Phase Diagram of a Polarized Fermi Gas

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Abstract: The polarized Fermi gas, which has recently been realized in ultra-cold atomic gases where the number of atoms in the two spin states is different, has attracted many experimental and theoretical works. This paper introduces the phase diagram of the polarized Fermi gas at $T = 0$ and at finite temperature as well. Open questions that haven't been solved are pointed out and available experiments are presented.

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1 Introduction

1.1 BEC

The first experimental discovery of Bose-Einstein condensation (BEC) in dilute vapors of alkali atoms has opened up the exploration of quantum phenomena in the area. Great achievements have been made in the study of superfluid features, the investigation of coherence phenomena in atom laser configurations, et al. [1] Fig. 1 shows the first picture of BEC.

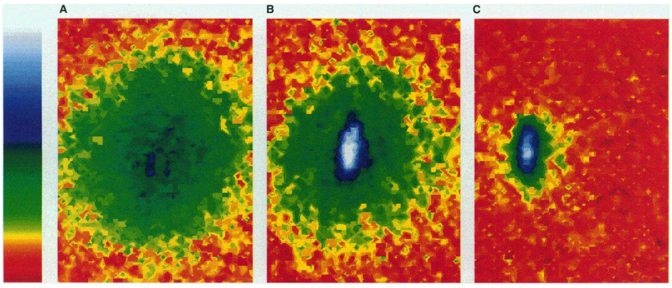

Figure 1: The first realization of BEC in trapped atomic gas. (1995 Anderson) False-color images display the velocity distribution of the cloud (**A**) just before the appearance of the condensate, (**B**) just after the appearance of the condensate, and (**C**) after further evaporation has left a sample of nearly pure condensate. The elliptical condensate fraction indicates that it is a highly nonthermal distribution. [2]

1.2 Ultracold Fermi gas

Trapped Fermi gases were first cooled to below the Fermi temperature in 1999 by the group at JILA. Quantum degeneracy was observed as a barrier to evaporative cooling and measurements of the momentum distribution and the total energy of the Fermi gas revealed the quantum statistics. [3] A convincing proof of superfluid behaviour was provided by the observation of vortex lattices in a strongly interacting, rotating Fermi gas of ^{6}Li atoms on both sides of the Feshbach resonance. [4] Fig. 2 shows the observed vortex lattices in the BEC-BCS crossover.

1.3 Polarized Fermi gas

In ultracold Fermi gases, it is possible to imbalance the numbers of atoms in different spin state. The obtained polarized Fermi gas owns much richer

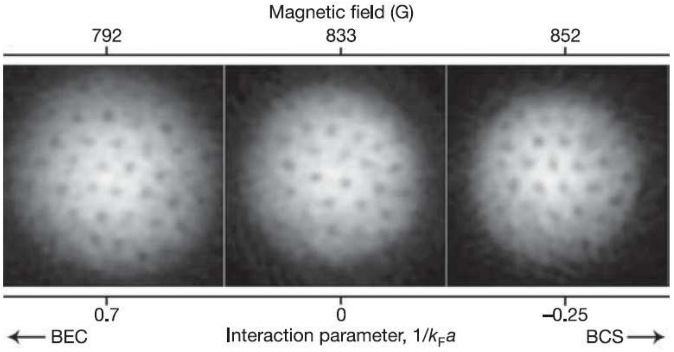

Figure 2: Vortex lattices in the BEC-BCS crossover. The dark dots are the vortices which form latices and exist throughout the BEC-BCS crossover. [4]

features like the exotic superfluid phase and we are still not sure about the phase diagram in the region near unitarity. In this paper, I discuss the phase diagram of polarized Fermi gas. Theoretical understanding is presented first and experimental achievements are discussed later.

