The Superconductor-to-Insulator Transition in Resistively-Shunted Josephson-Junction Arrays

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Abstract

This essay describes the, analytical theory, experimental observations and potential applications of superconductor-to-insulator quantum phase transitions in arrays of resistively-shunted Josephson junctions and superconducting islands dissipatively-coupled to an external source of single-particle excitations.

I. Introduction

The advancement of computer technology relies on the understanding of fundamental quantum systems. Quantum computation may be the future of computer information sciences, so determining the dynamics of coherent quantum systems is important. In addition, computer capacity is increasing at an astonishing rate, due to the miniaturization of the feature size of computer components. This advanced progress can only hold for so long, as quantum decoherence effects begin to blur the reliability of electronics devices below a critical feature size, adding to the need to understand the quantum properties of such systems. Quantum phase transitions (QPTs), or transitions from one quantum ground state to another, are of great experimental and theoretical interest, as they are a perfect playground for the study of such systems. One well-known example of a QPT is the superconductor-to-insulator transition (SIT) in disordered, two-dimensional systems. The SIT in arrays of dissipatively-coupled Josephson-junction Arrays (JJAs) is of particular interest, because Josephson-junctions are ideal candidates for use as quantum bits in quantum computation applications. In addition, this SIT can be exploited for use in a variety of microelectronic applications.

The superconducting phase is characterized by the complex order parameter,

$$\psi(r) = \sqrt{n}e^{-i\varphi(r)},$$

where n is the number density of Cooper-pairs in the superconductor and φ is the phase describing the superconducting state [1]. It is important to note that the eigenstates of a superconductor are ones of constant phase, as the number density and phase of the state are conjugate variables of one another, obeying the corresponding commutation relation,

$$[\varphi, n] = i.$$

Processes that cause fluctuations in the coherence, or phase, can destroy superconductivity.

Experiments have shown that in certain granular thin films, superconductivity can only be achieved once the sheet resistance of the films is reduced below a critical value, R_Q, above which they are normal metals [2]. It is thought that the reason for this transition is that dissipation, $\alpha \propto R^{-1}$, suppresses fluctuations in the phase, stabilizing the superconducting state [2, 3, 4]. Decreasing this dissipation below a certain critical value induces a transition to an insulating state. This phenomenon is not restricted to granular superconducting films, but can be generalized to a series of superconducting islands that are Josephson-coupled to one another with shunt resistances between them [5]. The transition in such arrays of Josephson-junctions is the focus of this paper.

Studying this transition in these systems may shed some light on dissipative processes in other superconducting systems, such as high-temperature superconductors or other systems displaying similar quantum phase transitions. These systems are interesting not only because they are rich in physics, but also because they have many practical applications. If the dissipation is a tunable parameter in real time, then a current-switching device can be created through induction of this QPT for a myriad of applications within microelectronics devices [6, 7]. Using graphene as a tunable source of dissipation is one way to realize this; its highly-adjustable electronic properties make it ideal for this purpose.

II. Theoretical Model

A Josephson-Junction (JJ) is the name for a junction consisting of two superconductors separated by a thin layer of either a normal metal or an insulator. If this layer is thin enough, then the leakage of the wavefunctions characterizing the two superconductors into this barrier allows for the tunneling of Cooper-pairs across it. After assuming that the wavefunctions of both SCs are eigenstates of phase, solving the Schrodinger equation for the superposition of these wavefunctions within the barrier region in a constant potential yields the following relations:

$$I = I_c \sin(\Delta\varphi)$$

$$V = \frac{\hbar}{2e}\Delta\dot{\varphi},$$

The current flowing between the two SCs is then strictly a function of the phase difference between the two, while the voltage difference is a function of the change of this phase difference with time. Thus, in the presence of an applied voltage, the phase difference will oscillate in time, while a supercurrent will flow back and forth between the two SCs via Cooper-pair tunneling.

