Geomagnetic reversal through critical models

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Abstract

Geomagnetic polarity reversal intervals display power law distribution functions, which indicate a critical phenomenon as the mechanism of their source. The geodynamo is assumed to be a system of magnetic spins in a critical phase-transition state. Seki and Ito developed two models to simulate these reversals: the N-disc coupled model and the Coupled map lattice model both derived from the Rikitake Dynamo system and are both reviewed here. The simulations were done in two dimensions in a square lattice with periodic boundary conditions. The major difficulty is simulation times, which hinders a full three dimensional model.

Introduction

The earth's magnetic field generates the magnetosphere which shields the earth from the stream of charged particles in solar wind. The geomagnetic field has reversed repeatedly during the Earth's history. The duration of reversal (10^3 years) is much less relative to the duration of constant polarity (10^5 - 10^6 years). The last reversal that occurred was around 700,000 years ago. Geophysicists are able to study these reversals by measuring the magnetic direction recorded in minerals in rock sediments and are able to track reversals several hundred million years ago. The time interval of the reversal is highly irregular with intervals of 10^7 years though the number of intervals decrease greatly as the interval length increases and The average interval between polarity reversals is about 7×10^5 years.

The mechanism behind the reversal and the wide variation in reversal interval is not completely understood but is believed to be associated with the MHD of the outer core. Dynamo theory studies how the motion of an MHD fluid can give rise to the generation of a macroscopic magnetic field. However, the fluid motion in the outer core is turbulent and forms eddies which makes it very difficult to model the dynamo using three-dimensional hydrodynamic equations and too time consuming to evolve the system over a time scale comparable to a reversal interval.

The simplest model that resembles a geodynamo was that proposed by T. Rikitake in 1958 that models the geodynamo a two disc coupled dynamo system. The model consists of two identical single Faraday-disc dynamo coupled together (figure 1). The current in the dynamo coils (x, y) are related to the angular velocities of the dynamo discs (z, z') given by the differential equations

$$\dot{x} = -\mu x + zy$$

$$\dot{y} = -\mu y + z'x$$

$$\dot{z} = \dot{z}' = 1 - xy$$

$$z' = z + \alpha$$

The parameter μ represents the resistive dissipation and α is the difference between the angular velocities of the two dynamo discs.

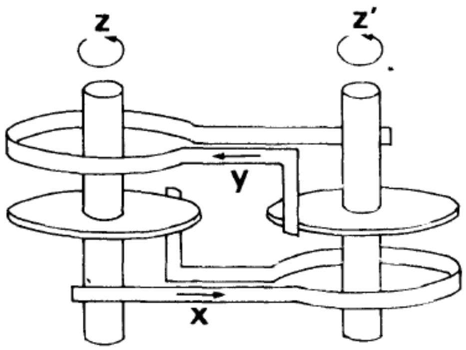

Figure 1: two disc-dynamo: The discs are coupled in the following way. Disc 2 rotates in the magnetic field caused by the current in disc 1 and vice versa.

The Rikitake system has two equilibrium points $(\pm K, \pm 1/K, \mu K^2)$ in the phase x-y-z phase space. $K$is given by

$$-\alpha = \mu(K^2 - K^{-2})$$

The Rikitake system is extensively used by physicists to model the geodynamo due to its simplicity. However, the Rikitake model exhibits chaos of the Lorenz type characterized by irregular flipping between two unstable fixed points and has been investigated from a dynamical point of view to the richness of the behavior presented by its solutions

Power law distribution

The challenge of studying geomagnetic reversals was determining the cumulative distribution of the polarity reversal time interval. Naidu showed that a gamma distributions was good fit for the observed time intervals over the last 65 m. years. Gaffin analyzed the geomagnetic polarity reversal record for the last 165 m. years and found that the cumulative distribution of the intervals follow a power law but concluded that the Poisson distribution provides a better description of the distribution of interval length. Ito noted that the scaling in the power law distribution was indicative of a critical phenomenon.