2 Theory

2.1 Phase diagram of a polarized Fermi gas at $T = 0$

First, let's consider the case when $T = 0$. For the moment, we don't take into account exotic superfluid phases like FFLO state, Sarma state or states with a deformed Fermi surface, then a qualitative phase diagram is shown in Fig. 3, where the polarization P is

$$P = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}. \tag{1}$$

We can see that the phase diagram is divided into three parts: a normal phase, a superfluid phase, and a separation phase. Next, I will give a discussion in different region of the phase diagram.

2.1.1 BCS regime

In the regime of small and negative scattering lengths, the system can be understood by BCS theory if $P = 0$. The BCS superfluidity arises from the pairing of particles of different spin occupying states with opposite momenta,

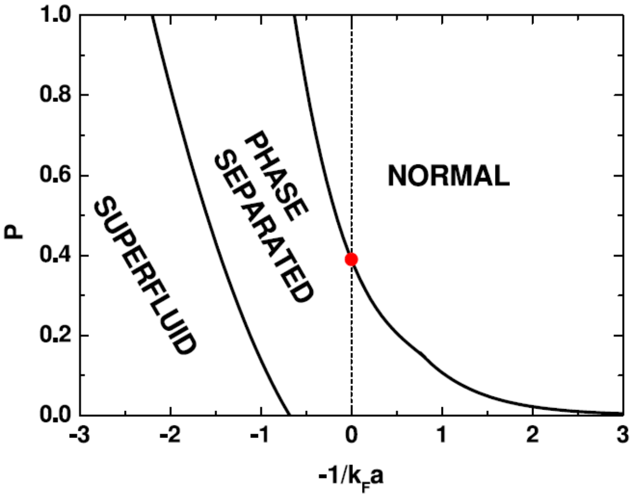

Figure 3: Qualitative phase diagram as a function of the interaction strength $-1/k_F a$ and of the polarization P. The Fermi wavevector corrsponds here to the total average density: $k_F = [3\pi^2(n_\uparrow + n_\downarrow)]^{1/3}$. Notice that the possible occurrence of the FFLO phase is not considered on this diagram. [1]

close to the Fermi surface. [1] As P increases from 0 to a finite value, the two Fermi surfaces mismatch and cooper pairs with zero total momentum become difficult to form. When P is too large, the system undergoes a quantum phase transition towards a normal state. This transition takes place when the gain in the grand-canonical energy associated with the finite polarization of the normal phase is equal to the energy difference between the normal and the superfluid unpolarized states. In the BCS regime one finds the critical condition

$$h \equiv \frac{\mu_\uparrow - \mu_\downarrow}{2} = \frac{\Delta_{gap}}{\sqrt{2}}, \tag{2}$$

where $\Delta_{gap} = (2/e)^{7/3} E_F e^{\pi/2k_F a}$ is the BCS gap. The corresponding critical value of polarization P is

$$P_c = \frac{3}{\sqrt{8}} \left(\frac{2}{e}\right)^{7/3} e^{\pi/2k_F a}. \tag{3}$$

For $P > P_c$ the system is normal and corresponds to a uniform mixture of the two spin components well described by the non-interacting model. For $P < P_c$ the system is instead in a mixed state, where the unpolarized

BCS superfluid coexists with the normal phase which accommodates the excess polarization. In this mixed state, the chemical potential difference of the normal phase retains the critical value irrespective of polarization, a decrease in P being accounted for by an increase in the volume fraction of the superfluid phase which eventually occupies the entire volume for $P = 0$. [1]

2.1.2 BEC regime

When the scattering length is positive and small, bosonic dimers are formed with pairs of atoms in two spin states. The energetically favourable phase in this deep BEC regime consists of a uniform mixture of a superfluid gas of bosonic dimers and of a normal gas of spin polarized fermions. It is expected in this regime that the normal uniform gas exists only for P = 1, corresponding to the fully polarized ideal Fermi gas. When $P \simeq 1$, there are a small number of bosonic dimers in a fully polarized Fermi gas sea. It's found that the relevant condition for the solubility of the mixture reads