A JJ acts as a capacitor; it stores charge on its "plates" up to a value of $2e$ (the charge of a Cooper-pair) before the pair tunnels through. This phenomenon, called Coulomb blockade, has a charging energy, E_C, associated with it. The larger this value is, the higher the tendency the system has to build up charge (particle number) at the plates, and thus, cause fluctuations in the phase of the wavefunction. A high charging energy can be a mitigating factor for superconductivity. The Josephson coupling energy, E_J, on the other hand, tends to stabilize the phase by increasing the coherence between the superconductors. Hence, the ratio $\frac{E_J}{E_C}$ is a determining factor in whether the system is in the superconducting phase or the insulating phase.

However, it has also been noted that another parameter, the dissipation α, can also determine the superconducting/insulating behavior of the junction. Increased dissipation suppresses fluctuations in the phase, and at a critical value, a transition from a finite to a zero-resistivity state can be achieved by tuning the dissipative parameter, even if when $\frac{E_J}{E_C} \ll 1$ [3]. To understand the origin of this behavior, it is instructive to analyze a double-well potential in the presence of dissipative-coupling to the environment.

A Josephson-junction can be adequately modeled by a double-well potential with generally unequal depths, with the parameter Δu representing the difference in depth of the two wells [3]. The minima of each well correspond to one of two quantum ground-states of the JJ at zero temperature. The phase characterizing the JJ eigenstates is the argument of the potential. By incorporating the fluctuation-dissipation theory developed by Caldeira and Leggett [8], it can be shown that as the parameter $\Delta u \rightarrow 0$ in the zero temperature limit, the tunneling rate between the minima vanishes if the dissipative strength (shunt resistance) is above (below) a critical value. This tunneling rate can be related to the fluctuations in the phase of the JJ, implying that dissipative processes tend to suppress phase fluctuations.

This analysis can be generalized to a JJA by modeling each element as an ideal Josephson element modeled as an inductor with inductance

$$L = V\dot{I} = \left[\frac{2e}{\hbar} I_C \cos(\varphi)\right]^{-1},$$

a shunt resistor with resistance R_S, and a capacitor with capacitance C (Figure 1) [4].

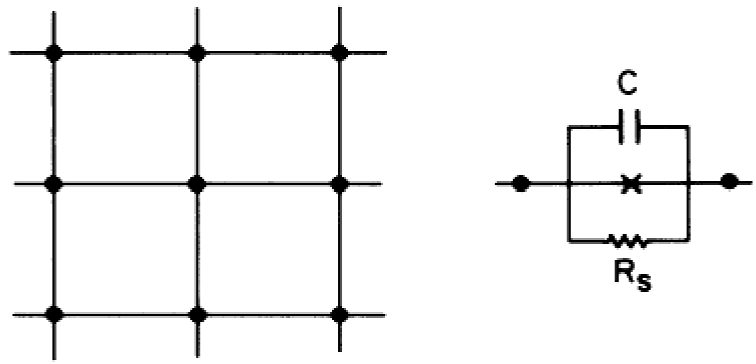

Figure 1: (Left) A 2D Josephson-junction array, with each intersection containing (right) an ideal Josephson-element, a capacitor, and a shunt resistor in parallel [4].

We can use uncertainty arguments to further illustrate the suppression of phase fluctuations with increased dissipation [2]. Since particle number (or charge) and phase are conjugate variables, they satisfy their own uncertainty relation.

$$\Delta\varphi\Delta n \gtrsim \frac{1}{2}$$

The width of the resonance frequencies in a parallel LRC circuit is given by

$$\gamma = [RC]^{-1}.$$

For an accurate measurement to be performed, the time interval must satisfy

$$\Delta t < \gamma^{-1}.$$

In this time, an uncertainty in phase

$$\Delta\varphi = \gamma\Delta t$$

will develop. Now, the uncertainty in the energy is related to the uncertainty in charge, or the number of particles, stored at the capacitor plates. Note it is twice the charging energy of a normal capacitor, due to the model that was chosen in this analysis [9].

$$\Delta E = \frac{(2e)^2(\Delta n)^2}{C}$$

$$\gtrsim \frac{(2e)^2}{4C(\Delta\varphi)^2}$$

$$= \frac{(2e)^2}{4C(\gamma\Delta t)^2}$$

$$= \frac{e^2R^2C}{(\Delta t)^2}$$

From the energy-time uncertainty relation, we have

$$\Delta E \gtrsim \frac{h}{\Delta t}$$

$$= \frac{e^2R^2C}{(\Delta t)^2}$$

$$\Rightarrow \frac{e^2R^2C}{h} = \Delta t < \gamma^{-1} = RC$$

$$\Rightarrow R < \frac{h}{e^2}$$

This means that if R is less than some critical threshold, the phase-number picture is valid. However, if this is not true, then the fluctuations in phase become large, destroying the phase-coherent state.

