# 1/f like noise and self organized criticality

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## Abstract

In this paper, the concept of ubiquitous 1/f like noise and the idea of self organized criticality as the explanation are introduced using simple models. Both theoretical and experimental investigations are discussed to further explore the concept of SOC and test its validity.

# Ubiquity of 1/f like noise

## 1/f like response

Properties of a physical system can be revealed by analyzing its responses against external perturbations. The way in which a system responses can be classified into several major categories. Here are some examples:

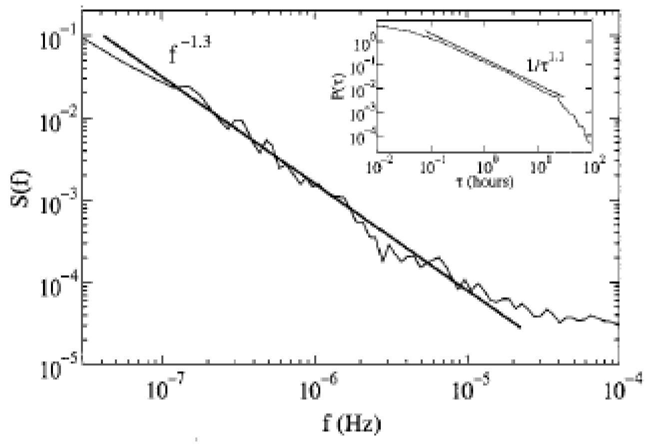
(i) The simplest type is "linear response". This kind of response always happens when a simple system is perturbed by a small external field. For example, a copper will response to a static electrical field by conducting electrical current. The current density J is proportional to the field strength E, and the coefficient is conductivity σ, which is determined by the system itself.

(ii) However, when we consider a complicated system or a large perturbation, the response is not so simple: suppose we heat a cup of water at the bottom, if the temperature difference of water at different position is small, conduction will be the major way of heat transfer and the heat current is approximately propositional to the gradient of temperature; When the temperature difference is large, convection will predominate in heat transfer. Sometimes the water flow is completely disordered, and sometimes regular patterns will form (like Rayleigh-Benard Convection).

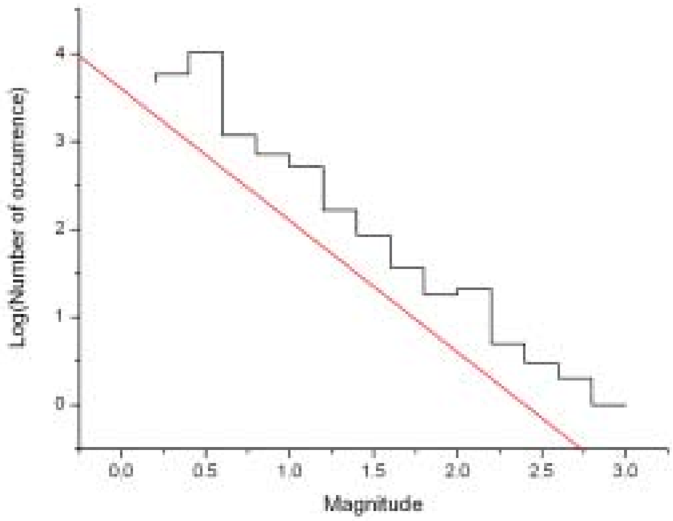
In this paper, we will analyze a special kind of system which is a complicated system (particularly spatially extended, like a pile of sand) and whose responses to small perturbation do not have characteristic length or characteristic time. The responses contain a serial of events at all length and time scale. Their distribution vs. time or length abbeys power law, which means there is no expectation value of time or length of the responses. Particularly, the distribution of the energy released in the events has the form $1/f^{\alpha}$, with $\alpha \approx 1$, so it is called "1/f like" noise.