$$k_F \leq \frac{4\pi}{2^{1/3}9} \frac{a_{dd}}{a_{ad}^2}, \tag{4}$$

where a_{dd} and a_{ad} are the dimer-dimer and atom-dimer scattering length respectively. By using the values $a_{dd} = 0.6a$ and $a_{ad} = 1.2a$, one finds that the uniform phase exists for $1/k_F a > 2.2$. In the opposite regime of a fully paired molecular condensate with a small number of unpaired fermions ($P \simeq 0$) a similar analysis yields that the uniform mixture is in equilibrium for $1/k_F a > 0.68$. This Bose-Fermi picture, however, looses its validity as one approaches the resonance region and more detailed analyses are needed to understand the phase diagram of the system close to the unitary regime.

2.1.3 Unitarity

To determine the energetically favourable configuration in the unitary regime is a more difficult problem. It's still an open question whether the unpolarized superfluid and the polarized normal gas would co-exit like in the BEC regime or they would separate. For simplicity, we base our scenario on the phase separation between an unpolarized superfluid and a polarized normal gas. Although there's no formal proof that this is the right one, the results are in good agreement with the experiments. The energy of the unpolarized superfluid phase and the polarized normal gas can be calculated by methods like QMC and the critical polarization P_c is then found to be 0.39, which is shown as a red dot in Fig. 3.

2.1.4 Open questions

There're still several open questions for the phase diagram of a Fermi gas at $T = 0$. One question concerns the FFLO state: one can show that in the BCS limit, the FFLO state exists for $0 < P < 1.13\Delta_{gap}/E_F$. As P is increased from zero, first there is phase separation between the BCS and the FFLO superfluids and the two phases co-exist up to some intermediate value of P where the FFLO state occupies the entire volume. At $P = 1.13\Delta_{gap}/E_F$ the system finally becomes normal. But near unitarity, where $k_F|a|$ becomes large, the concept of Fermi surface loses its meaning and thus it remains a question whether the FFLO phase survives. And it's also a question whether it can be realized in trapped configurations.

2.2 Phase diagram of a polarized Fermi gas at finite temperature

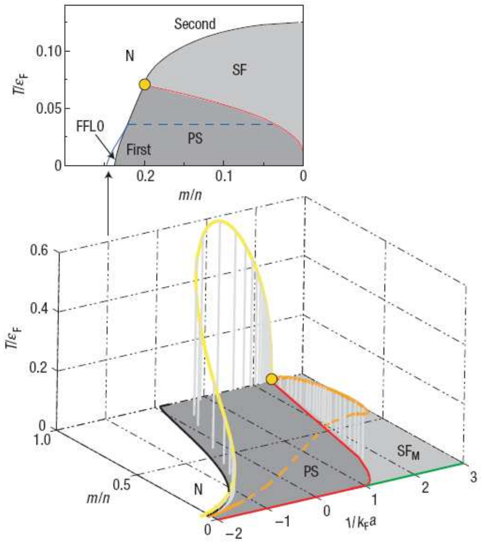

Figure 4: Finite-temperature phase diagram as a function of polarization m/n and interaction $1/k_F a$. [5]

The phase diagram at finite temperature is illustrated in Fig. 4 as a function of the polarization, the interaction strength and temperature. The yellow line represents the locus of tricritical points calculated in the mean-field approximation, whereas the orange tricritical line corresponds to mean-field theory plus pair fluctuations. The fluctuation correction breaks down in the unitarity regime $-1 < 1/k_F a < 1$, and is thus shown as a dotted line. The slice at $1/k_F a = -1$ is based on a mean-field calculation and it shows the region of phase separation terminating in a tricritical point (yellow circle) at finite temperature, followed by a second-order phase transition from the SF (superfluid) to N (normal) state. Note that the boundary between the FFLO and N states (blue line) defines a small region of FFLO phase confined to the BCS side of the crossover, as explained in the text. The presence of FFLO also divides the PS region into two different states: above the dashed blue line there is the usual mixture of SF and N phases, whereas below the line it consists of SF and FFLO phases. [5]