For a more conceptual description, imagine a current incident upon one such element [4]. As mentioned before, in the absence of dissipative coupling (shunt resistance), the competition between the charging energy and the Josephson-coupling energy determines whether the JJA will be in a superconducting or insulating phase. If a JJ is superconducting, a Cooper-pair of charge $2e$ can tunnel across it. Now, imagine the addition of the shunt resistor R_S. Incident charge has the choice of either tunneling through the Josephson element in a unit of charge $2e$ or by traveling through the shunt resistor. In one cycle of phase (i.e. with a 2π phase-slip), the shunt resistor transfers a charge

$$Q_s = \int I(t)dt = \int \frac{V(t)}{R_S}dt = \int \frac{\hbar\Delta\dot{\varphi}}{2eR_S}dt = \frac{h}{2eR_S} = \frac{2eR_Q}{R_S} = 2e\alpha$$

where R_Q and α are the critical resistance and the dissipation, respectively:

$$R_Q \equiv \frac{h}{(2e)^2} \sim 6.45k\Omega$$

$$\alpha \equiv \frac{R_Q}{R_S}$$

It is energetically favorable to minimize the charging energy of the capacitor, and thus, to minimize the unit of charge transferred across the junction. If $\alpha > 1$, then the smallest unit of transferrable charge is $2e$, the charge of the Cooper-pair. The system will then be superconducting with no resistance at zero temperature. However, if $\alpha < 1$, i.e. if $R_S > R_Q$, a charge of $2e\alpha$ will be transferred through the shunt resistor with a corresponding phase-slip of 2π; all current will pass through the shunt resistors, and the system will be resistive with $R = R_S$. The dissipative source minimizes the charging energy of the capacitor and hence suppresses phase fluctuations, stabilizing the superconducting state.

III. Experimental Evidence and Applications

Such transitions were found in granular Sn thin films when the thickness was varied among different samples [2]. Samples with sheet resistances slightly above R_Q were found to be in a metallic state at $T < T_C$ for bulk Sn, while samples with resistances slightly less than R_Q were resistanceless (Figure 2). These results propelled the authors to say that each grain within the film was locally superconducting and Josephson-coupled to one another. Grain boundaries acted as shunt resistors, thus, when the thickness of the films were increased, the average shunt resistance between Josephson elements was lowered. When this decreased past the critical resistance,$R{\sim}6.45k\Omega$, the coherent superconducting phase was destroyed.

The generalization of these findings to the above description should not be made hastily, as the variations in grain size introduce

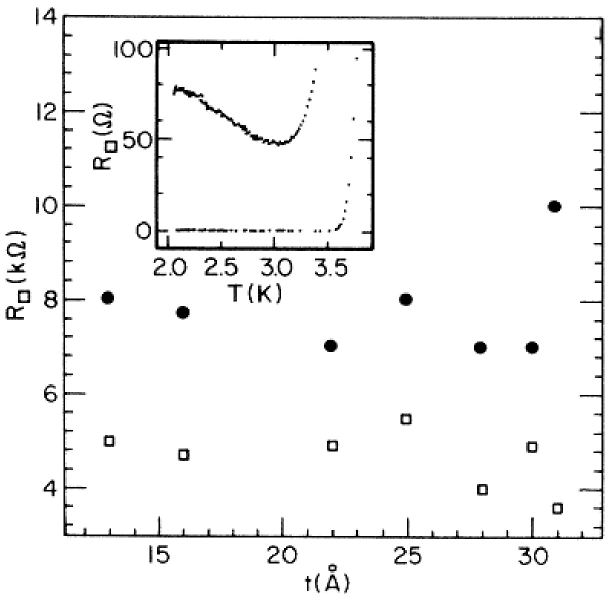

Figure 2: Resistance at T=20K for samples of varying thickness. Each vertical pair of points represents an individual sample before addition of a very small layer of Sn (black circle) and after (open square) to lower the resistance below R_Q. The inset shows one such pair at varying temperatures, displaying a normal, metallic state (above) and a superconducting state (below) [2].