## Earthquakes as an example

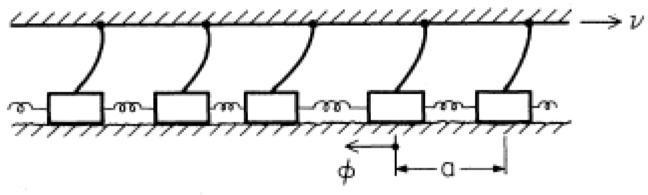
Earthquakes are caused by the movements of continental plates and oceanic plates. Due to the movements, energy and stress will accumulate at a certain spatial point. When the rocks can not withstand the external force, they will deform, but the way they deform is different from a rubber band does under stretching. They release energy and stress by discrete, impulsive events spanning a broad range of size to achieve the deformation required by external restrictions, and every event corresponds to an earthquake. It is just like crumpling a piece of paper, which will form creases of all size. The power spectrum of earthquake signal has 1/f like noise [1]:



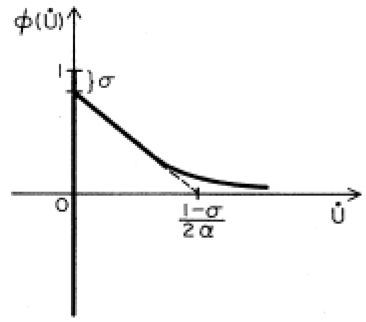

There is an easy to see the power law behavior in earthquake without doing complicated data analysis – to plot the logarithm of number of occurrence versus the magnitude. I used the data collected by Calpine Geysers seismic network in 1995 at http://quake.geo.berkeley.edu to draw the following graph. Since the earthquake magnitude is a logarithmic measure of the energy released by the earthquake, this can be viewed as a log-log plot. The red line has exponent α=1.5, and the power law behavior extended nearly three decades. (This behavior is called Gutenberg-Richter law [9])



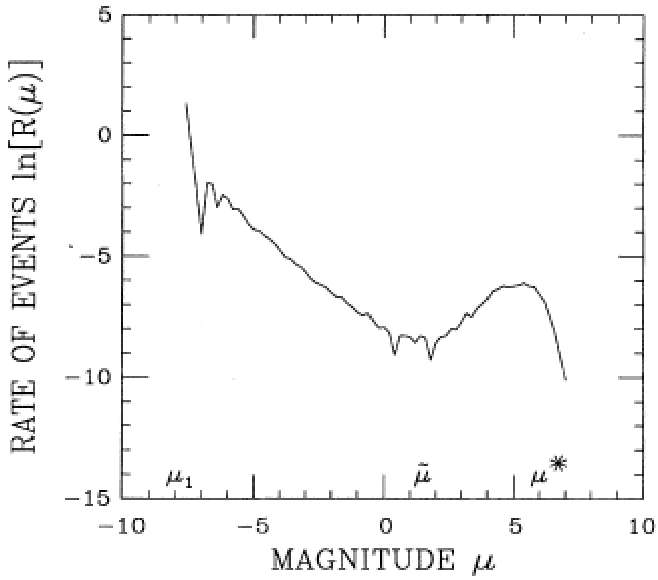

Analyzing a simple model can help to visualize the process that causes the rocks to release energy via events at different scale. In 1967, Burridge and Knopoff introduced a simple model (Burridge-Knopoff model [2]) to describe a fault, which has been extensively studied [3][4][5]. (See the following graph from [5])




The system consists of a chain of elastically coupled blocks; each connected to its two neighbors by harmonic springs and is pulled individually forward by leaf spring that moves at a constant speed. Note that the friction law has a nontrivial property, which is weakened at high velocity (illustrated in the figure from [5]).




When the friction can not hold the blocks, they will move forward a certain distance to release the energy accumulated in the springs. One can imagine: due to the coupling between the blocks, the number of blocks involved in a single slip event has a very broad distribution. If one of the springs connecting the blocks is tightly compressed, one of the blocks will reach the threshold to slip earlier than others. This is a small event. If a large number of the springs are similarly compressed or stretched, many blocks will slip at the same time, which is a large event. Actually, the frequency distribution of the slip events (earthquakes) of magnitude μ has a power law behavior (figure from [5]). A nice simulation program is also online at [http://simscience.org/crackling/Advanced/Earthquakes/TheEarthCrackles.html](http://simscience.org/crackling/Advanced/Earthquakes/TheEarthCrackles.html)