3 Experiment

The superfluidity in a polarized Fermi gas was first established in 2006 by M. W. Zwierlein et al., who observed vortices in rotating clouds. It was found that the superfluidity in the resonant region is extremely stable against population imbalance. And as the polarization increases, the quantum phase transition to the normal state was observed. [6]

The phenomena of phase separation has also been observed. [7, 8] In Fig. 5, it is shown that excess atoms in the majority spin state reside in a shell surrounding an inner core of unpolarized pairs. [7]

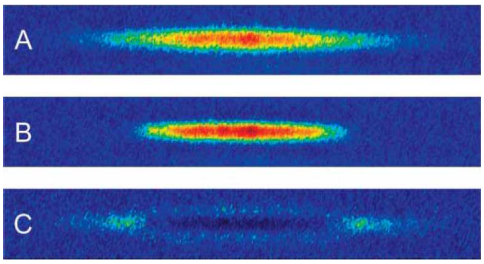

Figure 5: Absorption images showing phase separation near unitarity and $P = 0.14$. (**A**) Majority spin state, $|1\rangle$. (**B**) Minority spin state, $|2\rangle$. (**C**) Difference distribution, $|1\rangle - |2\rangle$, corresponding to the excess unparied $|1\rangle$ atoms. These excess atoms reside in a shell surrounding an inner core of unpolarized pairs. [7]

Recently, the phase diagram of a polarized Fermi gas with resonant interactions has been established by Shin et al. at MIT. [9] The tricritical point at which the critical lines for first-order and secodn-order phase transitions meet was identified. However, exotic superfluid states such as the breached-pair state in a stronger coupling regime and the FFLO state have not been observed. Fig. 6 shows the phase diagram that was obtained.

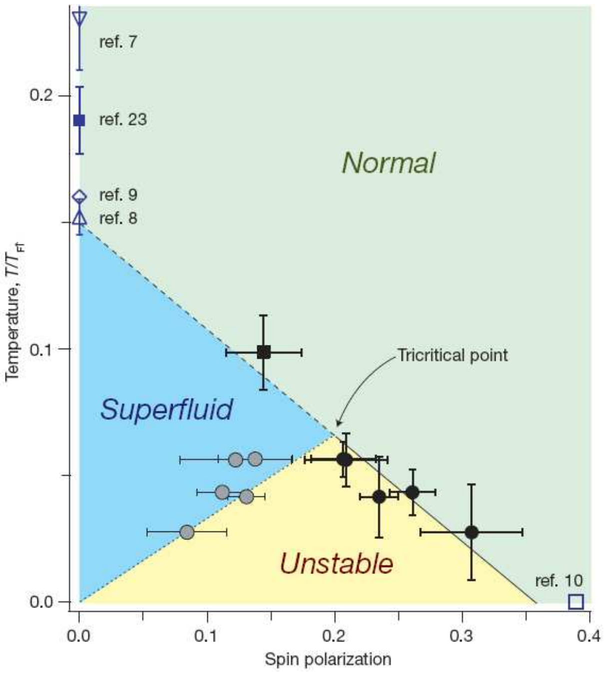

Figure 6: Phase diagram for a polarized Fermi gas with resonant interactions. The critical polarizations P_c (black solid circles and square) and P_s (grey solid circles) are displayed along the local $T/T_{F\uparrow}$ at the phase boundary. The yellow area ($P_s < P < P_c$) represents a thermodynamically unstable region, leading to the phase separation. [9]

4 Conclusion

The polarized Fermi gas provides a clean system to understand various novel quantum effects including exotic superfluid phase, BEC-BCS crossover et al. Many theorists and experimentalists have been drawn to this area and people are making progress towards a better understanding of the system. Still, there're several problems that haven't been solved, like a better theoretical description of the phase near unitarity, experimental observation of FFLO state, et al.

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