Two-Phase Diagram of Earthquake Distributions

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Abstract: This paper explains a model that shows that the conventional debate over whether earthquakes obey a Gutenberg-Richter Distribution or a Characteristic Distribution at a specific fault could be unfounded. The model, in the mean-field approximation, predicts a mode-switching phase and a Gutenberg-Richter phase that gives rise to the possibility of time-dependent seismic behavior. A qualitative phase diagram is obtained in parameter space and a second-order transition point is identified. Comparison with paleosiesmic data is made and discussed.

1 Introduction

Regardless of Voltaire's declaration that "Opinion has caused more trouble on this little earth than plagues or earthquakes," loss of life due to opinion cannot be stopped with the advent of innovative science. Earthquake prediction has remained elusive amongst scientists, but provides motivation for trying to understand earthquakes' underlying physical mechanism. In relation to this, in the mid-twentieth century, Gutenberg and Richter proposed their well-known empirical law to describe the number of observed earthquakes to their magnitudes,

$$\log_{10}(n(>M)) = a - bM, \tag{1}$$

where $n(>M)$ refers to the number of events with magnitude greater than M, a is a constant and b is also a constant that is approximately equal to 1. This kind of equation is referred to as a frequency-size distribution as it relates the number of earthquakes observed to their magnitude. Seismic patterns that obey this power law fall into the category labeled the Gutenberg-Richter Distribution. This implies that earthquakes of all sizes are seen between the largest magnitude earthquakes. There is another view, however, within the seismic community. The idea amongst the opposing school is that there exists a period of relaxation or quiescent behavior between the largest magnitude earthquakes. During the relaxation period, earthquakes do occur, but their magnitudes are relatively weak and limited to foreshocks, aftershocks and smaller background events. This model of seismic activity is referred to as the Characteristic Distribution. A diagram of the two distributions are shown below[8]:

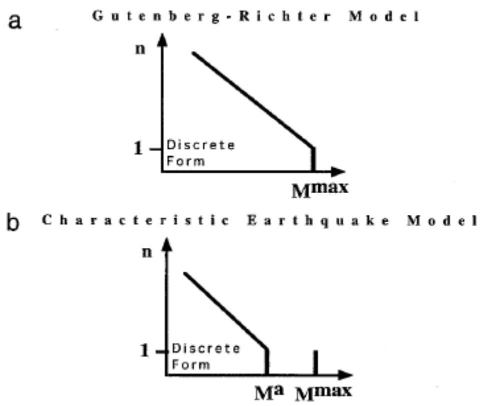

Figure 1.1: Log-Log Plot of Number of Earthquakes, n, versus Magnitude, M, for Gutenberg-Richter and Characteristic Distributions (M^a measures the magnitude of the largest aftershock)

Recently, data has been obtained from individual fault zones, suggesting that the aforementioned distributions are geometry-dependent[8]. It was found that faults with irregular geometries follow the Gutenberg-Richter Distribution up to the largest magnitude earthquakes, while fault zones with regular geometries obey the power-law behavior only for low-level earthquakes. These smaller events occur in periods of time between intervals of much larger magnitude earthquakes. There are bascially no observations of intermediate sized earthquakes at these faults.

In this paper, a theoretical model developed originally by Ben-Zion and Rice and improved upon by Dahmen *et al.* is examined[2][3]. The model permits faults that solely obey Gutenberg-Richter behavior, but these are stationary on time scales. The interesting results of the model yield a spontaneous switching back and forth from a Characteristic phase to Gutenberg-Richter phase and vice-versa[1]. This suggests that the question as to whether or not earthquakes obey the Gutenberg-Richter or Characteristic Distribution could be incorrectly posed. The question fails to address the role that time plays in a particular fault's seismic activity. The model seeks to unify observations of the Gutenberg-Richter Distribution and the Characteristic Distribution under one umbrella, as they are currently accounted for by separate theoretical frameworks[9]. This is accomplished by identifying a mode-switching phase and a Gutenberg-Richter phase for small earthquakes and identifying a second-order critical point. Because, according to the model, the time scale it takes to cycle between the two phases is on the order of thousands of years, the avaliable data is limited, but it does appear to corroborate the time-dependent predictions of the model. Most of the data obtained is from paleoseismic, historical and geological records from the Middle East, specifically Israel, and East Asia. The data seems to indicate that time does play an important role at some faults, as one can see periodic episodes of different distributions of earthquake activity. The model, comparison with data, and results are dicussed below.