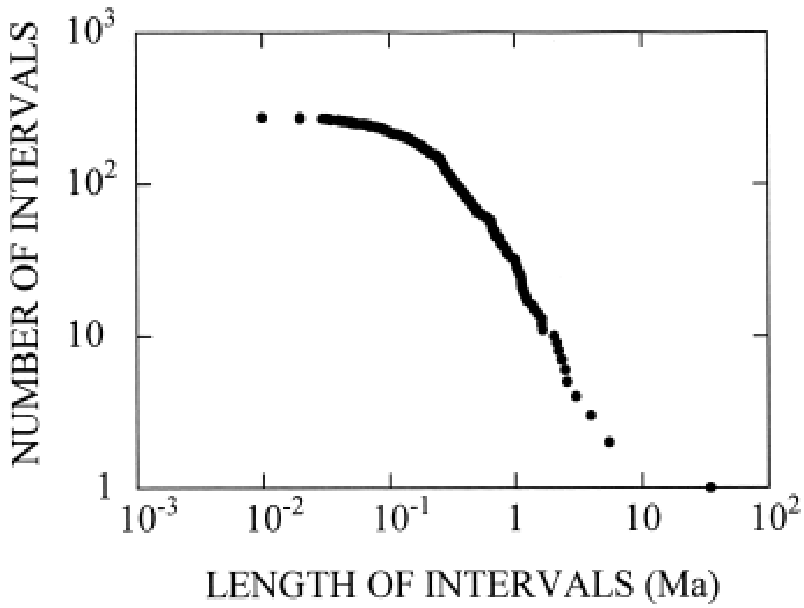

Figure 2: log-log plot of the cumulative distribution of polarity reversal length for the past 165 m. years.

The power law distribution has the form

$$P(x) \propto x^{-\alpha-1} \quad (0 < \alpha < 2)$$

From figure 2, the slope of distribution is -1.5 so α =1.5. The n-order moment is given by

$$\langle x^n \rangle = \int_{-\infty}^{\infty} P(x) x^n \, dx$$

The n-order moment is infinite when n is larger that α. Since α =1.5, the variance in divergent. variance should approach infinity as the time series become large while the mean value is expected to fluctuate strongly. This indicates that the process is statistically non stationary but dynamically stationary, occurring in a single state.

N-disc dynamo model

the two disc Rikitake dynamo was studied in detail and was found that the model predicts a cyclic variation of the dipole during stable polarity periods. The N-disc dynamo improved this weak point and exhibited chaotic behavior and a statistical nature of polarity reversals similar to that for the earth's magnetic field. However, solving the N-coupled disc dynamo model near the critical phase transition requires a large computing time to reach a steady state due to the critical slowing-down effect.

The model proposed by Ito and Seki uses a stochastic N-disc dynamo to approximate the turbulence in the outer core. In addition, the turbulent eddies are approximated by magnetic spins. The nearest neighbor interactions are similar as in the Ising model and extend over four nearest neighbors. The energy is given by

$$E = -\sum_{i,j} S_i \, S_j$$

Q2R cellular automata was used in the simulations. In Q2R, all of the spins are flipped simultaneously while in the canonical Ising model, only one randomly selected spin is flipped at a time.

To conserve energy, a particular spin is flipped if and only if it has the same number of up and down neighbors. The system is modeled in two dimensions one a 64×64 square lattice using periodic boundary conditions. As the number of spin ups is increased, the total energy if the system increases and the fluctuations in magnetization become large and more rapid and polarity reverses (figure 3).

