

PHYS 563: Term paper
Nonlinear magnetization and the “vortex liquid”
state above T_c in cuprate superconductors

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Abstract

This essay describes the observations of the magnetization M above the critical temperature T_c in cuprates. The nonlinear behavior of M suggests the existence of a “vortex liquid” region above T_c , which is a region of a strong superconducting fluctuations. This conclusion is consistent with the Nernst signal and other experimental data. At least in two families of cuprates, the low field magnetization exhibits non-analytic divergent behavior $M \sim H^{1/\delta}$, $\delta > 1$. The possible explanations are based on the Kosterlitz-Thouless (KT) transition and incompressibility of vortex liquid are considered in this paper.

Introduction The phenomenon of superconductivity is truly one of the most important discoveries of the XX century. It took about half of the last century to discover the microscopic theory of superconductivity and it seems that the mystery of high-temperature superconductors might also remain unsolved for a long time. Even though it is two decades after the discovery of cuprates, there is still debate about the origin of the highly anomalous “normal” state above the superconducting transition temperature. The main questions are: i) what is the extent of the temperature and magnetic field regime in which superconducting fluctuations are observed; ii) how to characterize the regime of superconducting fluctuation theoretically. It is worth noting that the region of strong superconducting correlation is also called “vortex liquid”. The reason that these problems are difficult is that other types of order parameters fluctuations are also significant, thus it is highly complex to disentangle the contribution from the different kinds of fluctuations.

The evidence of incipient order above T_c and of a single-particle gap that persists into the normal state was obtained from STM and ARPES measurements [2]. However

it is quite difficult to tell the difference between a superconducting gap and a density-wave gap.

Numerous experiments that making use of the Nernst effect have demonstrated an enhanced Nernst signal in hole-doped cuprates at temperature T significantly above the superconducting transition temperature. The Nernst effect is a thermoelectric phenomenon in which a voltage transverse to a temperature gradient is created in a conducting sample subjected to a magnetic field. The region of the enhanced Nernst signal for $T > T_c$ is supposed to be a continuous extension of the vortex-liquid state below T_c . That is why this region is referred as “vortex liquid”. In this state the enhanced Nernst signal originates from phase slippage caused by singular phase fluctuations of the pair condensate [1]. According to the phase-disordering scenario [6], the disappearance of the Meissner effect at T_c is caused by the loss of long-range phase coherence, rather than the vanishing of the pair condensate. However, the persistent short-range phase stiffness still supports vorticity and produces a large, strongly temperature dependent Nernst signal in some region above T_c . The Nernst signal, unfortunately, is highly sensitive not only to superconducting fluctuations, but also to any type of order that leads to a reconstruction of Fermi surface. That is why there are other plausible interpretation than the fluctuating vortex-liquid interpretation [1].

The method that is not subject to the above mentioned shortcomings is the measurement of the magnetization. Even if the superconducting correlation length is finite, the fluctuation diamagnetism can be large in comparison to the “background”, because the diamagnetic response of an ordered superconductor is many orders of magnitude stronger than that of any other known state of matter. To top it all, as opposed to the Nernst effect, the magnetization is a thermodynamic quality, thus it is not subject to the uncertainties in interpretation that are inherent to dynamical and nonequilibrium properties.

Li *et al.*[1] reported the results of a major experimental study of the magnetization of several important families of cuprate high-temperature superconductors over a broad range of temperatures and magnetic fields in their recent paper. They found that for a wide range of temperatures above T_c there is a strong, nonlinear in H , diamagnetic response. From the data, they extracted the field-dependent onset temperature T_M , below which the diamagnetic response was observed. The nonlinearity and the high magnitude of the diamagnetic response is strongly suggestive about its superconducting nature. To top it all the onset temperature T_M of the diamagnetic response demonstrate a strong correlation with the onset temperature of the Nernst signal and thus cleans up an ambiguity in the interpretation of the Nernst effect experiments. Li *et al.* also found an interesting feature of the low field magnetization in this and one of their previous papers [3]. It exhibits a non-analytic behavior, in some