2 The Model

To build the model, the heterogeneous fault zone is taken to be in the (x, z) plane and divided into N discrete cells. Thus, the N brittle patches are mapped onto the plane in between the two tectonic blocks that move with velocity $v\hat{\mathbf{x}}$ far away from the fault. The figure is below[9]:

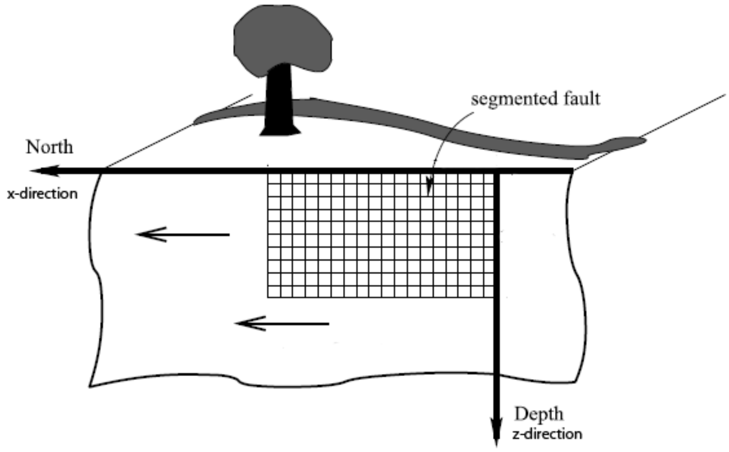

Figure 2.1: The Tectonic Plane is Divided into N Cells

The local stress on each cell is given by the following relation:

$$\tau_i = J/N \sum_j (u_j - u_i) + K_L(vt + u_i), \tag{2}$$

where u_j is the offset of cell j in the x-direction, J/N is the coupling due to elasticity and K_L is the effective loading stiffness of the bulk that surrounds the fault patch. Now, a mean-field approximation is made, and its validity is justified by the fact that elastic stesses are long-range for two dimensions and higher. Hence, we may write Eq.(2) as:

$$\tau_i = J\bar{u} + K_L vt - (K_L + J)u_i, \tag{3}$$

where $\bar{u} = (\sum_j u_j)/N$. The physical interpretation of this equation is as follows. In the intial state, the fault is relaxed. The external loading increases the stress on cell i, τ_i, linearly as a function of time. When the stress at cell i reaches a certain threshold level denoted $\tau_{s,i}$, called the local static failure threshold, the cell slips and the stress reduces to an arrest stress, $\tau_{a,i}$. In this model, arrest stresses vary from cell to cell and are randomly selected from a parabolic distribution. Similarly, the failure stresses are selected from a distribution as outlined in the next section. This is done to adequately represent the macroscopic spatial inhomogeneities of the fault system. Now, since the stress in a given cell drops from $\tau_{s,i}$ to $\tau_{a,i}$ during failure, the surrounding cells must absorb the released stress. We note the respective change in stresses by:

$$\delta\tau_i = \tau_{s,i} - \tau_{a,i} \tag{4}$$

$$\delta\tau_j = (c/N)(\tau_{s,i} - \tau_{a,i}) \qquad i \neq j, \tag{5}$$

where $c \equiv J/(K_L + J)$ defines the conservation parameter, which measures how much stress remains in the overall system after cell i slips. Eq.(4) gives the stress loss in cell i, whereas Eq.(5) gives the stress redistributed to the other cells. At this point, the model does not yet sufficiently take into consideration the weakening effects of the earthquake. In other words, when a cell fails, it is assumed to weaken due to the slipping process. Therefore, the failure threshold is reduced from its static failure $\tau_{s,i}$ threshold to a dynamical failure threshold, $\tau_{d,i} \equiv \tau_{s,i} - \epsilon(\tau_{s,i} - \tau_{a,i})$. The parameter ϵ is bounded by 0 and 1, i.e. $0 \leq \epsilon \leq 1$, and measures the importance of the dynamical weakening effects. It is immediately clear that the dynamical weakening has an affect only if a given cell is to fail more than once. In this model, an earthquake is observed if the stress transfer defined in Eq.(5) causes the neighboring cells to fail, and an avalanche effect is observed. This avalanche effect is observed until all the cells come to a value of $\tau_i \leq \tau_{s,i}$. This model also assumes that the external loading remains constant during the occurence of an earthquake, because the time scales of the earthquakes are short compared to the extrenal loading. Another assumption that goes into this picture is that between earthquakes, the cells are to heal causing the threshold for failure to return to the static value, $\tau_{s,i}$.

