

# **Directed Percolation: The Road to Experimental Realization**

Brian Wolin

## Abstract

The goal of this work is to introduce a 2007 experiment which provides the first unambiguous and robust experimental realization of a system exhibiting critical behavior in the directed percolation universality class. To this end one must first introduce the directed percolation model as a fundamental and characteristically non-equilibrium universality class. Then I will describe a number of proposed experiments which should show DP behavior. Next, previous experiments that have actually been performed will be detailed, with special attention given to their conclusiveness in representing DP behavior. Finally, the experiment of interest will be reviewed and the reasons for its particular success examined.

## Introduction

The aspiring physicist will toil long and hard mastering the concepts and techniques of equilibrium statistical mechanics. Unfortunately the vast majority of interesting systems found in nature are not in equilibrium. In this essay I will attempt to scratch the surface of this vast new horizon by discussing a relatively simple and particularly interesting class of critical phenomena called directed percolation (DP). In terms of its pedagogical usefulness, the directed percolation model may be thought of as the Ising model for non-equilibrium statistical mechanics. Luckily the non-equilibrium concepts of phase transitions, universality, and scaling are fundamentally the same as in equilibrium, so one need not start entirely anew. According to Odor, non-equilibrium systems can be classified into two groups: systems which have stationary states given by a Gibbs-Boltzmann distribution, but which are prepared out of equilibrium and systems which do not satisfy detailed balance and may or may not have steady states [1]. Directed percolation falls into the latter category and is thus a genuinely non-equilibrium class.

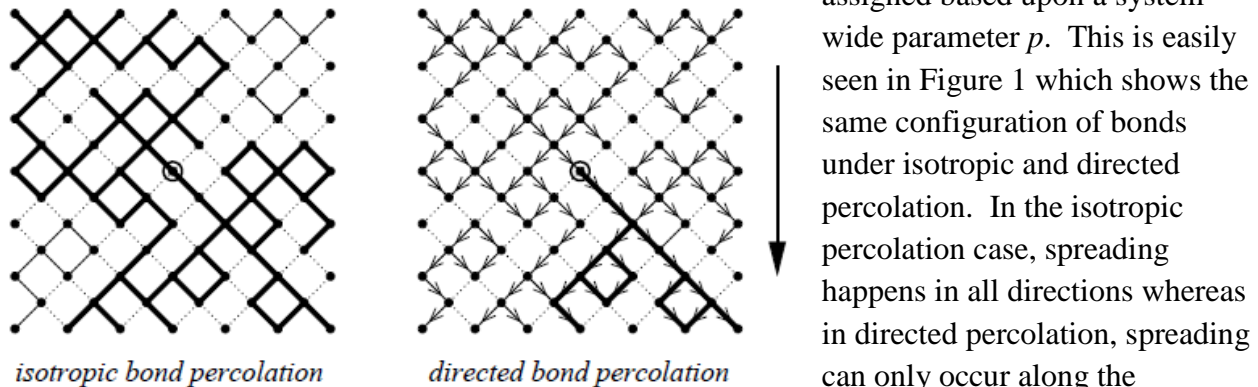
The DP class is the simplest model which describes transitions in systems with absorbing states. That is, models with phases that, once reached, cannot be left again. Envision a plastic cable-tie; the one-way fastening system means that once the tie is closed it cannot be undone. Perhaps the simplest way to understand the directed percolation universality class (and the way for which it is named) is to describe it in terms of a spreading process, though this by no means captures the rich variety of models that exhibit DP behavior. Some non-conserved agent spreads through a background medium based on certain rules. If the agent persists the system is in the survival phase while if the agent entirely disappears it cannot come back and the system is in the absorbing, extinction phase. This transition is continuous in that moving toward the extinction phase the density or proportion of agent dwindles continuously to zero instead of making a discrete jump.

A very well supported conjecture by Janssen and Grassberger states that any model featuring a fluctuating phase and a unique absorbing state, a positive one-component order parameter, short-range interactions, and no additional symmetries or quenched disorder should belong to the DP universality class [2,3] The generality of this statement means that physical systems exhibiting DP critical behavior should be very common in nature and easily made into experiments. However actually achieving this has not been so easy. In this paper I will review the process leading up to the first unequivocal realization of DP class critical behavior in experiment. To that end I will first describe directed percolation in more detail, including its critical behavior and the important role of quenched disorder. Next will be a very small selection of the myriad proposed experiments which should show DP behavior. Then I will describe some experiments that met with varying success in observing and measuring DP dynamics. To finish I will report the experiment which is the culmination of this endeavor and discuss briefly the keys to its success.