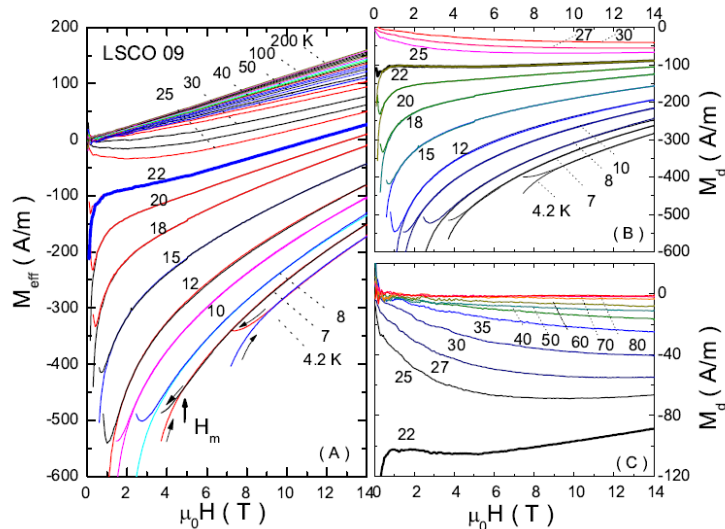


Figure 1: Figure from Li *et al.* [1]. Magnetization curves of sample LSCO 09 with Sr content $x = 0.09$ and transition temperature $T_c = 24K$ measured in magnetic fields H up to 14 T. (A) The (total) effective magnetization M_{eff} vs. H at temperatures $4.2 \leq T \leq 200K$. (B) The diamagnetic magnetization M_d vs. H at temperatures $4.2 \leq T \leq 30K$. (C) Curves of M_d vs. H at $22K \leq T \leq 80K$ displayed in expanded scale. In LSCO 09, the diamagnetic signal persists to more than 60 K above T_c . In (B) and (C), the bold curve is measured at the separatrix temperature $T_s = 22$ K.

range of temperatures above T_c that leads to a divergent susceptibility.

In this paper, we, at first, consider the experiment of Li *et al.* and its implications. Second, we discuss the divergent behavior of the magnetic susceptibility in the low fields and two possible explanations of it. One is based on the Kosterlitz-Thouless transition in a layered superconductor with zero Josephson coupling between planes [5]. The other was proposed by P.Anderson [4] and it treats the vortex liquid as an "incompressible superfluid". After that we discuss the results and suggest how the phase diagram of a cuprate superconductor should look like.

Torque magnetometry experiment. Nonlinear diamagnetic response above T_c and its implications [1, 3] The magnetization is measured by making use of torque magnetometry technique. In this method the sample is glued to the tip of a thin cantilever, with H applied at a small angle to the crystal c -axis. Because of the 2D electronic dispersion in cuprates, the diamagnetic currents are largely confined to the

a - b plane, which makes torque magnetometry a very sensitive probe of diamagnetism in cuprates. The deflection of the cantilever gives a torque signal which corresponds to the effective magnetization M_{eff} .

In order to show the analysis of the data, lets first examine the dependence of $M_{eff}(H)$ for the sample of $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ with $x = 0.09$ (LSCO 09), shown on Fig.1. For this sample $T_c = 24\text{K}$. One can note that at temperature $T > 100\text{K}$, M_{eff} is strictly linear in H and paramagnetic in sign (see panel A). This behavior corresponds to the dominance of the anisotropic Van Vleck paramagnetic susceptibility $\Delta\chi_p$, which has a weak T dependence given by $\Delta\chi_p = A + BT$, with $A \gg BT > 0$ at 200 K. Below 100 K, M_{eff} begins to display a weak diamagnetic contribution. The temperature at which the diamagnetic contribution appears is identified as the onset temperature T_{onset} . M_{eff} becomes increasingly nonlinear in H as T decreases from 60K to T_c (24 K). Below T_c , the diamagnetic term becomes so dominant in magnitude that M_{eff} has large negative values, in spite of the paramagnetic Van Vleck term. Li *et al.* assume that the paramagnetic background $\Delta\chi_p H$ follows the trend that is seen at $T > T_{onset}$ and consequently the diamagnetic term M_d is related to M_{eff} by

$$M_{eff}(H) = M_d(H) + \Delta\chi_p H = M_d(H) + (A + BT)H.$$

Hereafter, we consider mostly $M_d(H)$, subtracting the background Van Vleck term from M_{eff} .