Now, to simplify the upcoming statistical analysis of the model, it is useful to introduce normalized stresses,

$$s_i \equiv 1 - \frac{\tau_{s,i} - \tau_i}{\langle \tau_{s,i} - \tau_{a,i} \rangle} \tag{6}$$

$$s_{a,i} \equiv 1 - \frac{\tau_{s,i} - \tau_{a,i}}{\langle \tau_{s,i} - \tau_{a,i} \rangle} \tag{7}$$

$$s_{d,i} \equiv 1 - \frac{\tau_{s,i} - \tau_{d,i}}{\langle \tau_{s,i} - \tau_{a,i} \rangle} = 1 - \epsilon(1 - s_{a,i}), \tag{8}$$

such that failure occurs when $s_i = 1$ and $\langle s_a \rangle = 0$. The angled brackets ($\langle\rangle$) indicates an average over all cells.

The statistical analysis undertaken in obtaining the stress distributions and earthquake sizes is rather involved, hence the results are summarized and discussed below.

2.1 Properties of the Gutenberg-Richter Phase

The Gutenberg-Richter phase is possible in both segments of the phase diagram in Figure 3.1. To obtain a distribution for the earthquake sizes in the Gutenberg-Richter phase, it is necessary to engage in statistical analysis. To account for the heterogeneities of the fault, an exponential distribution is used to compare the stress differences between cells, X_n, and the stress redistribution after failure of the first n cells, Y_n. It is important to note that the distribution of stress differences in the Gutenberg-Richter phase is $P(X_n) \propto \exp(-X_n)$, and it takes into regard the fact that each cell does not fail more than once during an event. Hence, the dynamical failure value does not play a role in obtaining the earthquake size distribution. The means and variances of these two distributions (P(X_n) and P(Y_n)) are combined (since they obey linearity) to give a mean, μ, and variance, σ^2, for $Z_n \equiv X_n - Y_n$. The physical interpretation of Z_n can best be understood through examining its behavior. For $X_n < Y_n$, the redistributed stresses are greater than the stress gaps, which only occurs during an earthquake. For $X_n > Y_n$, there redistributed stress is not enough to push the remaining cells "over the edge" to cause the avalanche effect leading to an earthquake. It can readily be seen therefore that the point $Z_n = 0$ is the point that determines whether or not an earthquake will occur. The problem thus reduces to a biased random walk problem where $Z_n = 0$ corresponds to the point of first return to the origin after a certain number of steps. The final form of the probability distribution of the earthquake sizes is given by:

$$p^{(GR)}(n) = \frac{A^{(GR)}}{n^{3/2}} \exp\left\{-\frac{n(1+n/N)}{n_{cGR}}\right\}, \tag{9}$$

where n_{cGR} is a cutoff size that is approximately given by $n_{cGR} \approx 2(1-c)^{-2}$ and $A^{(GR)}$ is a normalization constant. The cutoff size is obtained from the mean and variance of the biased random walk problem in the following way $n_{cGR} = 2(\mu^2+\sigma^2)/\mu^2$, which gives the expected dependence on the conservation parameter. This probability distribution gives the number of cells that will fail, n, during an event, i.e. the size of the earthquake. The cutoff size, n_{cGR}, diverges in the limit that $c \nearrow 1$.

There remain two important results that need to be discussed concerning the Gutenberg-Richter phase. One is the stress distribution which is given by:

$$p^{(GR)}(s) = \int_{-\infty}^{s} \frac{\rho(s_a)}{1-s_a}\, ds_a, \tag{10}$$

where $\rho(s_a)$ is the parabolic probability density of the arrest stress and the factor $ds_a/(1-s_a) = P(s \leq s_i \leq s+ds)$ comes from the fact that the stress in a given cell can lie anywhere between the arrest stress and the failure stress, i.e. $s_{a,i} \leq s \leq 1$. The other important result is the persistence time, which approximates the time it takes for the earthquake distribution to switch from the Gutenberg Richter phase to switch to the Characteristic phase. The relation is given by:

$$T_{(GR)\to(CE)} \approx T_0 \frac{C_{(GR),(CE)} N^{1/2}}{n_{c,(GR)}^{1/2}} \exp\left\{\frac{(1-\epsilon)(1-\epsilon+c)}{c^2 n_{c,(GR)}} N\right\}, \tag{11}$$

where $T_0 \equiv \langle \tau_{s,i} - \tau_{a,i} \rangle / (K_L v)$ is the average time it takes a cell to go from its arrest stress to its failure stress due to the external loading, and $C_{(GR),(CE)}$ is a factor that is on the order of unity. It is important to note the dependence of the mean persistence time on the conservation parameter. These results enable one to determine where the mode-switching occurs. In the Gutenberg-Richter phase, if one observes an earthquake in which the number of cells that fail is greater than $(1-\epsilon)N/c$, this may alter the frequency-size distribution to a Characteristic one. This is because the likelihood of all cells failing is dramatically increased as the initially failed cells reach their dynamical stress value. This means that a significantly large earthquake in the Gutenberg-Richter phase may lead to a transition into the Characteristic phase.

2.2 Properties of the Characteristic Phase

Since much of the statistical analysis has already been summarized in the previous subsection, and the method is the same for this phase, the main results are quoted and comparison with the Gutenberg-Richter phase discussed below.

For the Characteristic phase, one must take into consideration the effects of the dynamical stress value because the cells are no longer likely to fail only once as in the Gutenberg-Richter phase. Therefore the distribution of the stress gaps, $P(X_n)$, is no longer of the same form as in the Gutenberg-Richter phase. In the Characteristic phase, $P(X_n) \propto \exp(-X_n/(1-\epsilon))$. However, the distribution of the redistributed stresses after the failure of the first n cells, $P(Y_n)$, remains the same. Following the same procedure as in the previous part, one obtains for the distribution of earthquake sizes:

$$p^{(CE)}(n) = \frac{A^{(CE)}}{n^{3/2}} \exp\left\{-\frac{n(1+n/N)}{n_{cCE}}\right\}, \tag{12}$$

where the cutoff size is given by, $n_{cCE} \approx 2(1-\epsilon)^2/(1-\epsilon-c)^2$. The stress distribution is given by:

$$p^{(CE)}(s) = \frac{1}{1-\epsilon} \int_{(s-1+\epsilon)/\epsilon}^{s} \frac{\rho(s_a)}{1-s_a} \, ds_a. \tag{13}$$

Upon examining the size distribution, $p^{(CE)}(n)$, one can see that this diverges in the limit that $c \searrow (1-\epsilon)$. Unlike the cutoff counterpart in the Gutenberg-Richter phase, though, this divergence cannot be observed. This is because the Characteristic phase becomes vulnerable to a transition into the Gutenberg-Richter phase. When $c > c^* \equiv (1+\epsilon)^{-1}$, one can observe an instability from the Characteristic phase into the Gutenberg-Richter phase. For all, $c < c^*$, though, the Gutenberg-Richter phase is the only persistent phase. Upon examining the total redistributed stress per cell, S, one can immediately see why this is the case. It is given by:

$$S = \frac{c(r - (1 - \epsilon))}{1 - c}, \tag{14}$$

where $r = n/N$ is the fraction of failed cells. To ensure that the stress distribution retains the profile given by Eq.(13), it is required that $S \geq 1$. Therefore, when $r \geq r^* = 1/c - \epsilon$, the Characteristic phase stress distribution, Eq.(13), cannot be maintained, and the Characteristic Distribution is vulnerable to cross-over into the Gutenberg-Richter Distribution. The relationship between r^* and c^* is given by $1 - r^* = (c^*)^{-1} - (c)^{-1}$. This means that when a significant number of cells fail during background activity, the formerly bundled stress distributions will be caused to break apart. This will consequently lead to a transition into the Gutenberg-Richter phase.

