The Superconductor Insulator Transition

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Abstract

As the thickness or magnetic field is varied in thin films a transition between superconducting and insulating phases takes place. This quantum phase transition obeys scaling relations and occurs at seemingly universal critical resistance. This paper reviews experiments that demonstrate this transition and measure the critical exponents. The microscopic theories are outlined and lead to different predictions.

Introduction

In a disordered thin film of a material that superconducts in the bulk, resistance is strongly dependent on film thickness. As the temperature is lowered, above a critical thickness the film resistance drops almost to zero. Below that thickness resistance rises exponentially as a function of length. This is a dramatic signature of the superconductor-insulator transition (figure).

As the figure suggests, this is a quantum phase transition. It takes place at absolute zero and is governed by quantum rather than thermal fluctuations. The parameter that brings the system to criticality is implicitly part of the Hamiltonian (thickness or magnetic field in this case), and the phase transition is due to a qualitative change in the nature of the ground state. Like ordinary phase transitions correlation lengths and times diverge near the critical point and define critical exponents which have been measured in a wide range of materials.

Why 2D?

In zero external magnetic field a 2 dimensional system will undergo a Kosterlitz-Thouless transition which results when vortices become bound at low temperatures. True long-range order is impossible due to phase fluctuations. With the addition of impurities this transition temperature might be lowered but still remain finite while the superconducting T_c is driven almost to zero. What should we expect to see as near the critical point?

Theories

The dirty boson picture due to Fisher [5] and others says that Cooper pairs persist in both the superconducting and insulating phases. The former is seen as condensate of Cooper pairs with localised vortices (pinned by disorder). The latter is a condensate of vortices with localised Cooper pairs (inspired by the KT transition.) Exactly at the transition both the Cooper pairs and vortices are mobile, an example of *duality*.

The above phase only picture deals only with the fluctuation in the the phase of the macroscopic order parameter $\Psi = \psi(r)e^{i\theta}$. From the point of view of field theory, the fermion degrees have been integrated out leaving only effective bosons (Cooper pairs). Another set of models describe fluctuations in the amplitude $\psi(r)$. Cooper pairs break up in the insulating phase due to an enhanced Coloumb interaction. A key prediction of this theory is the disappearance of the energy gap in the insulating phase which has been observed [3].

Experiments

In a typical experiment, such as the one in [1] a thin film of a superconducting metal like Bi is evaporated onto a cold substrate. The film thickness is gradually increased in increments of 0. 1-0. 2 Å. Resistance is recorded as each successive layer is deposited for a range of temperatures usually going no lower than . 1K. For a square piece of the film, the normal resistance is just the resistivity divided by the width of the film. Hence the *square* resistance is the most useful quantity for the graphs. R_{square} is also measured against a perpendicular magnetic field of upto 12 kG.