Now we further inspect $M_d(H)$ for LSCO 09, which are displayed at selected T in Fig. 1 B (4.2-30 K) and Fig. 1 C (20-80 K). As shown in Panel B, M_d is nonlinear in H over a broad range of temperature. The curve at 22 K has a flat profile and corresponds to a "separatrix" temperature T_s . Below T_s , M_d takes on very large, negative values at small H . It is noteworthy that the low temperature curvature of M_d vs. H curves changes from negative below T_s to positive above T_s . Panel C displays M_d vs. H curves of LSCO 09 sample at $T \geq T_s$ in expanded scale. The curves remain diamagnetic, displaying strong nonlinearity. Such a nonlinear diamagnetic response above T_c suggests the presence of local supercurrents as well as finite pair amplitude in the pseudogap state.

In order to check, if the diamagnetic signals above T_c can be suppressed by an intense magnetic field, Li *et al.* make extended measurements on LSCO 09 up to 33 T. In this case, M_d - H curves display a broad minimum. For higher H , M_d tends to zero as H is increased, which is a sign of superconducting fluctuation. It should be noticed that the minimum the M_d - H curve increases rapidly with T (from 8 T at 25 K to 33 T at 40 K).

YBCO. Optimally-doped YBCO ($\text{YBa}_2\text{Cu}_3\text{O}_7$ with $T_c = 92\text{K}$) is distinguished as the cuprate with the smallest resistivity anisotropy and the largest interlayer (c-coupling)

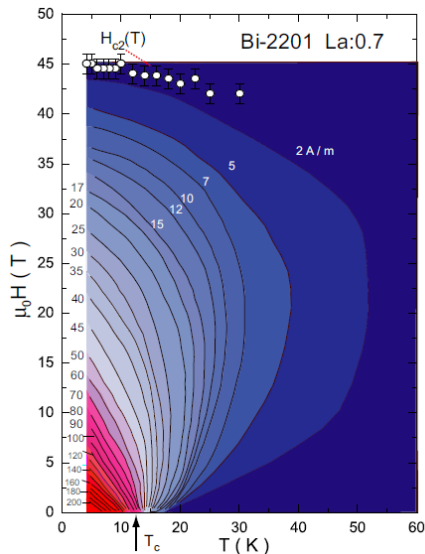


Figure 2: Figure from Li *et al.* [1]. Contour plot of the diamagnetic magnetization $|M_d(T, H)|$ of UD Bi 2201, with La content $y = 0.7$ and $T_c = 12K$ (arrow). The spacing between adjacent contour lines is 10 A/m for $T < T_c$. The upper critical field H_{c2} (defined by extrapolating $M_d \rightarrow 0$) is plotted as open circles.

energy. Because the coherence-length anisotropy $\xi_a/\xi_b = 3 - 5$ is only moderate, the vortices have the largest stiffness modulus along \mathbf{c} among cuprates. Here ξ_a and ξ_b are the coherence length along the axes \mathbf{a} and \mathbf{c} , respectively. Optimally-doped YBCO should be the least susceptible to the phase disordering mechanism for the distraction of long-range phase coherence at T_c and hence the best candidate for Gaussian fluctuations among cuprates [1].

Nevertheless, the experiments suggest that T_c in optimally-doped YBCO is also dictated by large phase fluctuation. The M_d - H curves are similar to those in LSCO 09. Only the magnitude of M_d is different at comparable H and T . The curves displaying significant diamagnetism surviving to intense fields at temperatures up to 40 K above T_c , which is a strong evidence that we are observing the phase-disordering mechanism, rather than Gaussian mean-field fluctuations.