## Directed Percolation

### *The Model*

Directed percolation is easily conceptualized as a generalization of isotropic percolation and was originally defined as a geometric model for conductivity in a randomly connected medium [4]. Connections between lattice sites are given a preferred direction so flow can only occur between lattice sites that are connected *and* connected in the correct orientation. Bonds are assigned based upon a system-wide parameter  $p$ . This is easily seen in Figure 1 which shows the same configuration of bonds under isotropic and directed percolation. In the isotropic percolation case, spreading happens in all directions whereas in directed percolation, spreading can only occur along the direction indicated by the arrows which correspond to the larger arrow at the right. Note that the cluster of connected sites in



**Figure 1: Isotropic and directed bond percolation on a 2-D lattice. Closed bonds are represented by solid lines in the left-hand figure and arrows in the right-hand figure. The arrow at the right shows the direction of percolation. The circled point represents the origin of percolation.**

directed percolation is a subset of the cluster in the isotropic case. One may think of water spreading in a piece of paper on a flat surface as being an example of isotropic percolation where if one turned the paper vertically (ignoring capillary action) the action of gravity changes the behavior to directed percolation.

The dynamic, non-equilibrium interpretation of DP comes when the preferred direction is associated with time instead of with space. Figure 1 then describes a (1+1)-dimensional system where the horizontal position is the spatial coordinate and the vertical position indicates time. Now instead of thinking about open channels between sites one can think of sites themselves as active or inactive. In this new dynamic picture the rules can be generalized to assign active status based stochastically on the configuration at any given time.

To give some terminology to describe the microscopic dynamics occurring here, it is easiest to think of the active sites as the subjects a reaction-diffusion scheme. Active sites can become inactive (self-destruction), change positions in the lattice (diffusion), induce inactive sites to become active (offspring production), and destructively combine with other active sites (coagulation) all based on the configuration of open bonds. Self-destruction occurs when all forward moving bonds are closed, diffusion when only a single bond is open, offspring production when multiple bonds are open, and coagulation when multiple bonds lead to the same site. In this new dynamic picture the rules can be generalized to assign active status based stochastically either based on the entire configuration at a given time or in random sequential

order. Thus a whole range of different processes can be covered which all fall into the directed percolation universality class.

### *Absorbing State & Initial Conditions*

The entire goal of the directed percolation model is to capture the dynamics of systems involving absorbing states that can be entered but never left. Key to this is the subtle asymmetry of the rules described above. Notice that multiple active sites may coagulate into a single site and that active sites may spontaneously become inactive, but inactive sites can never impromptu activate. As it turns out the essential nonlinear element of the directed percolation model that results in critical behavior is the coagulation process [4]. Thus it follows that the initial conditions for any useful calculations or experiments must include active sites or else the system remains trapped in the absorbing state. Typical initial conditions for simulations are fully active, randomly distributed active sites, and a single active seed.

### *Critical Behavior*

Just as in equilibrium, non-equilibrium critical phenomena are also expected to exhibit phenomenological scaling laws with critical exponents. However since the time dimension is in general not equivalent to the spatial dimensions, a whole new set of exponents associated with this unique dimension is required. Additionally, the inclusion of an external field and finite size effects also introduce more critical exponents. It would be a rather tedious and unnecessary endeavor to explain in detail all of the critical exponents and scaling relations so I will explore only a few of the more interesting or important ones in this section, but all will be included along with their most accurate estimated theoretical values.

In the simplest cases universality classes with phase transitions into absorbing states can be characterized by four critical exponents:  $\beta$ ,  $\beta'$ ,  $\nu_{\parallel}$ , and  $\nu_{\perp}$ . Outside of the absorbing phase, the density of active sites eventually levels off to some stationary value. Naturally this value must be zero in the absorbing state and near the transition the stationary density decays as a power law with exponent  $\beta$ . A related parameter to the stationary active site density is the ultimate survivability probability  $P_{\infty}$  that a randomly chosen site will be active at infinite time. This value must also go to zero below the transition and does so with the exponent  $\beta'$ . One can show that due to the time reversal symmetry of directed percolation there is a scaling relation  $\beta = \beta'$  [5].