### 1/f like noise is ubiquitous

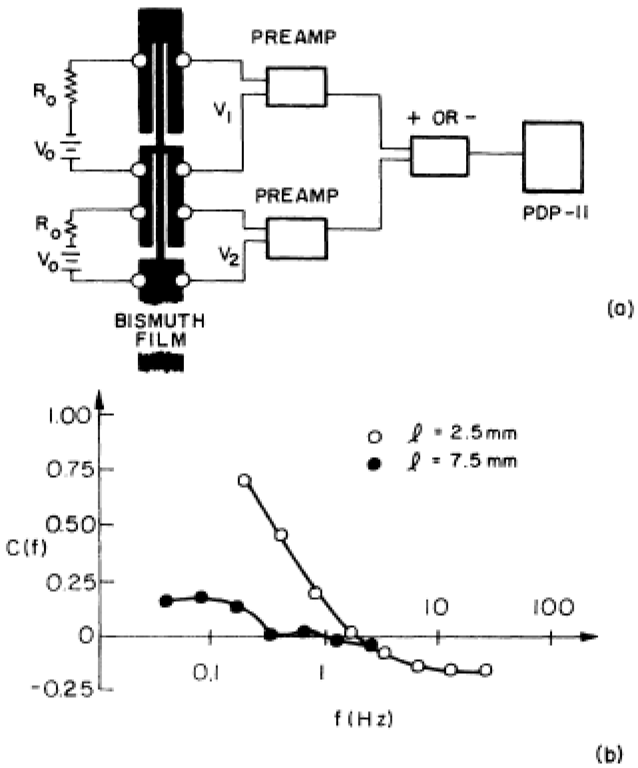
Power laws and 1/f like noise are found in many different phenomena: from earthquake to the noise in music [6]; from video broadcasts [7] to the power spectrum of ocean current velocity [8]; the number of systems in nature that present 1/f like noise is so huge that people think the presents of 1/f like noise is ubiquitous. Considering the appearance of power law and universality in the theory of critical phenomena, everyone will certainly make guess that there is also a deep reason for the ubiquity of 1/f like noise. Is the answer similar to the theory of critical phenomena? We will explore this topic in the following sections.

## Self organized criticality

### Relationship between noise at frequency f and spatial correlation

Analyzing the Burridge-Knopoff model of earthquake, we find out that the noise (earthquake signal) is produced by the movement of the blocks. Since movements with different spatial size are related to different noises, it is natural to ask whether the spatial correlation of the system is related to the property of the noise. In fact it is true. The noise at a given frequency f is spatially correlated over a distance L(f), which increases as f decreases [10][11].

Let's have a look at an experiment did on metal films [11]:




This experiment is designed to measure the frequency dependence of the correlation of the 1/f noise from two regions of a single film (figure a). Each part of the film is supplied a constant current by separate batteries and large resistances $R_0$. The noise voltage measured $V_1$ and $V_2$ are amplified with preamplifiers. The spectrums of their sum or difference are measured. Suppose $S_+(f)$ is the spectrum of $V_1+V_2$, and $S_-(f)$ is the spectrum of $V_1-V_2$, the fractional correlation is given by $C(f)=[S_+(f)-S_-(f)]/[S_+(f)+S_-(f)]$. If $V_1$ and $V_2$ are independent, then $S_+(f)=S_-(f)$, $C(f)=0$. If $V_1=V_2$, $C(f)=1$. In this experiment, as the frequency f is lowered, the value of C(f ) becomes nonzero at a point when the correlation length $\lambda(f)$ is comparable with the distance of the two regions where the noise voltages are measured. Figure b shows that low frequency noise is associated with large correlation length. This is an important observation: a system's power law behavior in frequency domain maybe related to the power law behavior of spatial correlation. This leads us to a possible explanation of the ubiquity of 1/f like noise – self organized criticality.

## The idea of self organized criticality

Self-Organized Criticality (SOC) was suggested by Per Bak, Chao Tan and Kurt Wiesenfeld in 1987 [12]. The title of the paper was "Self-Organized Criticality: An Explanation of 1/f noise." In this paper, Bak et al. argued that when a spatially extended system with many degrees of freedom is driven away from equilibrium by an external force, the stationary state is a state with power law spatial correlation. It will spontaneously evolve to such a "critical state" that lacks a characteristic length. They also argued that lacking a characteristic length will cause lacking a characteristic time, which will induce a power law behavior into the frequency spectrum [13].

