Application of Statistical Physics to Terrorism

Rodrigo Soto-Garrido

May 13, 2010

Abstract

This paper reviews two models used to study terrorism from a statistical physics point of view. The first model describes terrorism using percolation theory based on individual passive supporters. The second one is an opinion dynamic model with memory that is used to understand the inter-event time (time between terrorism events). The first model provides us with some recommendations to reduce the terrorism threat or at least confine it to a specific region, while the second model offers a good agreement between the empirical data and their predictions.

1 Introduction

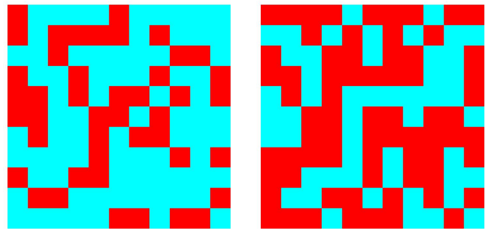
Terrorism has been one of the most significant threats to peace in the last few centuries and it has principally been studied by politicians and sociologists. The terrorist attack in the US on September 11, 2001 concerned millions of people around the world, and according to the experts was the first evidence of a worldwide spread of a terrorist group [1]. Also the last decade has been marked by several terrorist attacks, especially in some areas of the Middle East such as Iraq and Afghanistan. Because the attempts to reduce and understand terrorism have been not very fruitful in the past and because of the several applications of statistical mechanics, we are going to try a different approach and we are going to present two different models to study terrorism, both of them based in statistical concepts used in physics. The first model, due to S. Galam [2, 3, 4], studies some aspects of terrorism using percolation theory based on individual passive supporters. The second model, due to Zhu *et al.* [5], studies the inter-event time between terrorism events using an opinion dynamic model with memory that reproduces the empirical results very well.

2 Galam Model

This model focuses its attention in the spread of terrorism, and how it is related to the whole society, not only to the social behavior of the terrorist group. Even when the motivations of terrorism can be extremely complicated and difficult to model, the Galam model offers a good understanding using the physics of disorder. The basis of the model is percolation theory [6], and the spread of terrorism is a product of the passive supporters to the terrorist cause [2]. The model describes these passive supporters as people that even when they do not express explicitly their position, they share the terrorist cause and do not oppose the terrorist acts.

2.1 Percolation Theory

Before moving on, let us briefly review percolation theory. Percolation theory studies a system of a large number of connecting sites, where each can be occupied with some probability p. The goal in our model will be to determine the critical value of p (p_c) for which a cluster (group of adjacent occupied sites) can be formed crossing the entire system [6]. At this value of $p = p_c$ there is a phase transition (p_c is denominated as critical threshold) between a phase without percolation and a phase with percolation. To clarify this idea, see figure 1.



(a) (b)

Figure 1: Both figures show a lattice 11x11. The red sites refer to occupied sites and the cyan sites to unoccupied sites. The figure (a) shows the system for $p < p_c$ where there is no percolation, on the other hand figure (b) shows the system for $p > p_c$ where there is a red cluster that crosses the entire system from the bottom to the top.

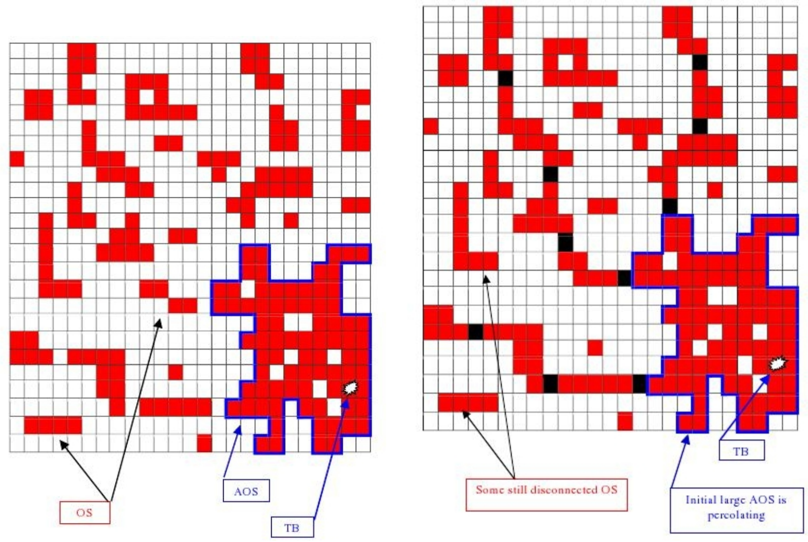
2.2 The model

The Galam model can be summarized as follows [2, 4]:

1. Consider the world as a grid or a lattice.
2. Each site has probability p of been occupied by a passive supporter. In other words, p is ratio between the number of passive supporters and the population.
3. A terrorist can only reach a target when there is a continuous path of occupied sites by passive supporters is formed from the terrorist base to the target. In other words, the terrorist can only execute a terrorist attack in the terrorist base cluster.
4. We designate the sites that are reachable to the terrorist as active open spaces (AOS) and the other sites that are occupied by passive supporters but cannot be reached by the terrorist as open spaces (OP).