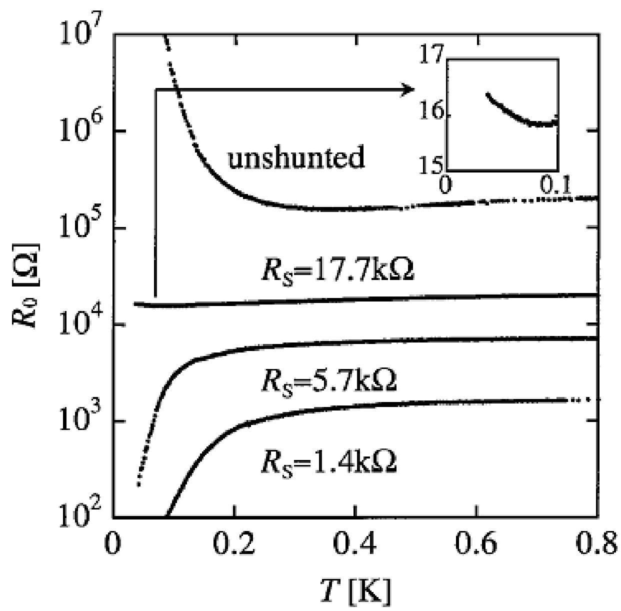

Figure 3: Zero-bias resistance as a function of temperature for JJAs with varying shunt resistances. Samples with shunt resistance less than R_Q extrapolate to zero resistance at low temperatures, while those with higher resistances behave as normal metals at low temperatures. [5]

a disorder, contrasted to the array of identical Josephson-junctions. However, many of the same features of the idealized theory seem to hold in the granular case. This uncertainty motivated researchers to perform measurements on an actual resistively-shunted 2D Josephson-junction array. Takahide *et al.* (2000) fabricated such a JJA with shunt resistances above and below the critical value [5]. Resistance measurements of various arrays with differing shunt resistances clearly showed a transition from a metallic phase to a superconducting phase as R_S was decreased below the critical value (Figure 3). Those samples above the critical shunt resistance had resistances equal to R_S. The fact that the resistance was equal to the shunt resistance for $\alpha < 1$ implies that the JJA was actually in an insulating state, while all current flowed through the shunt resistors. In the case of granular films, the superconducting channel of Cooper-pair tunneling became "insulating," while conduction through phase-slipped grain boundaries constituted the shunt resistance. Another insight given by Takahide *et al.* was that the transition due to adjusting the ratio $\frac{E_J}{E_C}$ was a consequence of the JJA. A single Josephson element should not have such a transition; rather, it should have a transition based only the ratio $\frac{R_Q}{R_S}$. This makes sense since Josephson-coupling should only hold for multiple Josephson elements.

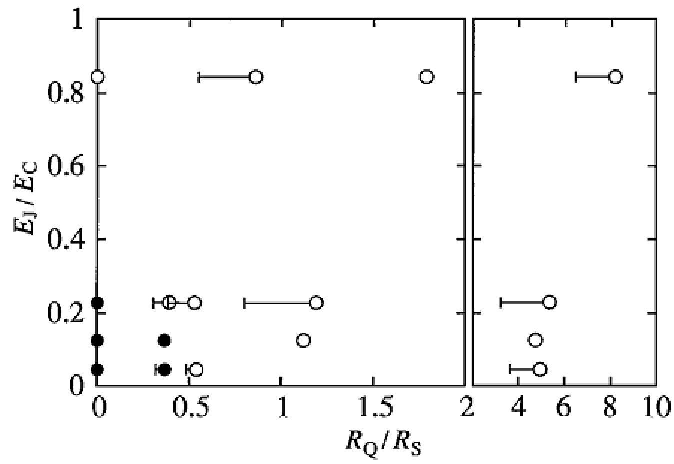

Figure 4: Phase diagram for the resistively shunted JJAs fabricated by Takahide *et al.* Open circles are superconducting states while closed circles are insulating [5].

