary to the left of the TCP point, that is for low T and high h_o. The remainder of the phase boundary indicates a continuous transition. As w is increased the phase boundary moves outward, since the symmetry broken regions expands as frequency of the external driving force increases while the relaxation time for the magnet stays constant.

Although no precise experimental work has been done to determine the phase boundaries, extensive Monte Carlo simulations of the Ising model have been done to determine the phase boundaries in finite dimensional systems [5]. Even though the shape of the phase boundary, as determined by Monte Carlo simulations, is different (concave instead of convex to the origin), there is a tri-critical point and the overall picture is consistent with the mean field phase diagram.

So far we have discussed dynamic transitions in magnets that are driven by a sinosodial driving fields ($h(t) = h_o sin(wt)$), however, the phenomena of a spontaneous symmetry broken phase is more general. Recent Monte Carlo studies have examined the effect of driving an Ising magnet with stochastically time varying fields (bounded by $-h_o/2$ and $h_o/2$) [7]. A symmetry broken phase was indeed observed for small T and small h_o, as is shown in Fig. 4 (from [7]). The physics here is the same, the symmetry broken state occurs due to two competing time scales, the only difference here is that the driving frequency is now represented by a whole spectrum of frequencies. Although one cannot tune the driving frequency in this case, one can increase the relaxation time of the magnet until it is unable to respond in sync with the stochastically time-varying driving field.

IV. EXPERIMENT

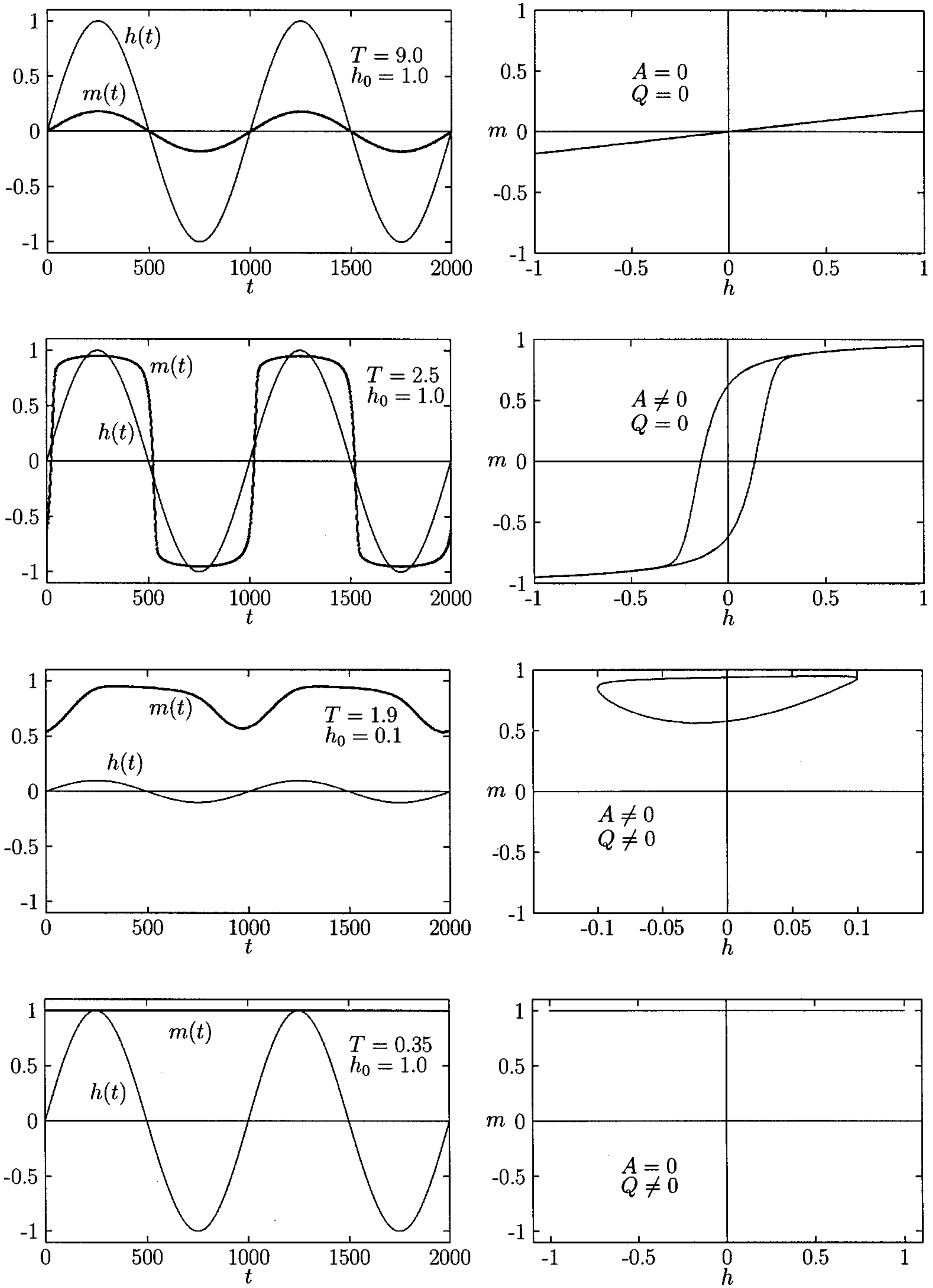
Most of the experimental investigations of dynamic hysteresis have been performed in ultrathin ferromagnetic flims (2D systems) [1,6,9]. In [6] a study of the frequency dependence of hysteresis was performed on an epitaxially grown ultrathin Co film at room temperature.