Now that the model is defined, let us see how it works. If $p < p_c$, the major part of the territory is safe from terrorist attack, since only the small area where the terrorist base (TB) is located is in real danger (see figure 2(a)). But when $p > p_c$, almost the whole territory is now in danger (see figure 2(b)).

Until now, it seems that everything fits maps very well onto a physical system, but the crucial difference is that we consider a physical system to be infinite. Nevertheless, the real world is not (the number of sites is not infinite, and it is limited by the population of the territory), so even when $p > p_c$ in a certain area, p can be less than



(a) (b)

Figure 2: Figure from Galam article [4]. Both figures show a lattice 25x25. The figure (a) shows the territory for $p < p_c$ where there is only a small area under terrorist threat. Otherwise figure (b) shows the territory for $p > p_c$ where almost all the area is in danger. The black sites represent the people that were not passive supporters in figure (a) and that now are passive supporters in figure (b) producing a percolation.

p_c in a large area which includes the first area. This is certainly what we expect, as the density of passive supporters should be largest near the terrorist base, decreasing as we get further from the TB. One example of this behavior is the Corsican case [4] where the terrorism only percolates in the island and not to all of France. This kind of terrorism is known as classical terrorism, where the terrorist threat is confined to a specific region (see figure 2). However, after 9/11 everything has changed, because for the first time there is a manifestation of a world percolation. All the world is under a terrorist threat, meaning that in our model, $p > p_c$ in the whole world.

2.3 The critical threshold

Until now we have described our model, and we have qualitatively showed the basic concepts behind it. We need to now calculate p_c to determine the passive supporters density at which the transition occurs.
The value of p_c depends on two quantities, the number of adjacent sites q (or nearest

neighbors) and the dimensionality of the space d. The approximate equation that relates these three quantities is given by [7]:

$$p_c = a[(d-1)(q-1)]^{-b}, \tag{1}$$

where a and b are constants, their values being $a = 1.2868$ and $b = 0.6160$. Now we need to specify the values of q and d. Let us first focus on q. It is obvious that the value of q depends on the region. In some areas it could be very large, for example in cities where the population density is very high, or it can be very small in small villages or small towns. To simplify the model it is necessary to take an approximate value of q for the entire system, and it seems that a number between 10 and 20 it is very reasonable, so let us assume $q = 16$ [4], so now we need to specify d to determine the value of p_c. It seems that the value of d is very simple to determine, since we live on the Earth's surface, we will expect $d = 2$, but let us think a little bit more carefully about this argument. It is completely true that we live on a two dimensional surface. Nevertheless, the dimensionality of the "Social Space" can be different. According to Galam and Mauger [3], the social space can have extra dimensions associated with different social aspects, such as religion, politics, history, economics, culture, etc. As we can see, in general, $d > 2$. Also note that according to equation 1, p_c decreases as d is increased. For example, when $q = 16$ and $d = 2$, we have $p_c = 0.243$, so to have a whole world percolation we need at least 24.3% of the world population as passive supporters. This means more than 1.6 billion passive supporters, a number that seems extremely high. However, if we include the extra dimensions due to social aspects and calculate the critical threshold for $q = 16$ and $d = 10$, we find that $p_c = 0.063$. Now, it is only necessary to have 400 million passive supporters to produce the whole world percolation phenomenon. 6% of the world population seems a reasonable number of passive supporters, so we can estimate that the real dimensionality of the social space for the whole world is around 10.