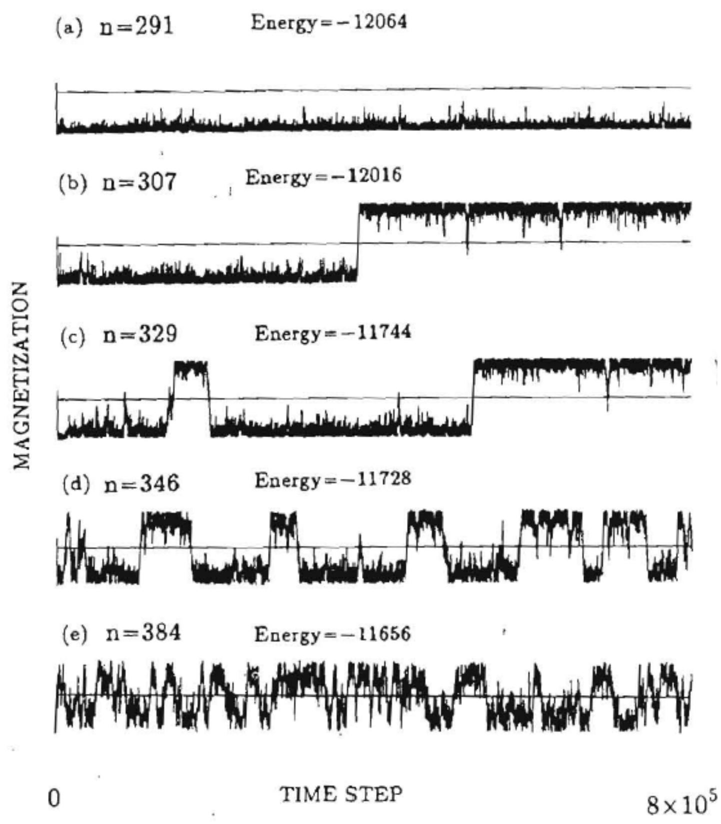

Figure 3: the fluctuation in magnetization for 8×10^5 time steps.

As energy is increased further, polarity reverses frequently and intervals of polarity reversal become short. The interval of polarity reversal is measured and integrated distribution is obtained. This is evident in figure 4, as the number of spins is increased the flat part (4a and 4b) becomes shorter and the distribution becomes linear at large time intervals (4c and 4d). The phase transition is evident when the polarity reversal intervals obey a power law distribution and thus figure 4c is a transition state. However, the power exponent obtained through the model is -0.5 while it is -1.5 for the geomagnetic data. The reason for the difference is that the model is too simplified although it still displayed a phase transition state. It should be noted that the power law distribution in bears a strong resemblance to the "1/f" noise spectrum associated with self organized criticality (SOC). Particularly, the coupled dynamo is similar to an SOC feedback mechanism.

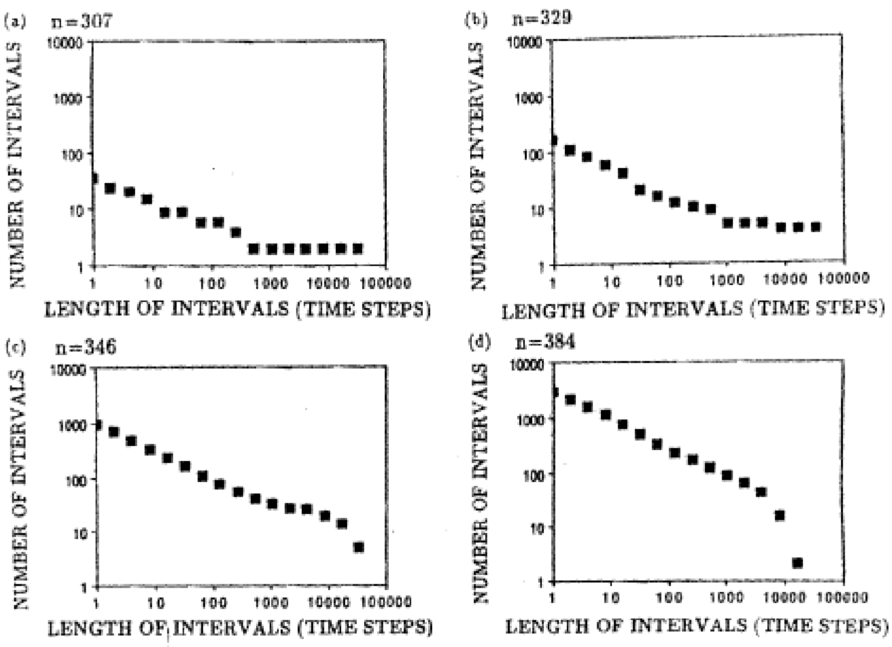

Figure 4: the distribution of polarity reversal intervals for 6×10^5 time steps for different number of up spins **a)** n=307 **b)** n=329 **c)** n=346 **d)** n=384