Since the film have a strongly preferred axis of magnetization (uniaxial anisotropy), with two opposing direction of spin orientation, it behaves like an Ising magnet. The external field was driven with a frequency between 0.1 to $500Hz$, and the amplitude (h_o) range was between 1 to $180Oe$. The hysteresis loop area A versus the frequency is given in Fig. 5(a)(from [6]). The insets are plots of hysteresis loops for both the symmetric phase (with $f = 85.0Hz$) and the symmetry broken phase (with $f = 350.0Hz$). Fig. 5(b) gives Q as a function of h_o for a fixed frequency, which becomes nonzero for small h_o as expected. Just as in the simulations discussed above, the area A also scales with both h_o and f [6].

We notice that the area A is not zero when $f \simeq 0$, suggesting that the hysteresis is not solely dynamical in origin. Although the issues of $A \neq 0$ at zero frequency has not yet been resolved, it is believed that it may be due to the inaccessibility of very low frequencies in experiment [1].

V. CONCLUDING REMARKS

Much has been brought to bear on the problem of dynamic phase transitions and dynamic hysteresis. Although the physics can be reduced to two competing time scales, from this emerges spontaneous symmetry breaking, scaling, and hysteresis. In other words, we have a relatively simple mechanism that leads to fascinating *emergent* states and behavior. There still remain open questions left to be answered, even though the field have not significantly advanced beyond the study of pure magnets. Experimentally, precise phase boundaries of the dynamical transition have yet to be determined. Also, it is unclear why A seems to be nonzero at $f \simeq 0$. Theoretically, studies have been limited to simple Ising systems. More complex many-body systems, such as perhaps the Random Field Ising Model (RFIM) as well as other systems are possible avenues for future research. There may be new emergent states and novel physics in systems such as the RFIM when driven by a time-varying field. More experimental studies and theoretical studies of more complex systems may lead to many new advances, not only new physics, but new technological applications as well.