2.4 Possible solutions

Let us now see what are the possible solutions to the terrorist threat. To avoid the whole world percolation we need $p < p_c$. For decades, the classical military action against terrorism has been to try to eliminate the terrorist group, and even when this action can produce immediate results, as soon as a new terrorist group emerges, the whole territory is again under terrorist threat. Another possible solution to satisfy $p < p_c$ is try to decrease the number of passive supporters, but this involves neutralization of millions of randomly located supporters around the world, a task that seems

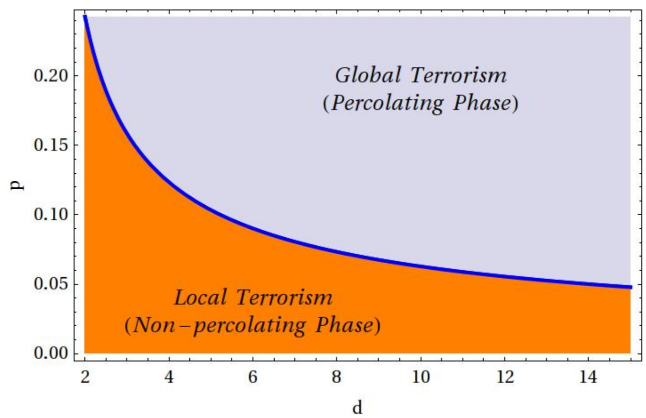


Figure 3: Phase diagram for the Galam model ($q = 16$). p represents the fraction of passive supporters around the world and d the dimensionality of the social space. The blue line, $p = p_c$, represents the phase boundary between the percolating phase and the non-percolating one.

extremely difficult and against world values of ethics and justice. The picture seems very bleak, but the Galam model can give us some ideas on how to try to solve the puzzle. Instead of focusing on the eradication of the terrorist group or the passive supporters, let us try to increase the value of p_c, so we can go from a phase of whole world percolation $p > p_c$ to a phase where $p < p_c$ without changing p (that means without hurting the passive supporters [3]). Thus, the terrorism is confined to a specific region (See figure 2).

From equation 1, we see that p_c is increased if we decrease q and/or d. Decreasing q seems very difficult, because it is related to geographical factors and population. We could also try to decrease d. As is described by Galam [1, 3, 4], we could decrease d by political actions reducing what he called "flags" (motives and ideals that the terrorist claims in his cause). According to Galam these flags create sympathies from the people, and they raise the number of dimensions in the social space, so the idea is to try to reduce the number of flags, decreasing d, and consequently increasing p_c, producing a phase transition between global terrorism (whole the world under threat) and local terrorism (only a confined region is under threat), as shown in figure 3.

3 The Inter-event Time Model

In the previous section we saw the Galam model and how it predicts the percolation of terrorist threats to the whole world. In the present section we are going to focus our attention to the the time between two successive terrorist attacks. To do that we are going to present the Zhu *et al.* model [5], that is based on an opinion dynamic model with memory effect. The model suggests that a terrorist attack is produced when the order parameter of the system reaches its critical value. The goal of this model is to have a good agreement with the empirical data.

3.1 The model

The model corresponds to an opinion dynamic model with memory. It has the following ingredients:

1. It presents the society as a 2D square lattice, where each site has 4 nearest neighbors (n.n). Each site represents an individual.

2. The interaction between the site and its n.n. represents the social interactions.

3. Before a terrorist event occurs, the individual supports (σ_i=+1) or rejects (σ_i=-1) a terrorist attack.

According to Zhu [5] the opinion of a particular person is given by:

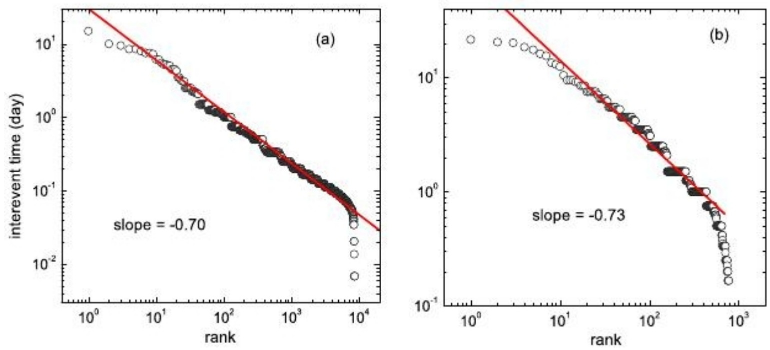
1. Influence of its n.n.: $W_1(\sigma_{i,t}) = \sigma_{i,t-1}\sum_{j=1}^{4}\sigma_{j,t-1}$, that means that $W_1 > 0$ the person agrees with his n.n., on the other hand $W_1 < 0$ the person disagrees.