The scaling analysis described below claims that,

$$R_{square} = R_c f(\delta T^{1/\nu z})$$

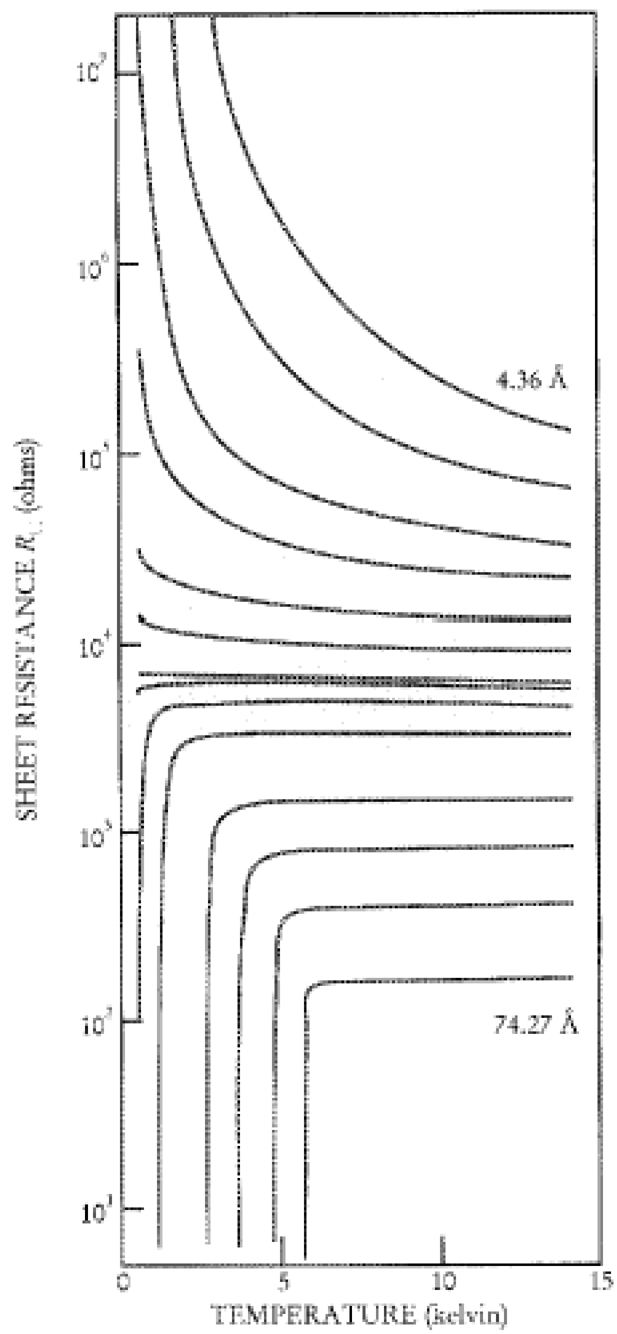

Figure 1: Temperature dependence as a function of film thickness. The line separating the insulating and superconducting families has a resistance close to $h^2/4e^2 = 6450\Omega$(the fundamental resistance for electron pairs). [1].

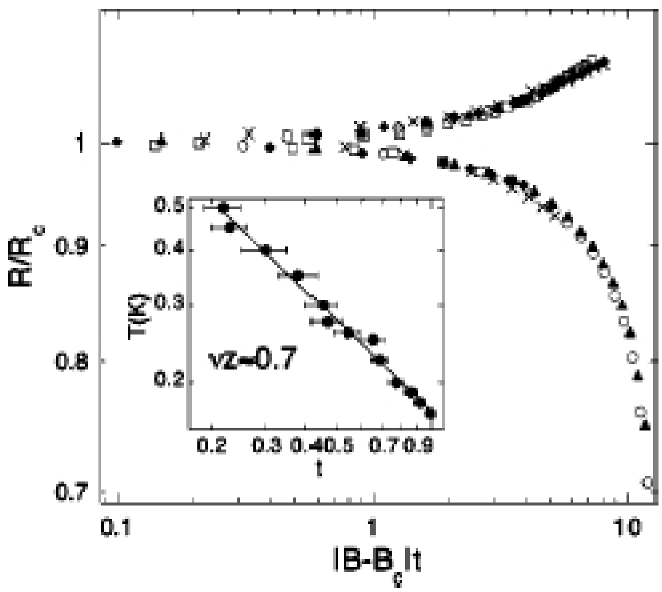

Figure 2: Normalised resistance per unit square as a function of $T^{-1/\nu z}|B - B_c|$. The data are taken from a variety of samples across a range of temperatures and clearly collapse into two curves. The upper curve represents the insulating phase while the lower one is superconducting. From the inset $\nu z \approx .7$ independent of thickness (provided the film superconducts in the first place). Taken from [1].

where δ is $|B - B_c|$ for transition tuned by the magnetic field and $|d - d_c|$ for the one tuned by the film thickness. One way to obtain the critical exponents is to fit the data to the relation,

$$R_{square} = R_c f(\delta t(T))$$

where t(T) is via a stastical minimization that gives the best collapse of the data for a range of values in δ. Then νz can be found from a log-log plot. The advantage of this method is it doesn't assume that the critical exponents exist. However the error bars in the exponents are hard to determine exactly.