The first consequence of the inequivalence of temporal and spatial dimensions in DP is that two independent correlation lengths diverge as the critical point is approached. The reason for two different correlation lengths is easily seen by referring back to Figure 1. By definition, in the isotropic case there is no preferred direction so the horizontal and spatial correlation lengths must be the same by symmetry. In the directed case the symmetry is broken so the directions are different. Associating the preferred direction with time leads to a temporal correlation length with critical exponent  $\nu_{\parallel}$  and a spatial correlation length with exponent  $\nu_{\perp}$ .

Secondly, one must also take into account scaling as a function of time. An important function that captures the dynamics of both active site creation and destruction is the pair-

connectedness function which is defined as the probability that two active sites at different locations and times can be connected by a directed path of open bonds [4]. Part of the scaling form of this function is the critical exponent  $\theta$ , called the critical initial slip exponent. For small times,  $\theta$  describes the power law growth of the average number of active sites.

One piece that has been missing so far from our model is an external field. The natural choice is to think of an external field as spontaneously driving inactive sites into the active state. This naturally destroys the absorbing state and thus moves the system away from criticality. For small fields the scaling behavior of the stationary density of active sites defines another critical exponent  $\sigma$ .

critical exponent	MF	IMF [155]	$d = 1$ [168]	$d = 2$ [125]	$d = 3$ [170]	$d = 4 - \epsilon$ [171]
$\beta$	1	1/2	0.276486(8)	0.584(4)	0.81(1)	$1 - \epsilon/6 - 0.01128 \epsilon^2$
$\nu_{\perp}$	1/2	1	1.096854(4)	0.734(4)	0.581(5)	$1/2 + \epsilon/16 + 0.02110 \epsilon^2$
$\nu_{\parallel}$	1	3/2	1.733847(6)	1.295(6)	1.105(5)	$1 + \epsilon/12 + 0.02238 \epsilon^2$
$z$	2	3/2	1.580745(10)	1.76(3)	1.90(1)	$2 - \epsilon/12 - 0.02921 \epsilon^2$
$\delta$	1	1/2	0.159464(6)	0.451	0.73	$1 - \epsilon/4 - 0.01283 \epsilon^2$
$\theta$	0	1/2	0.313686(8)	0.230	0.12	$\epsilon/12 + 0.03751 \epsilon^2$
$\gamma$	1	3/2	2.277730(5)	1.60	1.25	$1 + \epsilon/6 + 0.06683 \epsilon^2$
$v$	1	1/2	0.82037(1)	0.88	0.94	$1 - \epsilon/12 + 0.03317 \epsilon^2$
$\sigma$	2	2	2.554216(13)	2.18	2.04	$2 + \epsilon^2/18$

Figure 2: Critical exponents calculated for the directed percolation universality class. References correspond to those in [4].

Figure 2 shows estimates of the critical exponents for DP as calculated in mean field theory, improved mean field theory, and the best values resulting from field theoretic and renormalization group methods. One interesting fact is that the DP model is one of the few that have not yet been solved exactly in one dimension and consequently every value listed in the figure represents an estimate. Perhaps related to this fact is that all the critical exponents appear to be irrational.

### Quenched Disorder

Critical to the application of directed percolation models to real systems is an understanding of the effects of spatially and temporally quenched disorder. A key assumption of DP models is that diffusion probabilities are spatially and temporally invariant. But this is very much at odds with imperfect experimental conditions. Disorder may be quenched in that it is frozen into the system and doesn't vary based on the configuration, but may be spatial (varying in space but constant in time), temporal (varying in time but uniform in space), or spatio-temporal (varying in both time and space. For example in the case of diffusion through a porous medium, local variations in pore density would be spatially quenched disorder, varying temperature which effectively changes the bond parameter throughout the system would be temporally quenched disorder and a combination of both would result in spatio-temporally quenched disorder.

Both spatially and temporally quenched disorder will alter the critical behavior of a system. In the language of field theory, it can be shown that spatially quenched disorder causes a marginal perturbation of the DP dynamics while temporally quenched disorder is a relevant perturbation. However, spatio-temporally quenched disorder becomes an irrelevant perturbation. Disorder changing in time and space acts in the same way as the intrinsic noise of the system and does not affect critical behavior [1].