Bi 2201, Bi2212. The nonlinear diamagnetic signals above T_c are also observed in a single-layer $\text{Bi}_2\text{Sr}_{2-y}\text{La}_y\text{CuO}_6$ (Bi 2201) family. In this family, the transition temperature T_c depends on the La content y . The optimal doping (OD) corresponds to $y \sim 0.44$. Specimens with $y > 0.44$ are underdoped (UD), while those with $y < 0.44$ are overdoped (OD). The measurements were made in OD Bi 2201 samples and in

both OD and UD regions. Above T_c , the M_d - H curves in Bi 2201 are also similar to those in LSCO (Fig.1) and YBCO samples, except that the magnitude of M_d and field scales are slightly smaller than for LSCO. Above T_c , M_d attains a broad minimum at fields below 20 T, and then approaches zero at $H \geq 40T$. Like the curves for LSCO and YBCO, the the low-field M_d - H curves change a curvature sign across T_c . In order to suppress M_d we need to apply about 20 T. Above T_c , the complete suppression of M_d requires very high fields - comparable to those needed below T_c . An interesting weak field region is discussed under “fragile London rigidity”

Contour plot. An excellent way to view the nonlinear diamagnetic magnetization is the contour plot of M_d in the $T-H$ plane. Fig.2 displays the contour plot in single-layered UD Bi 2201 ($y = 0.7, T_c = 12K$). The value of $|M_d|$ is as indicated at selected contours. With H fixed (e.g at 10 T), $|M_d|$ decreases monotonically as T is raised from 4 K to 60 K. Just as in the Nernst signal, the diamagnetic signal in the T-H plane bulges out to temperatures high above T_c , with no obvious discontinuities or changes in slope. The highest temperature at which M_d is resolved is $\sim 50K$ (the onset temperature in this sample). The absence of a boundary implies that the vortex-liquid state below T_c evolves continuously to the diamagnetic state above T_c .

Onset temperature. An important question is how high in temperature does the diamagnetic signal extend above T_c . We plot M_{eff} measured in fixed H (14 T) versus T . Fig. 3(a) displays these plot for samples of Bi 2201 and LSCO in panels A and B, respectively. The high temperature part of the dependences may be fitted to the Van Vleck anisotropy term $\Delta\chi_p = A + BT$ (a straight line). We can determine T_{onset} quite accurately, provided a set of points above T_{onset} is sufficiently dense. In optimally doped samples of each family, T_{onset} extends above T_c by factors of 1.3 (YBCO), 1.4 (Bi 2212), 2.1 (LSCO) and 2.5 (Bi 2201). This is in dramatic contrast to the fluctuating diamagnetism which is observed in disordered low- T_c superconductors. For instance, the sample of disordered $Mg_{1-x}B_2Al_x$ ($x = 0.25, T_c = 25K$) has a broad transition width of $\sim 15K$ and sizeable diamagnetism exists in the narrow interval 28 - 32 K above its T_c . But even though the profile of M_d vs. H is roughly similar, the factor $T_{onset}/T_c < 1.3$.

In order to compare T_{onset}^M obtained here with the onset temperature of the vortex Nernst signal T_{onset}^ν , we plot the onset temperatures vs. doping x in the phase diagram for LSCO and Bi 2201 (Fig.3(b)). Notably, in LSCO, T_{onset}^M (open squares) is almost equal to T_{onset}^ν over the entire doping range. In Bi 2201 the temperature scales are also the same. The difference in the slopes might be explained by the fact that the measurements were made on different samples.