Another method calculates,

$$\frac{\partial R}{\partial K}_{K_c} \propto R_c T^{-1/\nu Z} f'(0)$$

where $K \equiv B$ or d and then a similar log-log. As shown in the next section, the nonlinear scaling of R with the electric field yields $\nu(z+1)$.

Results

It is easier to compare the scaling with B field across experiments. An example from [1] is shown in figure . Figure shows the E field scaling from the same reference.

Unfortunately experiments have contradicted each other over the last decade or so. The figures shown give $\nu.7$ and $z = 1$ for the B field tuned transition. Not only is this different from observed values for other

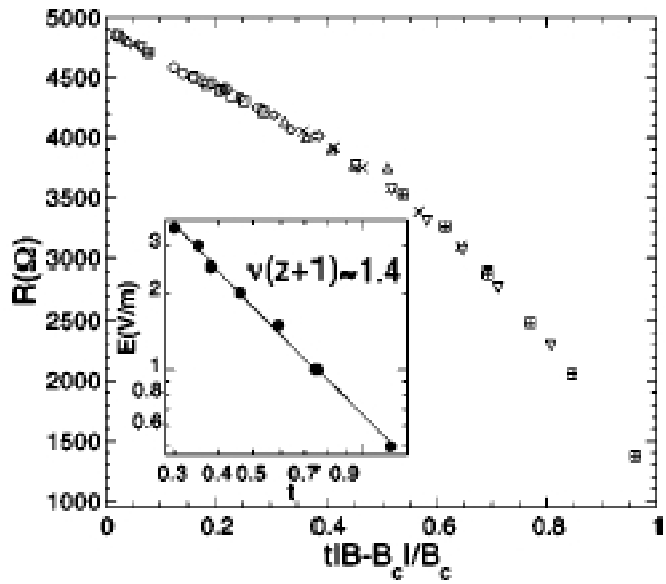

Figure 3: Resistance per unit square as a function of $T^{-1/\nu z}|B - B_c|$, now for a range of electric fields. This only shows the superconducting phase $\nu(z+1) \approx 1.4$ From [1].

materials (Indium oxides and MoGe), it varies from the thickness tuned exponents calculated from the same samples! It has been suggested that the root cause of these inconsistencies is the fact that the critical region in the insulating phase is different from the one probed. In that case, electron transport in the insulating phase might also be affected by modes that have nothing to with critical fluctuations.

Scaling

Near any critical point, whether quantum or classical, the correlation length ξ diverges. As some fundamental parameter K is brought to close to its critical value K_c then,

$$\xi \sim |\delta|^{\nu} \quad \text{where} \quad \delta \equiv |K - K_c|$$

defines the critical exponent ν. For a quantum transition, K is no longer the temperature but a parameter that goes into the Hamiltonian. For dynamical systems (where the 'dynamics' is actually in imaginary time) there is a correlation time ξ_τ which diverges as,

$$\xi_\tau \sim \xi_\tau^{z}$$

defining the dynamical critical exponent z. Most experimental evidence points to $z \approx 1$, a fact sometimes referred to as 'isotropy' in the time direction. Near the critical point ξ and ξ_τ are the only length and time scales respectively and the basic scaling relation is,

$$F(k, \omega, K) \sim \xi^{\tilde{d}} f(k\xi, \omega\xi_\tau)$$

where $\tilde{d}$ is the scaling dimension of an operator F that is measured at some wave vector k and frequency ω. To make both sides dimensionally correct we have to multiply by physical quantities relevant to the problem but they don't scale with δ and are hence ommitted. Exactly at the critical point F should approach a finite limit, which means that the ξ dependence should cancel out.

$$F(k, \omega, K_c) \sim k^{-\tilde{d}} f(k^z/\omega)$$

In other words k is the only length scale that remains.