### **Proposed Experimental Realizations**

The directed percolation universality class is particularly robust with regard to the choice of microscopic details and dynamical rules of a specific model [6]. Thus as one may expect, real-world systems that should exhibit DP behavior are myriad and easily imagined. This section aims to describe some of the more plausible or noteworthy models: disease epidemics, catalysis reactions, and turbulence.

Recently there has been a heightened public awareness of the threat massive disease pandemics: SARS, H1N1, West Nile virus, and mad cow disease to name just a few. It is easy to see that simple models of infection spreading fall into the DP universality class. The rate of infection between neighbors plays the role of the offspring production parameter, and recovery corresponds to the self-destruction parameter in the reaction-diffusion picture. Active sites represent infected individuals and inactive sites healthy ones. Obviously the absorbing state is achieved when there are no more infected sites and the disease is eliminated. One should note this toy model is too simple even to capture the concepts of death or immunization. In the DP model a site can change to healthy to infected and back without limitation whereas in reality recovery from infection typically results in immunity. Additionally, for sufficiently severe diseases infected individuals eventually die or become hospitalized and can no longer transfer the disease.

A great number of systems that have been proposed to obey directed percolation critical behavior involve the propagation or diffusion of some non-conserved agent, be it viruses, calcium ions, or kinetic energy of grains. However, the DP universality class is by no means restricted to such conceptually similar models. One good example of this is the DP critical behavior expected in catalysis reactions. The Ziff-Gulari-Barshad (ZGB) model describing the catalysis reaction of carbon monoxide and oxygen into carbon dioxide on a platinum surface has microscopic rules corresponding to a directed percolation model [7]. Here gas particles adhere to the surface which can be split up into a discrete lattice allowing only up to one occupation per site. Neighboring CO and O particles combine into CO<sub>2</sub> and desorb. Adsorption rates are set by the concentrations of CO and O<sub>2</sub> gas present and the two absorbing states are those wherein the entire surface is filled with one or the other particle and the catalysis reaction halts (called “poisoned” states). In this case only the high O<sub>2</sub> concentration results in a continuous DP class transition; the high CO case is actually predicted to be first order.

Pomeau conjectured in 1986 that the transition from laminar to turbulent flow via a process called spatio-temporal intermittency (STI) may fall into the DP universality class [8]. His claim has since been substantially supported by theoretical and simulation work in 1-, 2-, and (3+1)-dimensions. Spatio-temporal intermittency is a state characterized by stochastically fluctuating areas of order and disorder. With respect to turbulence STI describes the state in which laminar and turbulent regions are intermingled, and can be thought of as the behavior near a front separating turbulent and laminar regions when the front velocity goes to zero. In many systems, the Reynolds number which relates the characteristic flow velocity, length scale, and fluid viscosity acts as the percolation probability, though with a nonlinear correspondence [9]. Thinking of the turbulent patches as active sites, totally laminar flow constitutes the absorbing state. Turbulence transitions via STI can be realized in a plethora of experimental systems including pipe flow, driven liquid crystal systems, filtering, and ferrofluids under the influence of magnetic fields.

### Previous Experiments

Because of the ubiquitous nature of the directed percolation universality class and the great variety of experimental systems it is expected to describe, many previous experiments have been done attempting to measure DP critical behavior. Yet these have all met only partial success. In this section I will describe two experiments which are realizations of those listed in the previous section. Additionally I will comment on how well they match up to theoretically predicted behavior and what about them holds them back from robustly showing DP behavior.

#### *Catalysis Reactions*

The catalysis reaction of CO and O<sub>2</sub> to CO<sub>2</sub> in the presence of platinum described above has also been investigated experimentally by Ehsasi *et al.* [10]. A high purity platinum crystal was cleaned and placed in a controlled environment wherein the concentrations of CO and O<sub>2</sub> gas could be systematically varied. The rate of CO<sub>2</sub> production was measured using a mass spectrometer gas analyzer attached to the system and the concentrations of CO and O adsorbed on the surface were measured via low energy electron diffraction, Auger spectroscopy, and laser induced desorption.

The groups results are shown in the upper portion of Figure 3 and the theoretical predictions are shown below for comparison. The most obvious and important feature is that the continuous DP transition is not observed at all. In fact it appears that the absorbing state of total O coverage is never actually achieved for non-zero CO concentration. Correspondingly, the reaction rate shows a linear response to increasing CO concentration and clearly no power law behavior.