The last important result is the persistence time, or the time predicted for cross-over behavior, $T_{(CE)\to(GR)}$, as this is a measurable quantity predicted by this model. For the transition from Characteristic phase to the Gutenberg-Richter phase, the persistence time is given by:

$$T_{(CE)\to(GR)} \approx T_0 \frac{C_{(CE),(GR)} N^{3/2}}{n^2_{c,(CE)}} \exp\left\{ \frac{(c - c^*)(1 + (c - c^*)/(c^* c))}{c^* c n_{c,(CE)}} N \right\}, \tag{15}$$

where it is assumed that $c > c^*$, because it is only in this regime that the persistence time is defined. Because the persistence time is dependent on the conservation parameter, a fixed conservation parameter of $c = 0.73$ was estimated to make theoretical predictions. With these values, the approximate transition times are given by, $T_{(GR)\to(CE)} \approx 5{,}000$ years and $T_{(CE)\to(GR)} \approx 20{,}000$ years. It is important to note, however, that small variations in the conservation parameter (≈ 0.04) lead to variations in the persistence times of approximately 1,000 years. This sensitivity is demonstrably a limitation of this model.

3 Identification of the Critical Point

For small earthquakes, as seen in Figure 1.1, there is always a power-law behavior. However, once earthquakes start getting larger, one can observe either a truncated power-law behavior or Characteristic sized earthquakes. In this model, these two phases are distinguished by the presence of the dynamical failure stress, which is accounted for by the parameter ϵ. As $\epsilon \to 0$, the Characteristic Distribution approaches the Gutenberg-Richter Distribution. This suggests a critical point at $\epsilon = 0$, $c = 1$, as this corresponds to the diverging point for n_{cGR}. In this limit, one has rare earthquakes of gigantic proportion, but we are guaranteed Gutenberg-Richter statistics. In the context of critical phenomena, as $\epsilon \to 0$ we have an absence of a characteristic length scale, as the statistics are scale invariant over all earthquake magnitudes. This is because the Gutenberg-Richter relation is given by a power-law, unlike the Characteristic Distribution, which is characteristic of a second-order critical point. Now, it was seen above that the stresses on the fault clearly affected the outcome of the frequency-size distribution. If one looks at effect of the spatially averaged stress $\langle\tau\rangle$, which can be tuned by tuning ϵ, on the criticality of the system, we can realize three qualitatively different states. When compared to some critical mean stress, τ_{crit}, we see that $\langle\tau\rangle > \tau_{crit}$, $\langle\tau\rangle < \tau_{crit}$ and $\langle\tau\rangle \approx \tau_{crit}$ yield distinct behaviors. $\langle\tau\rangle > \tau_{crit}$ gives a supercritical fault that is given by a high mean stress that frequently produces large earthquakes and recovers slowly to repeated produce large events. This describes the Characteristic Phase, but is vulnerable to the mode-switching mentioned above. $\langle\tau\rangle < \tau_{crit}$ gives a subcritical fault where earthquakes are generally quite small and the system is always far from the critical point. This is the Gutenberg-Richter Distribution for small earthquakes. $\langle\tau\rangle \approx \tau_{crit}$ gives the second-order critical point where Gutenberg-Richter statistics are obeyed over the entire range of the magnitude of the earthquakes. These understandings in conjuction with the results obtained from the previous section allows one to achieve the qualitative phase diagram for these transitions given in the figure below:

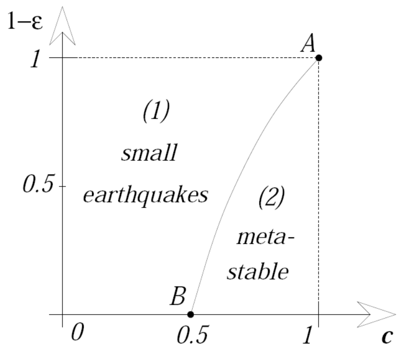

Figure 3.1: The Phase Diagram in Parameter Space. The point A on the diagram labels the critical point.

4 Comparison with Observations

While other data exists that suggest a time-dependence in earthquake activity, the data obtained by Marco *et al.*, Leonard *et al.*, and Kyung *et al.* span the largest periods of time and are therefore the most pertinent here [7][6][5]. The results obtained by Refs.[7] and [6] are taken in Israel around the Dead Sea Graben and the Arava Fault. We look first at the results obtained from the

Dead Sea Graben. The data gives a 50,000 year window into the seismic activity in this region. According to the results, each 5,000 - 10,000 years, there is a change in the seismic activity. Half of these periods are characterized by the presence of intermediate and large events, whereas the other half only exhibits earthquakes of relatively small size with very few larger events[7]. At the Arava Fault, one can see heavy faulting for a period of approximately 20,000 years, consistent with a Characteristic Distribution, with quiescent behavior on both time intervals surrounding the 20,000 segment. This corresponds approximately to 15,000 to 35,000 years ago. The data after 15,000 years ago corresponds more closely to Gutenberg-Richter behavior, although this last result is not definitive[6]. Figure 3.1 displays these results below[4]:

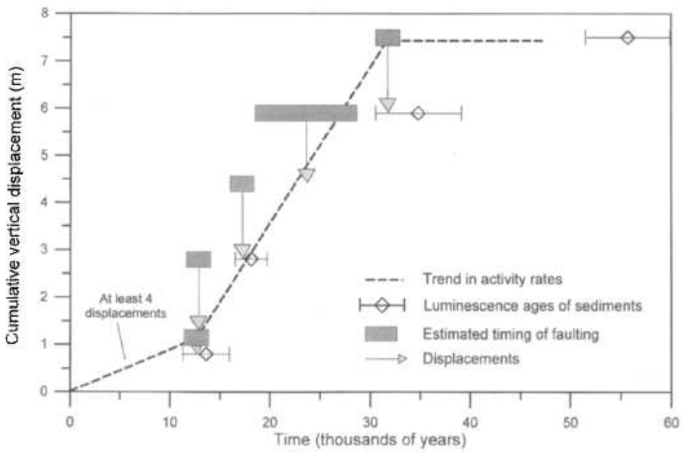

Figure 4.1: Plot of Arava Fault Activity Over 60,000 Years

Lastly, we look at the data presented by Kyung *et al*[5]. Their results take into consideration both volcanic and seismic activity, and they note that seismic and volcanic activity show very strong correlations. Hence, most volcanic eruptions only occur during periods of strong seismic activity. In this paper also, there is a notable time-dependence in the behavior of the seismicity. Although significantly shorter in time than the examined cases in Israel, there is a window of approximately 400 years where the volcanic and seismic activity was much more pronounced. Figure 3.2 summarizes their results below:

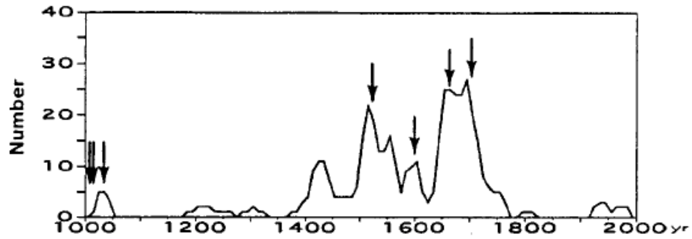

Figure 4.2: Number of Seismic and Volcanic Events versus Year (Arrows Indicate Volcanic Eruptions)

While the data is not conclusive evidence for the mode-switching model, there is clearly some time-dependent behavior in the seismic activity[5].

5 Conclusion

The model presented here predicts a transition from a Gutenberg-Richter phase to a phase that has a mode-switching behavior from Characteristic to Gutenberg-Richter behavior and vice-versa as earthquakes get larger in magnitude. This model also predicts a persistence time between for the mode switching. The comparison with the available geological data is difficult because of the lengthy persistence times, but there is clearly some time dependent behavior in the seismic activity, which did not previously enter the debate when discussing whether a fault obeyed a Gutenberg-Richter or Charactersitic Distribution. Even if future data serves to disprove the model put forward, the realization that time plays a role in a fault's seismic activity is a new stance in the exhaustive Gutenberg-Richter versus Characteritic Distribution debates. That being said, there is no definitively observed phase transition that can immediately validate this model. In fact, the second-order critical point can never be observed because it requires complete stress transfer, which is unphysical in nature. Another limitation in this model is that the arrest stresses and the static failure stresses were assumed to follow a statistical distribution and are therefore uncoupled. Depending on the size of the contact between two tectonic plates, however, this might not always be the case. Another assumption that went into the work is that the model was two-dimensional. For earthquakes above a magnitude of M6.3, however, it effectively becomes one-dimensional and the mean-field approximation is no longer valid. With those assumptions in mind, investigating this model's relevance should include an examination of the cell stresses and magnitude of the coupling constants at fault zones, since these parameters were phenomenological insertions into the theory. This will determine the conservation parameter more accurately and deem whether or not its use as a parameter on the phase diagram is warranted. Future work could involve the possibility of coexistence between the two phases. In the context of this model, this would correspond to a region of the cells obeying a Gutenberg-Richter behavior while another region of the cells are in the mode-switching phase. This would in effect lead to a frequency-size distribution that is some combination of the two.

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