Finite Size Scaling

We need to understand the kind if scaling relations that should be expected at $T \neq 0$. The main relation is,

$$F(k, \omega, K, T) \sim L_\tau^{\tilde{d}/z} f(k L_\tau^{1/z}, \omega L, L_\tau^{1/z}/\xi_\tau)$$

which holds for small δ and T. Setting $k = 0$ and applying this to the resistivity,

$$\rho = \rho_c f(0, \hbar\omega/kT, \delta/T^{1/\nu z})$$

where the last argument has been raised to the $1/\nu z$ power to make the δ dependence and f redefined to this end. Finally, putting $\omega = 0$ for the DC resistivity we see that,

$$\rho = \rho_c \tilde{f}(\delta/T^{\nu z})$$

where $\tilde{f}(0) = 1$. In the critical region we should observe data collapse as ρ only depends on the combination $\delta/T^{\nu z}$.

The other experimentally useful quantity is the relation of ρ to the applied voltage, or equivalently to the measured current. Here we are looking at the nonlinear response, assuming it is caused by critical fluctuations. Let the electric field introduce a new length scale l and its corresponding time scale that goes as l^z. The exact definition of l is not important. All we want is that l satisfy the following relation,

$$eEl \sim \hbar l^{-z}$$

which says that the energy gained by an electron moving through a distance l goes as the characteristic energy associated with frequencies l^z near the critical region. So,

$$l \sim E^{-1/(1+z)} \frac{l}{\xi} \sim \delta^\nu E^{-1/(1+z)} \sim \left(\frac{\delta}{E^{1/(1+z)}} \right)^\nu$$

Putting everything together,

$$\rho = \rho_c g(\delta/T^{\nu z}, \delta/E^{1/\nu(z+1)})$$

Of course measuring both $\nu(z+1)$ and νz gives ν and z separately.

It has also been argued that the critical resisitivity ρ_c is in fact universal or at least simply related to the famous Hall resistance h^2/e (or rather $h^2/4e$ since have pairing) in a sample independent manner. This has only been partly borne out by experiments.

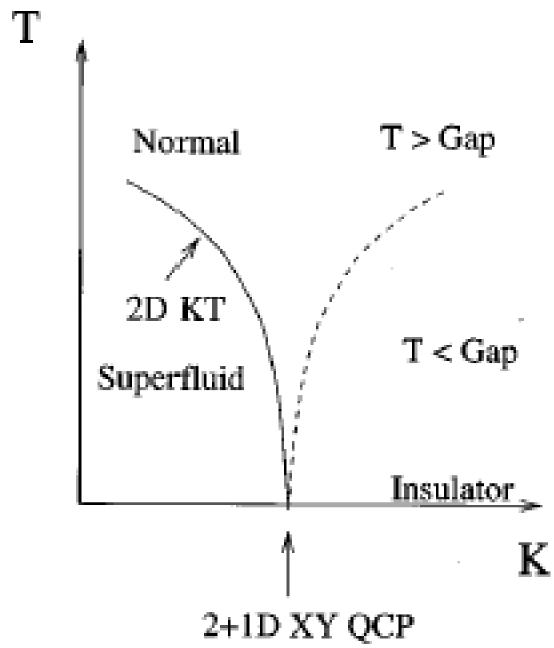

Figure 4: Phase diagram of the 2D XY model. The solid line represents the Kosterlitz Thouless transition. The dashed line indicates that there is crossover when the temperature is larger than the insulating gap. From [4]

Conclusions

Scaling analyses conclusively prove that a superconductor-insulator transition exists and is quantum in nature. But there are doubts about the underlying microscopic theory.

There is plenty of evidence to suggest that the phase only picture does not describe everything. The density superconducting gap as measured by the tunneling experiments seem to vanish in the insulating phase suggesting the breaking of Cooper pairs. The critical resistance has been found to deviate from its quantum value. A major worry while comparing experiments is determining to what extent the films are granular or homogenous. Uniformity and more measurements further into the critical region are the next challenges.

References

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