CML Model:

CML was first introduced by Kaneko in 1984 to systematically investigate high-dimensional, chaotic systems within a simple framework. It displays various complex collective behaviors and thus can be used as a model for problems such as turbulence, pattern formation and phase transitions in spatial structures. The model is described by a lattice where each site evolves in time through a recurrence equation of the form

$$X^{(t+1)} = F(X^t)$$

and time is discretized. In the case of polarity reversal, the Lorenz map of the Rikitake Dynamo system (figure 5 a) was discretized into a piecewise linear form (figure 5 b). The generic recursion relation becomes

$$S_{t+1} = \gamma(a, b)\, S_t + \delta(a, b)$$

Where γ and δ vary depending on the interval. The parameter a controls the condition of the laminar phase of motion (slope in the Lorenz map) and parameter b controls the width of the domain for each S_t and S_{t+1}. A discontinuity indicates a chaotic behavior, thus b controls the width of a chaotic burst. It should be noted that time and space are discrete while the state is continuous.

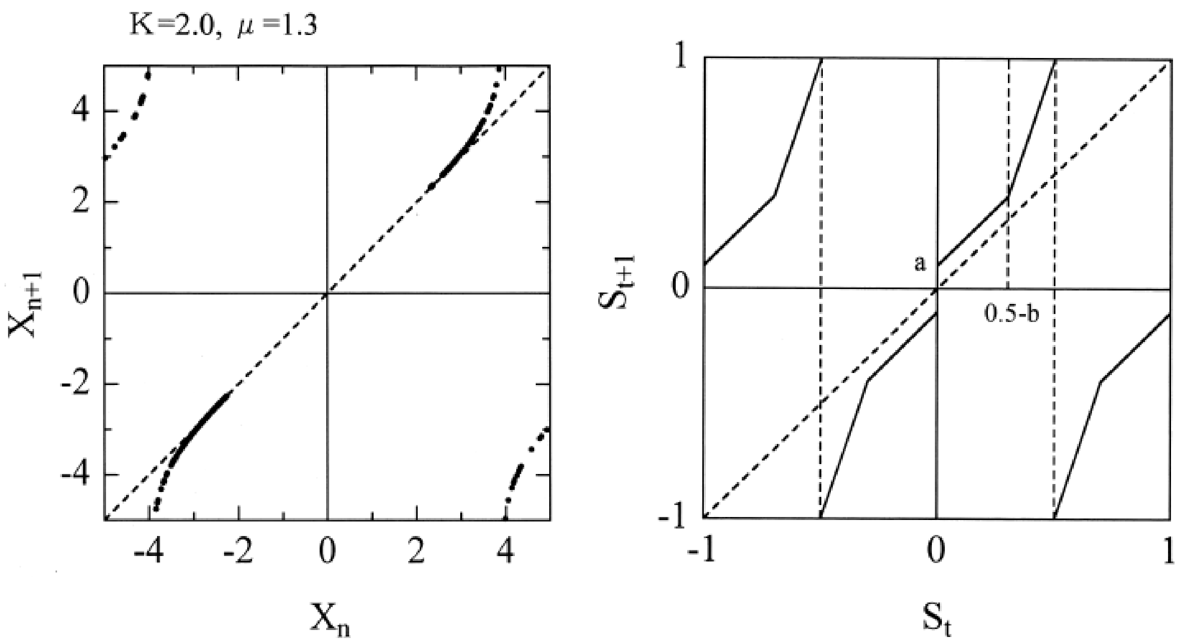

Figure 5: **a)** Lorenz map of the Rikitake Dynamo system for K=2.0 and μ=1.3. Xn represents the *n*-th local maximum of X_n and X_{n+1} is the *n+1*-th local maximum. **b)** Discretized Lorenz map in (a) used in CML model.