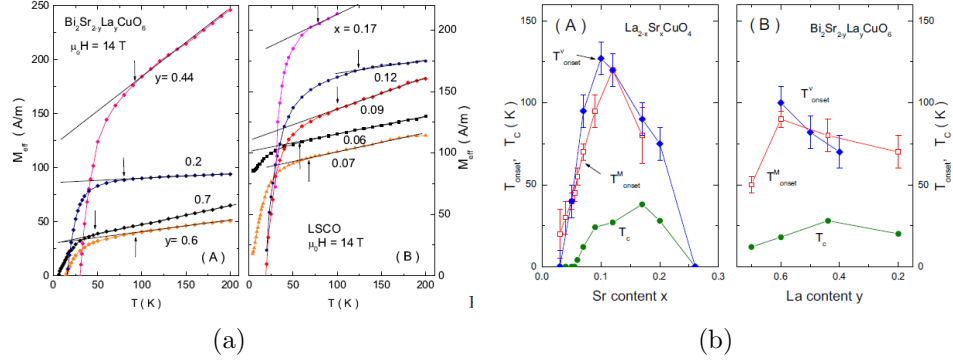


Figure 3: Figure from Li *et al.* [1].(a) Plots of the temperature dependence of $M_{eff}(T)$ in Bi 2201 (Panel A) and in LSCO (B), showing the onset of diamagnetism as T is decreased. In both panels, the value of M_{eff} measured at $H = 14$ T is plotted vs. T in samples with various doping levels x . In general, M_{eff} at high T varies weakly vs. T , as shown by the straight lines which are of the form $A + BT$. Relative to this linear background, M_{eff} shows a strong downwards deviation starting at the onset temperature T_{onset}^M (indicated by arrows). (b) Phase diagram comparing the onset temperatures for the Nernst and diamagnetism signals vs. doping x in $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ (Panel A) and in $\text{Bi}_2\text{Sr}_{2-y}\text{La}_y\text{CuO}_6$ (Panel B). The superconducting transition temperature T_c (solid circles) is plotted with the onset temperature T_{onset}^{ν} determined by the Nernst effect (solid diamonds), and T_{onset}^M determined by torque magnetometry (open squares). In Panel B for $\text{Bi}_2\text{Sr}_{2-y}\text{La}_y\text{CuO}_6$, a large La content y implies small hole carrier concentration (UD regime).

Fragile London rigidity. One of the most interesting features of the vortex liquid state above T_c is the fragile London rigidity, observable in the limit $H \rightarrow 0$. Lets investigate closely how M_d approaches zero in OPT Bi 2201 Fig.1 (Panel C). The M_d curves display a strong curvature as H approaches zero from either direction. As T decreases from 38 K to T_c (30 K), the zero- H slope rises sharply to a vertical line. The curve at T_c (25 K is the closest one to $T_c = 25\text{K}$ on this plot) seems to approach a logarithmic dependence vs. H . However mechanical noise precludes accurate measurements for $|H| < 300\text{Oe}$, thus precludes to investigate this feature in this particular experiment.

In one of his earlier experiments [3], Li *et al.* used high-resolution SQUID magnetometry to extend measurements in Bi 2212 down to 10 Oe. They discovered that over a broad interval of T (86-105 K) in OPT Bi 2212, the low- H M_d follows the power-law

dependence

$$M_d(T, H) \sim -H^{1/\delta(T)} \quad (H \rightarrow 0),$$

with an exponent $\delta(T)$ that grows rapidly from 1 (at $T \simeq 105K$) to large scales (> 6) as $T \rightarrow T_c^+$. This implies that the weak-field diamagnetic susceptibility

$$\chi = \lim_{H \rightarrow 0} M/H \rightarrow -\infty$$

is weakly divergent throughout the interval in T where $\delta > 1$. However, this divergence is extremely sensitive to field suppression. London rigidity seems to reflect the increasing tendency of the phase-disordered condensate to establish long-range superfluid response as $T \rightarrow T_c^+$. This feature has no analog in bulk samples of low- T_c superconductors, but may exist in a finite T interval above the Kosterlitz-Thouless(KT) transition in 2D systems such as $\text{Mo}_{1-x}\text{Ge}_x$ and InO_x .

The Kosterlitz-Thouless RG calculation of the magnetization[5]. In paper [5] Oganesyan *et al.* report an RG calculation of the magnetization of a two-dimensional superconductor in a perpendicular magnetic field near its Kosterlitz-Thouless transition and at lower temperatures.

To compute the free-energy and thence magnetization M they map the two-dimensional vortex problem onto a two-component Coulomb plasma whose Hamiltonian is given by

$$H = N_T E_{c0} + \pi \rho_{s0} \sum_{i < j} q_i q_j \ln r_{ij}^2 / a_0^2 + H_B. \quad (1)$$