The most exciting message in this article is that there are systems that do not need a tuning of parameters but spontaneously evolve to a critical point. After the 1987 article, there are thousands of articles published on SOC - the 1987 article accumulated about 2000 citations. However, there is controversy surrounding the 1987 article. Henrik Heldtoft Jensen, Kim Christensen and Hans C. Fogedby tried to repeat the same spectral analysis of the 1987 article, but found $1/f^2$ noise which is quite different from 1/f noise [13]. In fact, some recent researches argue that in some systems, SOC is not the underlying mechanism for the presence of temporal correlation between events[14][15], while other researches support that SOC can provide a dynamical mechanism which gives correlations between bursts leading to $1/f^{\alpha}$ noise[16]. As far as we know, SOC is still only a candidate to explain the ubiquity of 1/f like noise.

Let's have a look at the argument in the 1987 article: imagine an array of damped pendulums. The pendulums are connected by torsion springs which are weak comparing with the gravitational force. This is a good model to describe a spatially extended system. Now let this system be perturbed by a small external force. Suppose one of the pendulums rotates due to this perturbation, causing the forces on neighbor pendulums to change. Then, the changing of the forces on neighbor pendulums may also cause them to rotate. As this process goes on, the perturbation will propagate to elsewhere of the system. Then how far it will go? To answer this question, we can let the system start to evolve far away from equilibrium, where most of the spring's forces on the pendulums are at a large value, so that small perturbation will cause them to rotate. In that case, a single perturbation can propagate to infinity, changing the global structure of the system, and release large amount of potential energy stored in the springs. This is the equilibration process: When more and more energy is released, the global structure becomes more and more stable. The system reaches equilibrium precisely at the point when perturbation can not propagate to infinity. Hence, the correlation function in such a system must have a power law form. The whole picture is "the system approaches through a self organized process to a critical point with power law correlation functions for noise and other physical quantities" [12].

## SOC and the traditional critical phenomena

To further explain the idea of SOC, let's make an analogy between SOC and traditional critical phenomena [17]. The model we are going to discuss here is a pile of sand. The fundamental physics is that when we add particles to a sand pile, every particle will cause rearrangements of the sand pile to reach a stable state. When the pile is flat, only local rearrangements happen, and the pile will get steeper and steeper. If the initial state of the sand pile is steeper than a critical value $\theta c$, the sand will make rearrangement even if there is no particle adding to the pile. Hence, we can make an analogy between a pile of sand and the Ising model: the slope of pile $\theta$ corresponds to the reduced temperature t; the flow of sand corresponds to the magnetization m; the flux of adding particles corresponds to the external field h. When$\theta > \theta c$, there is spontaneous sand flow. This is the ordered phase of Ising model. When $\theta < \theta c$, there is sand flow only when we drive the system by adding particles to it. So this is the disordered state of Ising model. There difference between this two systems is that a sand pile always wants to evolve to a state with $\theta = \theta c$.

We can define several critical exponents to describe the non-equilibrium dynamical properties.

| Physical quantity | definition | Critical exponent | definition |
|---|---|---|---|
| Order parameter | Flow J | $\beta$ | $J \approx (\theta - \theta c)^{\beta}$ |
| susceptibility | $\delta J(x',t') = \int X(x',t';x,t)\delta J(x,t)dxdt$ | $\gamma$ | $X \approx (\theta c - \theta)^{\gamma}$ |
| External field | Flux of adding particles | $\delta$ | $J(\theta = \theta c) \approx h^{1/\delta}$ |
| | | $v$ | $\xi \approx (\theta c - \theta)^{v}$ |
| D(s) | Distribution of avalanches size | $\tau$ | $D(s) \approx s^{-\tau+1}$ |
| S(ω) | Power spectrum | $\varphi$ | $S(\omega) \approx \omega^{-\varphi}$ |
| l & t | Spatial and temporal scale of an avalanche | z | $t \approx l^{z}$ |