IV. Superconducting Islands

One of the main difficulties facing those studying this phase transition is the ability to adjust parameters in the same sample to induce this transition in real-time. Current techniques require fabrication of multiple samples with differing shunt resistances to see the transition as a function of dissipative strength. If it were possible to control the dissipation by tweaking a parameter of the measurement apparatus, more fundamental aspects of the transition could be studied, such as measurement of critical exponents.

In 2001, Feigelman *et al.* reported that a similar QPT to that in JJAs could be found in an array of superconducting islands coupled to a disordered, two-dimensional conductor [6]. The islands behave similar to the elements of idealized, resistively-shunted JJAs, with altered values of critical parameters, namely, the critical resistance:

$$Rc = \frac{\hbar\pi^2}{e^2[ln\, b/d]^2} \ll R_Q$$

where b is the island separation and d the island radius. These predictions motivated Lutchyn *et al.* (2008) to propose using graphene as the 2D conductor coupled to the islands (Figure 5) [7].

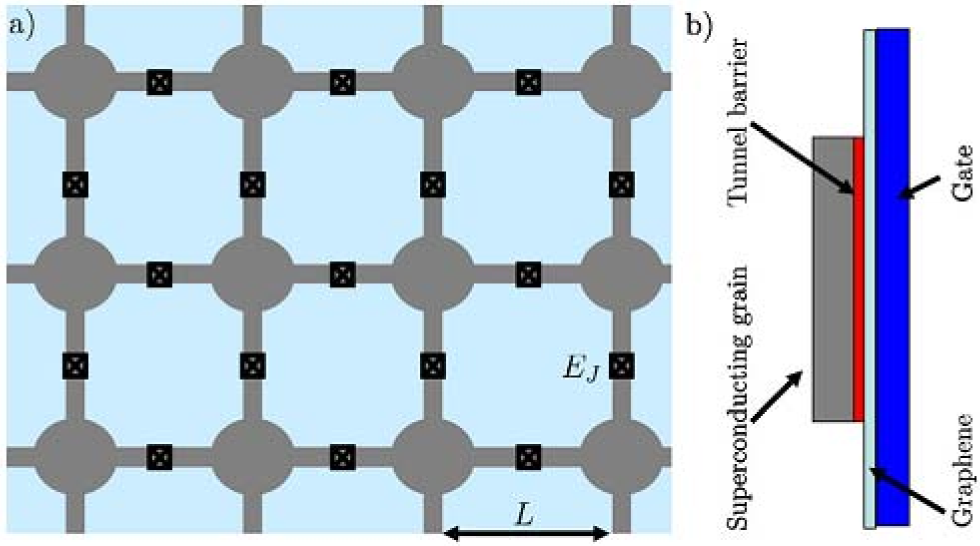

Figure 5: A superconducting island array dissipatively-coupled to graphene as proposed by Lutchyn *et al.* Here, the separation distance L is much longer than the correlation length, so there is no coherence within the graphene. The gate is used to adjust the density of excitation states in graphene, thus, adjusting its dissipation strength [7].

Graphene is an extremely interesting material because of its band structure. It is a zero-gap semiconductor, whose Fermi energy lies at the tips of a conduction and a valence cone. Applying a gate voltage transverse to the conduction plane will adjust the Fermi energy, altering the density of states at the Fermi level, and thus the carrier density and type [7]. Since the carrier density of states acts as a dissipative source of gapless, single-particle excitations, the dissipation strength can be adjusted just by altering a gate voltage. This enables the SIT in such arrays of superconducting islands to be seen in real time rather than by testing a range of samples (Figure 6). The advantages are obvious; discrepancies in different parameters between samples can be ignored as a possible source of the transition. It can be unambiguously probed as a function of just the dissipation strength. In addition, various aspects of the phase-transition itself could be analyzed. Lutchyn *et al.* (2008) have determined that the critical gate voltage is a function of the ratio $\frac{E_J}{E_C}$, and because systems can be easily fabricated for a given ratio, this theory can be accurately probed in the future. In addition, the ability to manipulate the resistance of a system this easily lends itself nicely for applications as a current-switching device in many different areas.