2. Memory effect: $W_2(\sigma_{i,t}) = \begin{cases} 1, & \sigma_{i,t-1}\sigma_{i,t-2} > 0 \\ 0, & \sigma_{i,t-1}\sigma_{i,t-2} < 0 \end{cases}$

Using these ingredients, Zhu [5] defines the probability that a person change his opinion by the following equation:

$$P(\sigma_i) = \begin{cases} (e^{-aW_1} + e^{-bW_2})/T, & W_1 > 0 \\ e^{-bW_2}/T, & W_1 < 0. \end{cases} \tag{2}$$

The term for $W_1 < 0$ is because he disagrees with his n.n., so memory dominates his opinion. In this model, a and b are parameters that, according to Zhu, indicate “social conformity psychology and self-affirm psychology respectively” [5]. T is in someway the temperature of the social system (a measurement of chaos, we can read off from



(a) (b)

Figure 4: Figure from Zhu article [5]. Log-log plot rank v/s inter-event time for Iraq (a) and Afghanistan (b).

equation 2 that $T \geq 2$).
Finally we define the order parameter as in magnetism (for example, the Ising model) $m = \mid \frac{1}{N}\sum_{i=1}^{N}\sigma_i \mid$. As we mentioned before the model is based on the idea that a terrorist attack occurs when $m = m_c$. Now that the model is defined, let us see what statistical properties of the inter-event time follow .

3.2 Terrorist attacks and Zipf's law

In his article, Zhu defines the inter-event time as the days between consecutive terrorist attacks (when there is more than one terrorist attack in a day, the inter-event time is 1/(number of attacks in this day)).
In this section we are going to present the empirical data and see what statistical properties it obeys.
The Zipf's law exponent is given for a probability distribution $p(x) = Cx^{-\alpha}$ as [5, 8]:

$$R^{-1/\alpha'} = \int_R^\infty p(x)dx = C\int_R^\infty x^{-\alpha}dx = \frac{C}{\alpha-1}R^{-(\alpha-1)} \sim R^{-(\alpha-1)}, \qquad (3)$$

where R is the rank given in our case by sorting the the inter-event time in decreasing order of frequency, numbering them 1, 2 and so on.
Now lets see if the inter-event time between terrorist attacks obeys Zipf's law. For this purpose we need to see what the curve looks like on a log-log plot. These graphs for the cases of Iraq and Afghanistan are showed in the figure 4.

We can see that the cumulative probability actually follows Zipf's law, where $\alpha =$

2.43(2.35) for Iraq (Afghanistan), that means that the probability distribution for inter-event time follows a power law, $p(t) = t^{-\alpha}$, is a good approximation.
Now let us see how the Zhu model can explain this behavior of the inter-event time. As we saw in the previous section, the model depends of four different parameters: a, b, T and m_c. Figure 5 shows the results of simulations when each of those parameters is changed independently of the others.

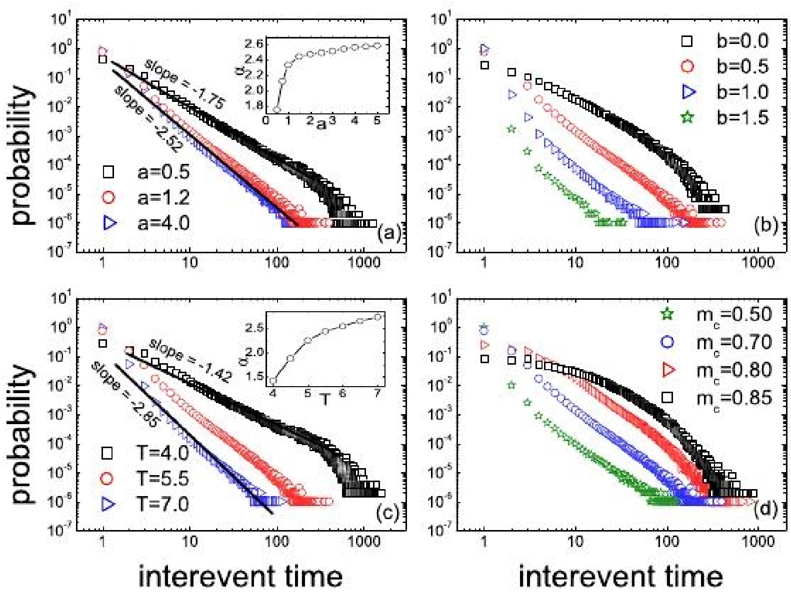


Figure 5: Figure from Zhu [5]. Values of parameters: $a = 1.2$, $b = 0.5$, $T = 5.5$ and $m_c = 0.7$. Figure (a) shows different curves when a is changed. The solid lines are fit and represent values of $\alpha = 1.75$ and $\alpha = 2.52$. Figure (b) shows different curves when b is changed. Figure (c) shows different curves when T is changed. The solid lines are fit and represent values of $\alpha = 1.42$ and $\alpha = 2.85$. Finally figure (d) shows curves for different m_c.