$h(t)$
$m(t)$
$T = 9.0$
$h_0 = 1.0$
t
$A = 0$
$Q = 0$
m
h
$m(t)$
$h(t)$
$T = 2.5$
$h_0 = 1.0$
t
$A \neq 0$
$Q = 0$
m
h
$m(t)$
$h(t)$
$T = 1.9$
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t
$A \neq 0$
$Q \neq 0$
m
h
$m(t)$
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t
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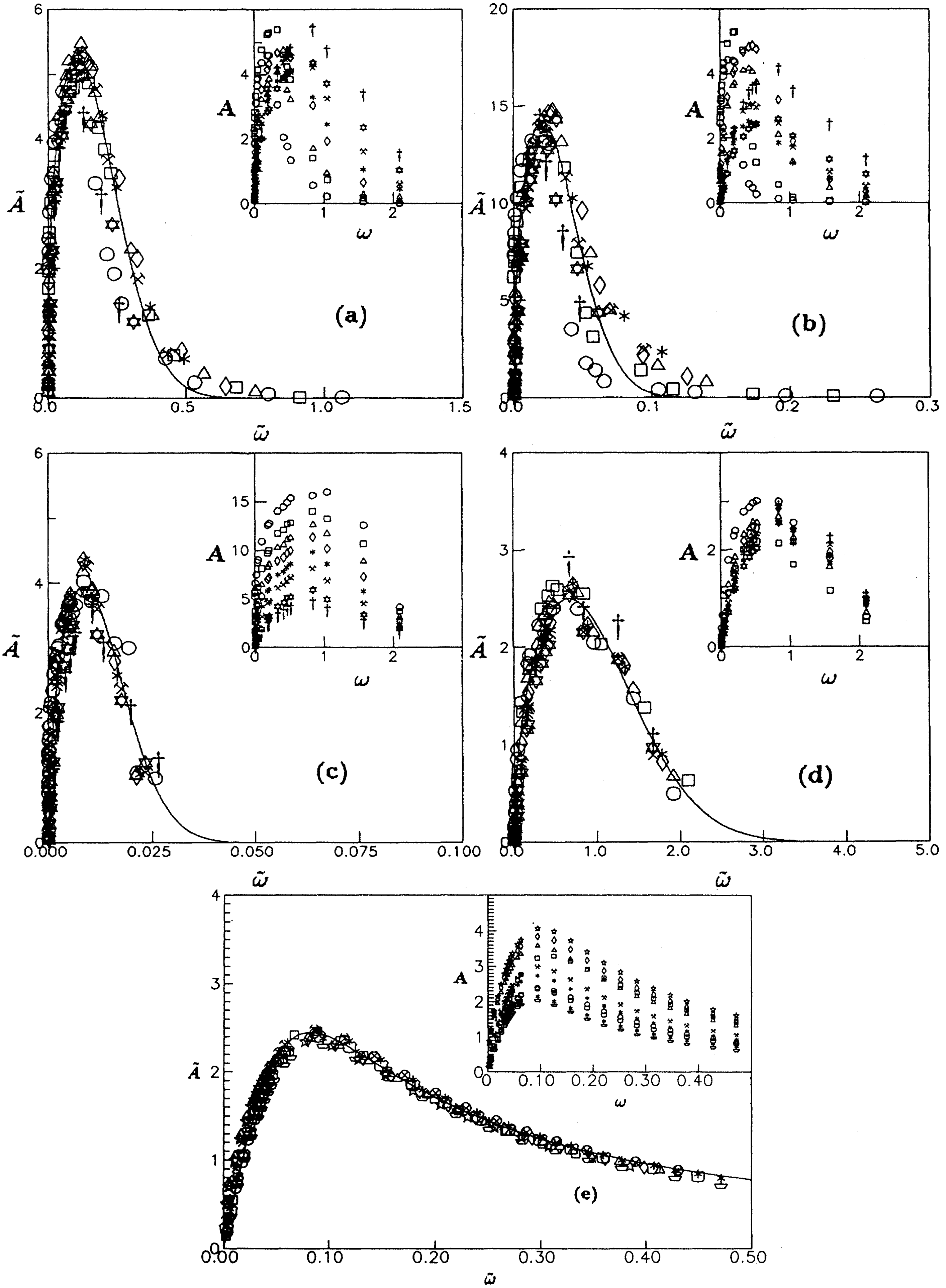
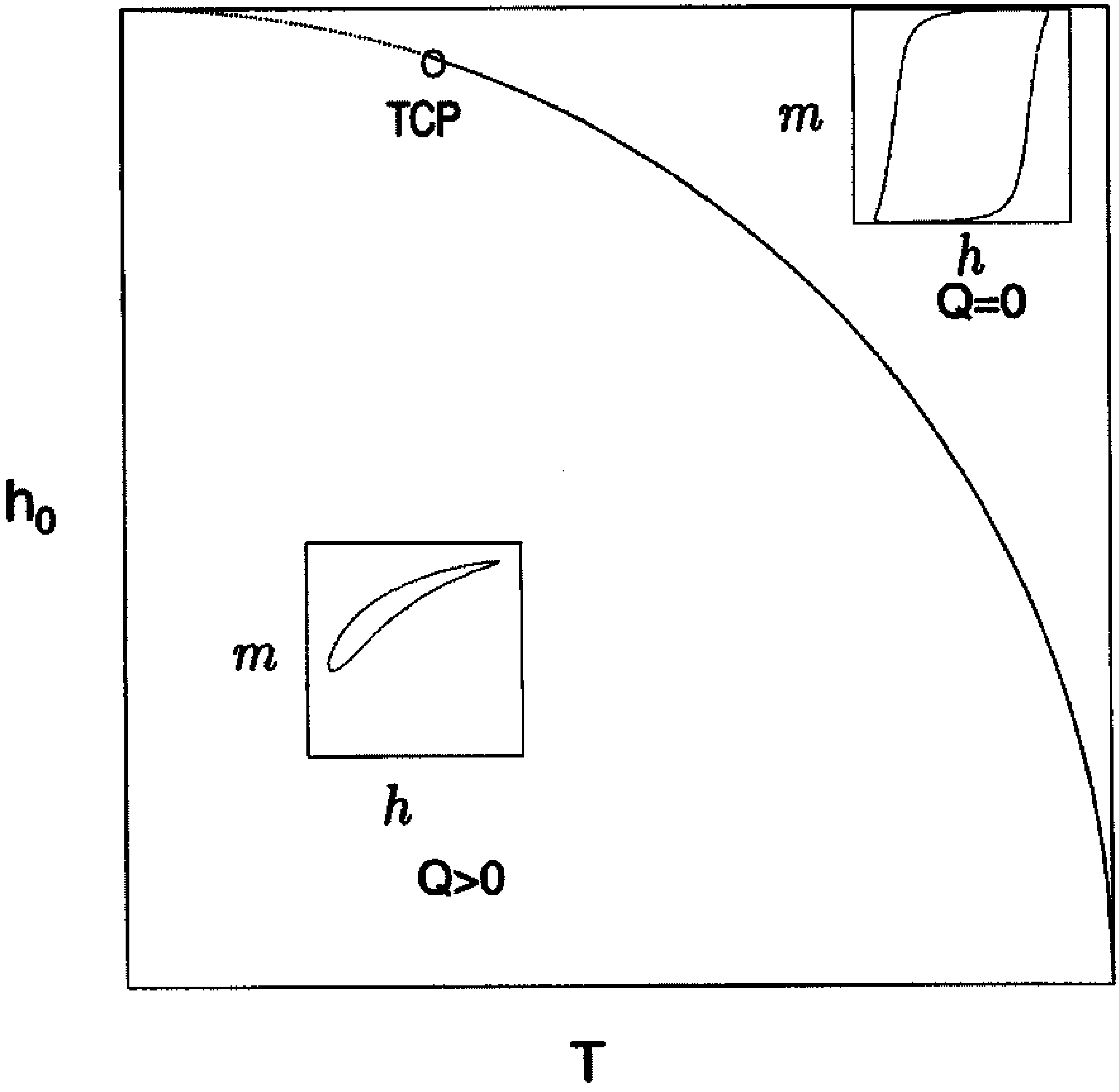
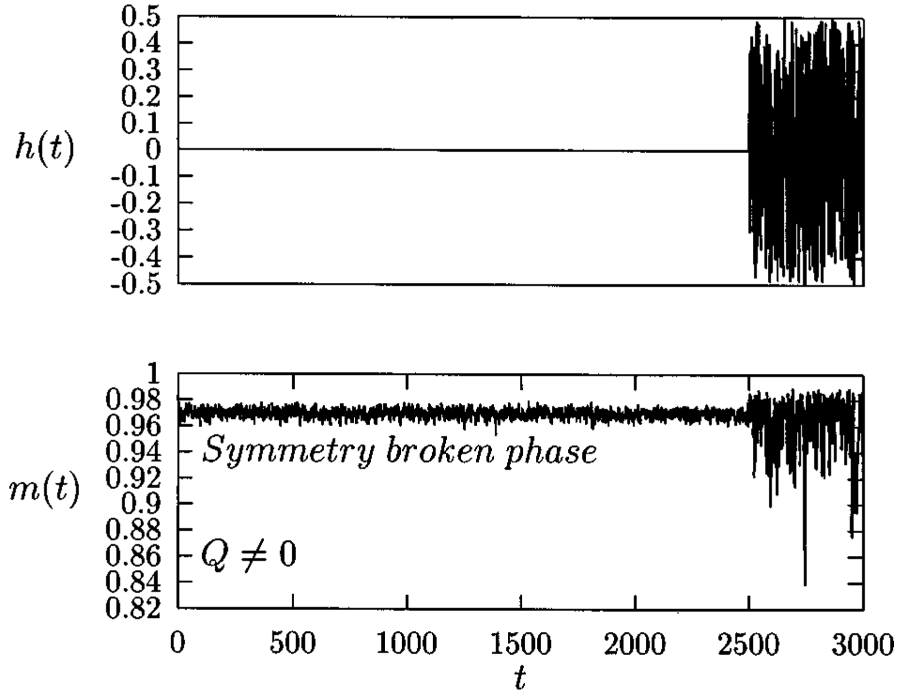
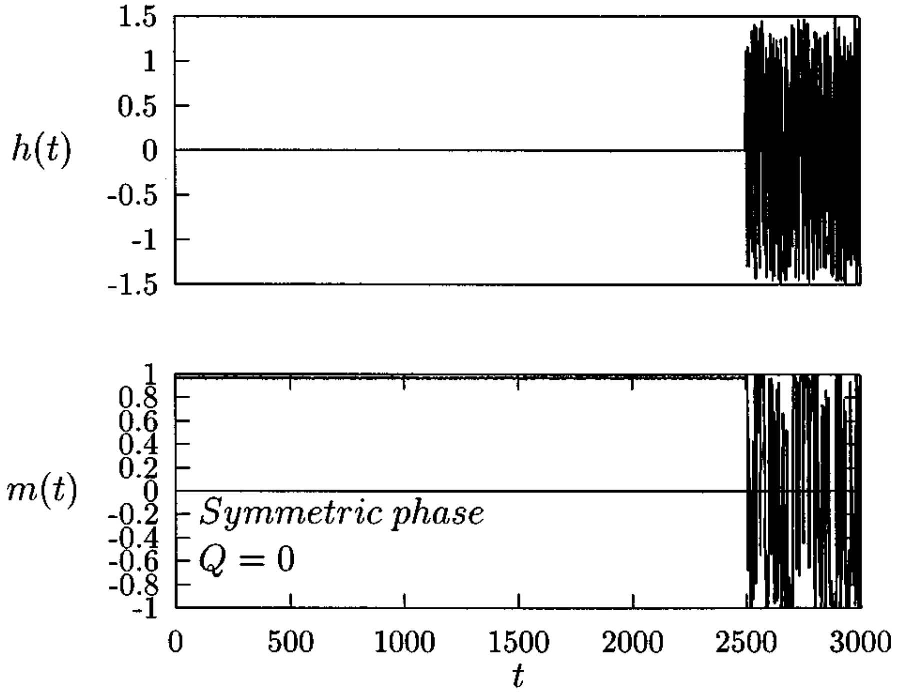
$Q \neq 0$
m
h