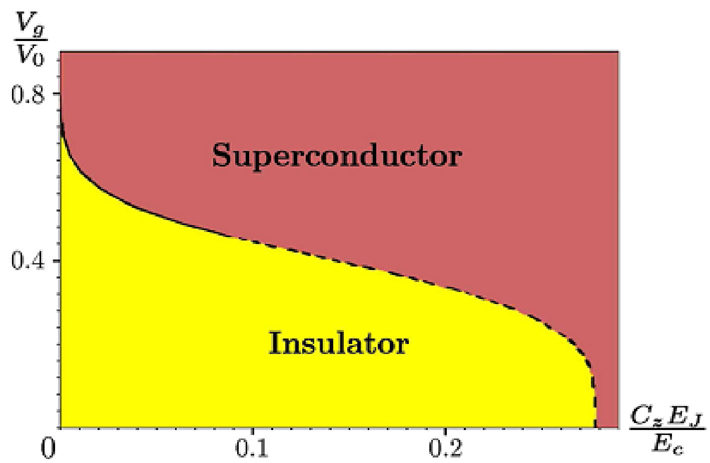

Figure 6: The phase diagram for an array of superconducting islands dissipatively-coupled to graphene as predicted by Lutchyn *et al.* Note that the dissipation strength is now completely specified by the gate voltage, allowing the transition to be driven in real-time [7].

V. Conclusion

Systems which act as resistively-shunted JJAs are quite interesting, because they allow us to view one of most fundamental aspects of a dynamical system, its quantum ground state. Even more interestingly, we have discovered a phase transition between two of these such ground states by applying a relatively simple model to a very complex, real life situation. Experimental systems have been created to best mimic the model, but real systems are usually much more disordered and complicated [10]. Real superconducting films have phases which include the interactions of vortices and antivortices, and their interactions with Cooper-pairs. Parameters such as the temperature and the external field dictate the overall phase of the system, which can be quite different from the strictly superconductor or insulator phases we have studied (Figure 7). However, our discussions of the role of dissipation in these systems is nonetheless relevant, as many of the same features we have seen in JJAs can be also seen in granular, disordered systems.

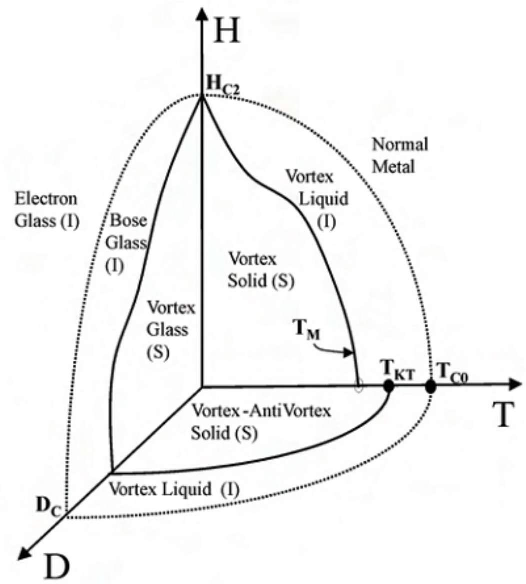

Figure 7: Phase-diagrams in disordered 2D superconductors, such as granular thin-films, are much more complicated, involving many different phases as a function of dissipation, temperature, and external field [10].

Another interesting point to note is that we have also demonstrated that in a resistively-shunted JJA, it is possible to simultaneously have both short-range order and zero resistivity. Fisher states that though the system is statically disordered, its dynamic properties, i.e. the resistance, determine the phase transition. In addition, the value of the critical parameters are independent of the static correlation length, further stressing the difference in the static and dynamical properties of a system [4]. Typical considerations view superconductivity as strictly a long-ranged order phenomenon, but we have shown that superconductivity can be sustained and even controlled in highly disordered systems.

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