Basing on this definition, one can also ask that how many exponents are independent, and what are the scaling laws. For example, the flow J caused by the field h below $\theta c$ is the average size of the clusters:

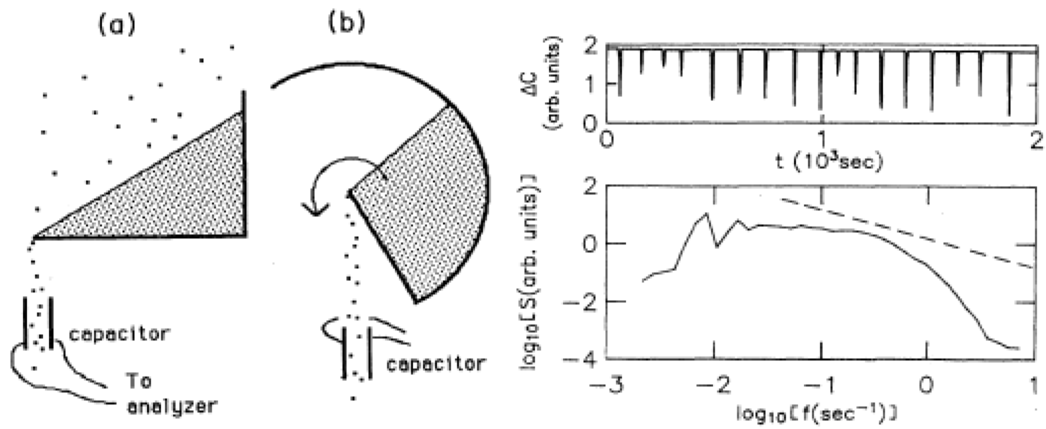
$$X = \int_0^{\xi^D} sD(s)ds = (\theta c - \theta)^{-(3-\tau)Dv} \qquad \therefore \gamma = (3-\tau)Dv$$

Following this analogy, SOC can be studied in the fashion of traditional critical phenomena, like Lee-Yang zeros [18], Scaling functions [24] and renormalization group [19].

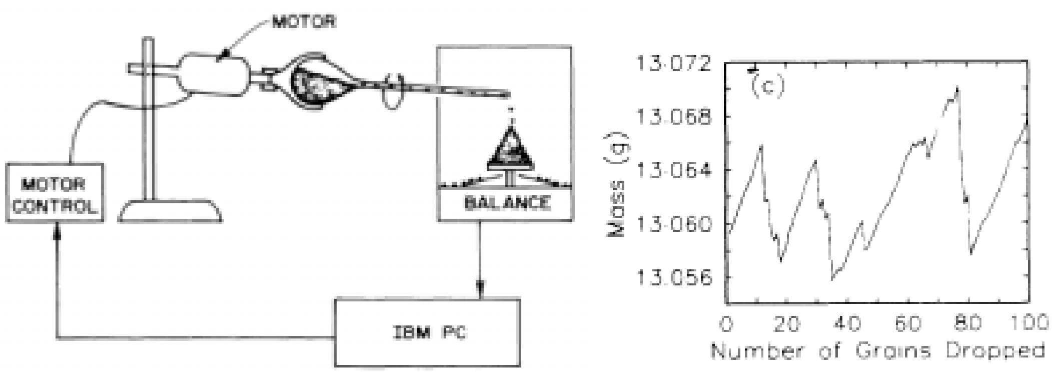
## Experimental tests

SOC is so attractive because it has the potential to explain the 1/f noises in a simple way without using any external fine tuning parameters. However, neither it is generally accepted that SOC implies 1/f noise [13][20], nor the systems used as examples of SOC (for example, the sand pile) robustly stay at SOC state [21][22].