(a)
(b)
(c)
(d)
(e)
$\tilde{A}$
$\tilde{\omega}$
A
ω

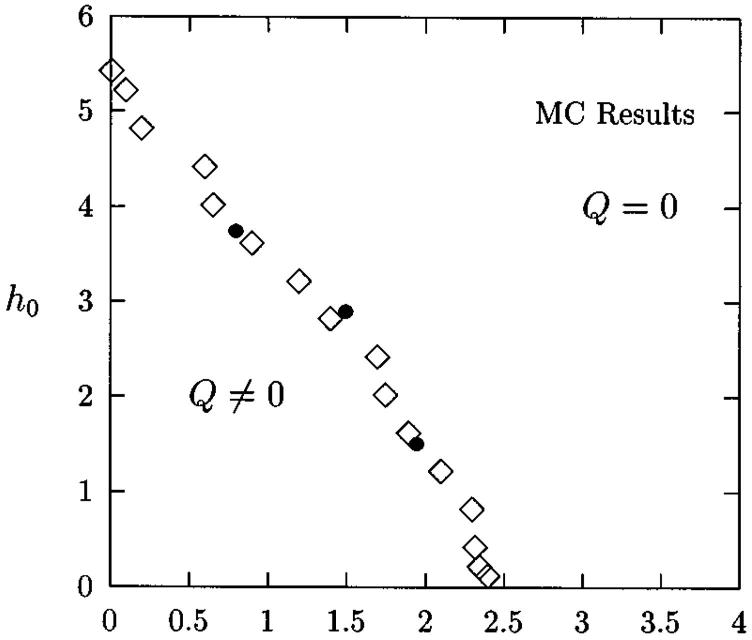

TCP
m
h
Q=0
h_0
m
h
Q>0
T



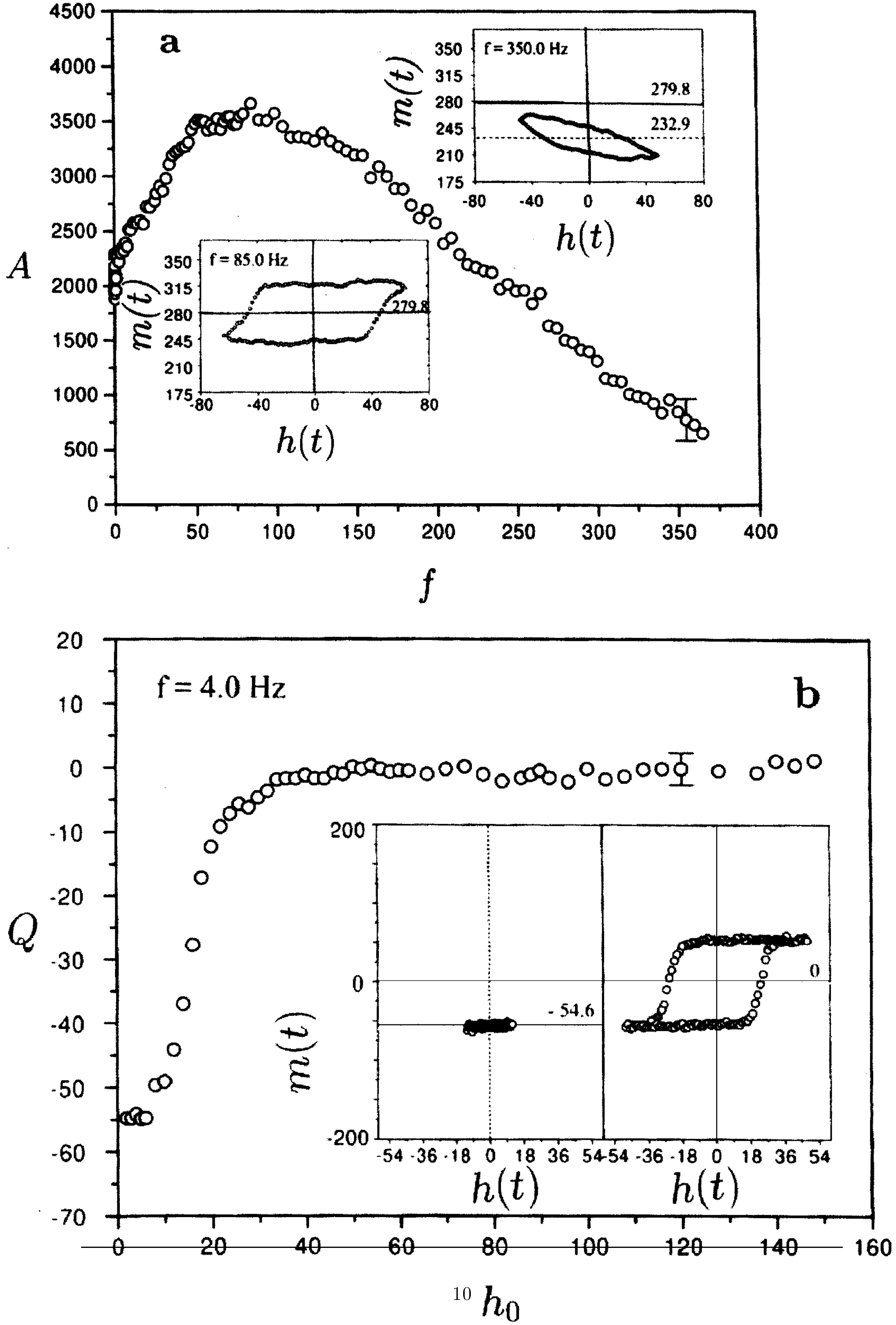
h(t)
m(t)
Symmetry broken phase
Q ≠ 0
0 500 1000 1500 2000 2500 3000
t



h(t)
m(t)
Symmetric phase
Q = 0
0 500 1000 1500 2000 2500 3000
t



MC Results
Q = 0
Q ≠ 0
h0
T


a
A
f
f = 350.0 Hz
279.8
232.9
m(t)
h(t)
f = 85.0 Hz
279.8
b
f = 4.0 Hz
Q
- 54.6
0
h_0

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