A two dimensional lattice with nearest neighbor interactions was simulated and the interaction was done over four neasrest neighbors with interaction intensity j. The simulation was done using 32×32 square lattice system with periodic boundary conditions. The system was evolved for 10^6 time steps. The parameters a, b and j were repeatedly varied and the polarity reversal distribution of CML model was compared to the geomagnetic data.

A larger j indicates a larger interaction between turbulent spins and a more ordered system. The system thus exhibits supercritical behavior (< T_c) and the power exponent is small. A larger b indicates more chaotic behavior and thus more frequent the polarity reversals. In addition, the power exponent becomes larger for larger b. The parameters b and j were varied until the simulated distribution polarity reversal coincided with the geomagnetic data in figure 2 with a power exponent of -1.5. The optimal values obtained for j and b were 0.05-1 and 0.4-0.45 respectively. The large value obtained for b and relatively small value for j indicates that each element is in a strongly chaotic state, which may suggest that the geodynamo is turbulent and is in a strongly chaotic state.

Finally, when j and b are fixed, a is varied (figure 6) and polarity distribution is examined again. For large a, the distribution is too steep, indicative of the CML system acting chaotically. For smaller a, the distribution appears similar to the geomagnetic result in figure 1. The change in shape of graph indicates a phase transition that is determined by a.

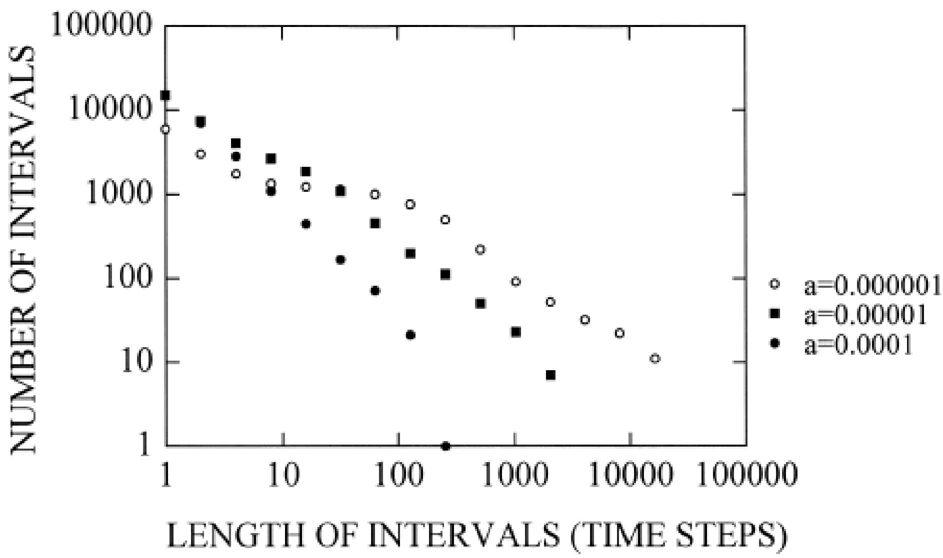

Figure 6: the distribution of the polarity reversal intervals for different values of a (b =0.4, j= 0.1)

The advantage of the CML model is that it requires much less computation time than the coupled disc dynamo model. In the CML model, spins do not have discrete values and can freely reverse their polarity. This is a major variation to the previous model since spins in the previous model only reversed polarity through interacting with neighboring spins. Thus autonomous reversal of polarity in CML is similar to the geomagnetic data where short term polarity reversals are more frequent.

Conclusion:

Though a full and accurate model of the geodynamo is a long time away, the use of critical phenomena in explaining the geomagnetic polarity reversal has shed new light on the dynamics of the process. Seki and Ito interpreted the power law distribution as evidence of a dynamical phase transition state in the geodynamo. The N-disc dynamo model based on the Rikitake Dynamo yielded a power law distribution similar to the geomagnetic results but with different exponents. The model was later improved using a coupled map lattice model instead which yielded an exponent similar to the geomagnetic results but only when the parameters are varied such that each element is in a strongly chaotic state. Future models might include a better alternative to the Rikitake system and include variables such as eddy turbulences and tectonics activity.

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