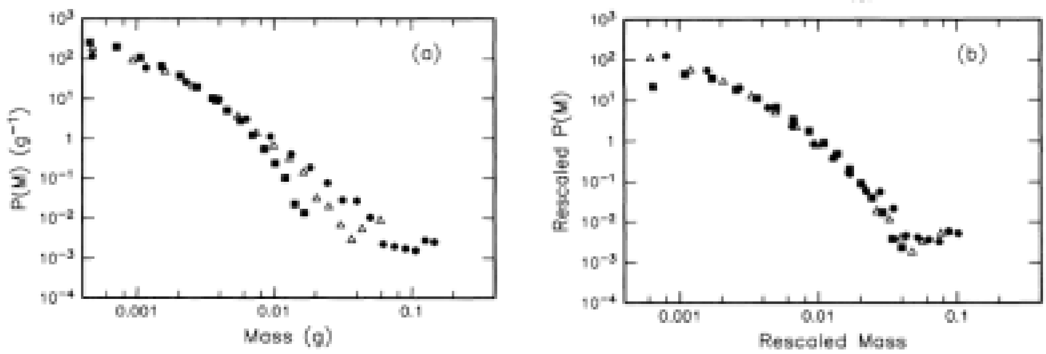
After the 1987 article, H.M.Jaeger, Chu-Heng Liu, and Sidney R.Nagel [21] did an experiment to test whether avalanches on a sand pile have power law distribution. They monitored the avalanches caused by both adding sand to the pile (figure a) and changing the slope of the pile (figure b). The avalanches are recorded by letting the sand drop through a capacitor, and monitored the changing of capacity. They showed us a sand pile does not always have 1/f noise.



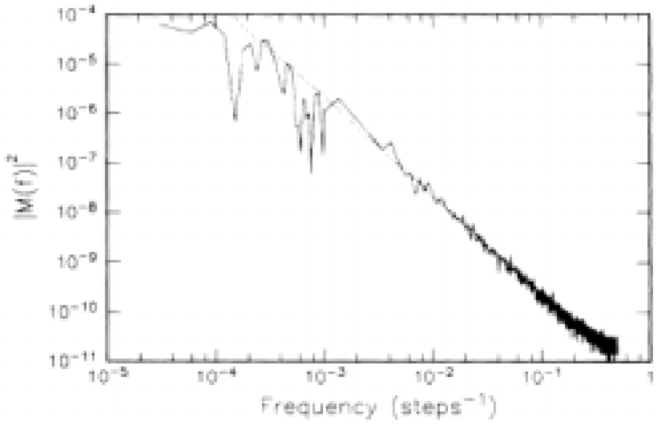

One year later, G. Grinstein et al. did an experiment and the result supports the existence of SOC. They used a rotating computer controlled funnel to add sand one by one to the top of a pile, and monitored the fluctuation in the mass of the sand pile. (The following pictures are the experimental configuration and mass fluctuation signal)



The distribution of falloff mass is approximately a power law, $P(M) \approx M^{-2.5}$, and more importantly, the distribution of avalanches has a scaling form: $P(M.,L)=(1/L^{\beta})g(M/L^{v})$, with $\beta=2v=1.8$. The experimental data lie almost exactly on the same universal curve $g(M/L^{v})$. (The following pictures are the distribution of mass and the data collapse)




However, instead of observing 1/f noise, they saw the spectrum falls of as $1/f^2$ which is consistent with the power spectrum of a weighted random walk. (the power spectrum: )




In fact, in the past 18 years, many experiments on various systems were did, like Imre M. Janosi and Viktor K. Horvath's work on water droplets [25] and G. W. Crabtree et al.'s work on vortices in superconductors [26]. However, those experiments did not result in a clear and consistent picture on SOC and the explanation of 1/f noise so far.

## Conclusion

Bak et al.'s original idea is that: (i) The concept of SOC is universal – spatially extended systems in nature are always in the SOC state. (ii) SOC causes the 1/f like power spectrum. These basic physical ideas are not hard to grasp when we consider simple models like a sand pile or the Burridge-Knopoff model. However, after 18 years of theoretical and experimental investigations, people still do not have a clear understanding on SOC. First, experiments and computer simulations have shown that many systems are at SOC state only under certain conditions, which means the concept of SOC does not have the universality claimed by Bak et al. Second, there are some systems that have the fingerprints of SOC but have $1/f^2$ noise instead of a nontrivial $1/f$ like noise. Nevertheless, Bak et al.'s idea is valuable in the sense that it provided people a way of solving such problems in a pre-established theoretical framework.

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