The number of vortices of charge $q_i = \pm 1$ is N_{\pm} . The total number of vortices is $N_T = N_+ + N_-$, and it is allowed to fluctuate by addition/removal of neutral vortex-antivortex pairs. However the net charge $Q = (N_- - N_+)L^2 B / \phi_0$ is constrained by the field B. E_{c0} is a bare vortex energy cutoff, a_0 is a ‘‘vortex core radius’’, ρ_{s0} is the bare superfluid stiffness (inverse dielectric constant in the plasma language), and L^2 is the area of the system. The density of the field-induced vortices is $n = B a_0^2 / \phi_0$. It is noteworthy, that Oganesyan *et al.* generalize standard Kosterlitz RG method to non-neutral situation. The brief description of the main steps of their calculation without giving details is as follows. The couplings of the Hamiltonian is renormalized upon increasing the cutoff to $a = a_0 b$ and integrating out neutral vortex-antivortex pairs with spacing less than a . Under RG flow, the density of field-induced vortices grows as $n(b) = n_0 b^2$. For $T \leq T_{KT}$ (T_{KT} is a critical temperature of the transition) thermally induced vortices become increasingly dilute and the system approaches a one component plasma. On the next step, the asymptotic behavior of magnetization derived from the exactly known free energy of a dilute one component plasma. The

simplified scaling relation for low-field M when $T \leq T_{KT}$ looks like.

$$|M| \sim (T_{2D} - T)\ln(H_c/|H|) \quad (2)$$

Actually, this relation is quite different from experimental data for $\delta(T)$, but it gives a divergent susceptibility.

Incompressible vortex fluid. [4]. According to P. W. Anderson [4], the nonlinear behavior of diamagnetic susceptibility is caused by the missing term in the conventionally accepted model Hamiltonian for quantized vortices in the Bose fluid. Anderson starts his consideration from a claim that it is appropriate to describe the phase above T_c as a vortex fluid and that the system is similar to supersolid He. He infers the next properties of a vortex fluid state. In most range of observation it is dissipative. It means that the random motions of the vortices constitute a thermal reservoir into which energy may be dissipated, and current-current correlations decay with time. It is also incompressible in the sense that inserting an extra quantum of vorticity costs an energy which is divergent in the distances between such extra vortices. Other theories of vortex-mediated transitions such as Kosterlitz-Thouless and Willians theories discuss only the questions of adding/removing of pairs of vortices of opposite sign or vortex loops. Anderson takes into account the addition of net vorticity and suggests that the response of a vortex liquid to this is anomalous.

Here we consider a 2D model, that should be simply generalized to 3D. In this model the current of vortices is simply the sum of those due to the individual vortex points and the energy is the integral of the sum of the square of the sum over all vortices.

$$J_i = \nabla\phi_i = q_i\hat{\theta}_i/|r - r_i|, q_i = \pm 1; \quad U = \frac{1}{2} \int d^2r (\sum_i J_i)^2. \quad (3)$$

There must be a lower cutoff around the vortex points a and an upper cutoff R for the sample as whole. After the integration of the energy we obtain

$$U/2\pi = \sum_i q_i^2 \ln(R/a) + \sum_{i \neq j} q_i q_j \ln R/r_{ij} = (\sum_i q_i)^2 \ln(R/a) - \sum_{i \neq j} q_i q_j \ln r_{ij}/a \quad (4)$$

If the system of vortices is neutral ($\sum_i q_i = 0$) then the self-energy of the vortices which diverges logarithmically cancels and we have the standard Kosterlitz-Thouless interaction energy result.

$$U_{K-T} = -2\pi \sum_{i \neq j} q_i q_j \ln(r_i - r_j)/a + \sum_j E_c, \quad (5)$$

where E_c is a core energy. However if we have a mismatch in + and - vortices (which corresponds in our case to an external B field) there remains an additional to Eq.3 divergent term, proportional to the logarithm of the upper cutoff radius. Anderson shows that this additional term is proportional to $n_V \ln(R_c^2/a^2) = n_V \ln(1/n_V a^2)$, where n_V is a density of "extra" vortices and R_c is approximately the distance between unpaired "field" vortices. The crucial point, which makes the vortex liquid incompressible, is that the additional energy term is not screened out by the thermally excited pairs above T_c . It is noteworthy, that this is in contrast to seemingly similar systems in electrostatics, where we have screening. To sum it up, according to Anderson, "the