We can see from figure 5 that all the parameters have to be tuned in such a way that the system presents a power-law behavior, so the probability for a given inter-event time is a combination of the social conformity, self-affirmation, chaotic degree and critical order degree. In this model we would like to have a smaller α, so the inter-event time is increased and the terrorist events occur more spaced in time, from figure 5. We can see that to decrease α we need to decrease T and (or) decrease a. Decreasing a means that the person tries to follow the other people, or in the model its n.n. Decreasing T is related to the decrease of social chaos, so when the chaotic

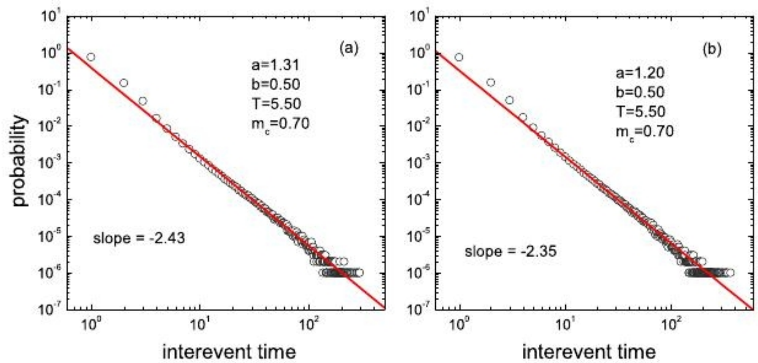


Figure 6: Figure from Zhu [5]. Simulations of Inter-event time distribution for Iraq (a) and Afghanistan (b). The figure also shows the parameters used to obtain the empirical results.

degree is smaller, the terrorist event are more spread in time.
To compare the model with the real data it is necessary to choose the appropiate values for the parameters. We can see that the model fits very well with the emperical results for some set of parameters (see figure 6).

4 Conclusions

Terrorism is a very complicated human phenomenon and its complexity from the sociological and psychological point of view can be very difficult to study. We presented two models using concepts of Statistical Mechanics and Phase Transitions to help us understand how terrorism spreads from a confined region to the whole world and how terrorist attacks are spaced in time in two of the most dangerous countries in the world.
In the Galam model [2, 3, 4] we saw how the raising of "flags" create extra social dimensions that decrease the critical threshold. We can see that it is actually the value of p_c which we have to control to avoid the percolation of terrorism to a larger area, as for larger p_c the terrorism is confined to a smaller region. From this idea we see that military action against terrorism is useless, and instead, neutralizing terrorism using repression creates new "flags" for the terrorist group; p_c is increased and terrorism spreads even more. Even when we cannot determine from the Galam model the political actions to confine the terrorist attacks, it gives us a good model to understand this collective human behavior and gives us some abstract ideas of how to fight against it.

As was mentioned, the Galam model is useful to determine how terrorism is spread in the world. However, it says nothing about the time dependence of the terrorist attacks in an active or dangerous region. In that sense we saw that the Zhu model [5] gives us a good understanding using an opinion dynamic model, where the interactions with the social environment and the memory effect of the person play an important role to determine the inter-event time. This model has results that fit very well with the real data. We can see from figure 6, the only different parameter for Iraq and Afghanistan is the social conformity a, which can be different due to the fact that they are different cultures.

Even when these two models are very useful to understand some aspects of terrorism it is still a lot of work to do. In the Galam model the society is simplified to passive supporters and people that don't support the terrorism, so the model is a two state model, but we could include other states, as active supporters or the terrorist group itself. Another aspect that has not been studied from a physics point of view is the formation of the terrorist group, how a terrorist group emerges in a certain region and how we can model this event. We can finally comment that the collective human behavior can be approximately modeled using techniques from statistical physics and simplified models, because even when the human being is very complex and his actions sometimes does not seem to follow any logic, a large group of individuals can be studied using statistics, as we saw for the specific case of terrorism. The same ideas can be used for other systems, such as politics, religion and group decision among others.

References

[1] *Econophysics and Sociophysics*. Edited by B. Chakrabarti, A. Chakraborti and Arnab Chatterjee. WILEY-VCH 2006.

[2] S. Galam, Eur.Phys.J.B 26 (2002) 269.

[3] S. Galam, A. Mauger, Physica A 323 (2003) 695.

[4] S. Galam, Physica A 330 (2003) 139.

[5] Jun-Fang Zhu, Xia-Pu Han, Bing-Hong Wang, 2009, arXiv:0910.3985v2.

[6] *Introduction to Percolation Theory* by D. Stauffer and A. Aharony. Taylor and Francis, London, 1994.

[7] S. Galam, Alain Mauger, Phys. Rev. E 53 (1996) 2177.

[8] M. E. J. Newman, Contemporary Physics 46 (2005) 323.