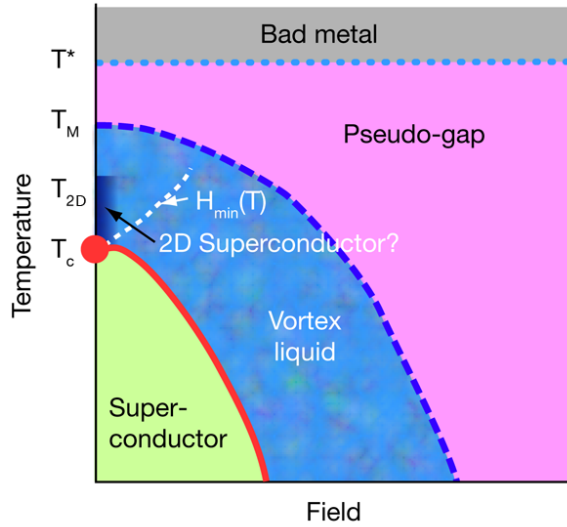


Figure 4: Figure from [2]. Schematic phase diagram of a cuprate superconductor plotted as a function of the applied field (H) and temperature (T). The region in which evidence of unusual, possibly 2D superconducting behavior is found in some cuprates is indicated by the dark blue region above T_c . T^* denotes the crossover between the bad metal phase.

anomalous response is not a critical phenomenon but an intrinsic property of the vortex liquid phase, and it should persist as long as there is a finite core energy for vortices".

Discussion The boundary between the normal and superconducting state (Fig.4) is defined as a line, across which the resistance vanishes. The other lines on a phase diagram are, as far as we know, crossovers from one state to another, consequently, they are more fuzzy. The region labeled "vortex liquid" is identified as a region

of strong superconducting fluctuations because of several characteristic features of magnetization curves: the magnetization is opposite in direction to H (diamagnetic), it is large compared to (for example) the Landau diamagnetism in conventional metals, and it is nonlinear function of H . T_M is identified as the point at which M_d vanishes, and where M_{eff} changes from being a linear function of H to nonlinear. The nonlinearity is an expected feature of a superconducting state. For small H , M_d grows, but after some characteristic value of the field H_{min} it should tends to zero. In other words, a M_d curve exhibit a minimum at some nonzero field H_{min} . Experiments suggest that $H_{min} \rightarrow 0$ as $T \rightarrow T_c$ in all samples, considered above excepts OPT YBCO. The observation of H_{min} requires very high magnitudes of a magnetic field (up to 45 T), because H_{min} grows rapidly as we increase the temperature above T_c . The important result is the smoothness of M_d across T_c , which suggests that the region of the phase diagram above T_c corresponds to an extension of a "vortex-liquid" with somehow degraded, by a disorder, long-range correlation.

It is worth to emphasize, that the "vortex liquid" state has two features: 1) In two families of cuprates (Bi 2201 and Bi 2212), a nonlinear behavior of the low-field magnetization was observed : $M \sim H^{1/\delta(T)}$, $\delta > 1$ for some range of temperatures $T_{2D} > T > T_c$, with a strongly dependent exponent δ . If this behavior really extends to arbitrary small H , this means that the susceptibility is divergent. At the present, we have measurements only down to 10 Oe [3]. Probably, we can make measurements for even lower values of the magnetic field by making use of the phase-locked cantilever magnetometry technique, which is currently employed by the group of R. Budakian at UIUC.

There are two main ideas of why the magnetization displays a non-analytic behavior. The first was proposed by Oganesyan et al. [5]. They noticed that a layered superconductor with zero Josephson coupling between planes displays has a diamagnetic magnetization at small H given by $|M| \sim (T_{2D} - T)\ln(H_c/|H|)$ below the Kosterlitz-Thouless transition temperature. This scaling relation, at least, produces a divergent susceptibility. However, an apparent problem is that even weak but finite interplane Josephson coupling leads to 3D superconducting transition with $T_c > T_{2D}$ [2].

2)It is no clear, why $H_{min} \rightarrow 0$ as $T \rightarrow T_c$. We need more experimental data to find a precise value of critical exponents. The present in [1] data is consistent with the scaling behavior $H_{min} \sim (T - T_c)^{2\nu}$, with $2\nu \sim 1$. This is not consistent with 3D XY(Kosterlitz-Thouless) scaling, but looks reasonable for 3D critical points.

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