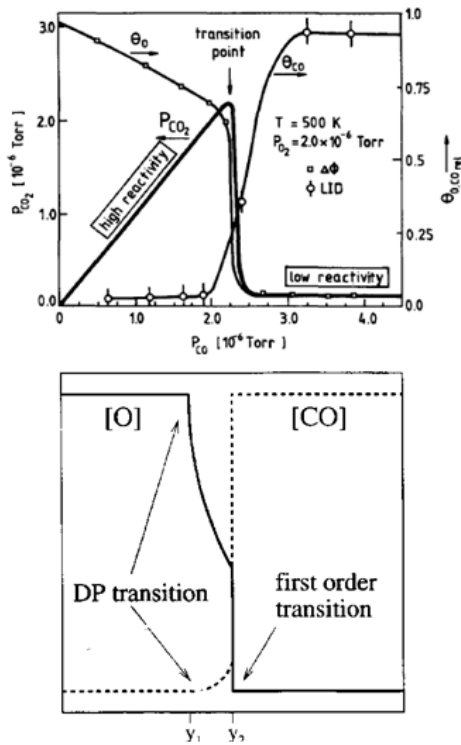


Figure 3: Comparison of experimental and theoretical results of a CO + O<sub>2</sub> catalysis reaction [10,6].

It seems that in this case the ZGB model is ultimately too simplistic to describe the real dynamics of this catalysis reaction. Evidence suggests that defects in the catalyst as well as structuring of adsorbed molecules create significant spatially quenched disorder. Also it seems likely that the O poisoned state is not a true absorbing state. Either O molecules thermally desorb from the surface (resulting in an effective external field driving the system away from the transition) or CO molecules can react directly with adsorbed O molecules. Exactly why the system does not exhibit the predicted critical behavior is still an open question, but in any case DP critical behavior is not realized.

*Turbulence in Ferrofluids*

As experimental and theoretical work progressed it became more and more apparent that road to realizing directed percolation critical behavior experimentally lay in Pomeau’s conjecture that a transition to chaos via STI should fall into the DP universality class. Unlike the catalysis reaction, turbulence experiments are macroscopic, and unlike an epidemic spreading experiment, they are easily repeatable and tunable. In this vein, in 2003 Rupp, Richter, and Rehlberg conducted an experiment with ferrofluid which showed a STI transition [11].

It is well known that when a sufficiently strong magnetic field is applied to a flat ferrofluid surface a pattern of liquid spikes is created. Rupp *et al.* created a ring of such spikes around the pole shoe of an electromagnet. Under the influence of an additional oscillating current spikes can move, combine, and split, resulting in a chaotic phase. The idea behind this experiment was to drive the system into the chaotic phase with a large excitation and then quench toward the STI regime. The group’s experimental setup is shown in Figure 4. A CCD camera recorded the motion of the spikes which were then digitized and analyzed.

The group investigated the scaling behavior of the mean chaotic fraction of the system, the correlation lengths in space and time, and the inactive intervals in space and time. With the notable

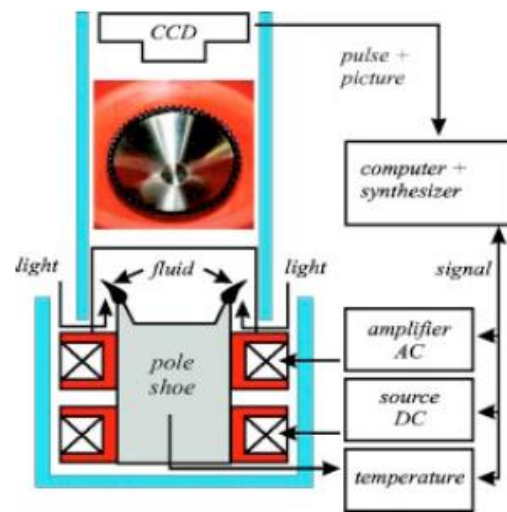


Figure 4: Experimental setup for (1+1)-D ferrofluid STI experiment [11].



exception of the time correlation length critical exponent, the results agree reasonably well with theoretical results. However the statistical errors, especially on the measurement of  $\beta$  are rather large and the value measured of  $\nu_{\parallel} = 0.62 \pm 0.14$  is quite different from theoretically calculated  $\nu_{\parallel} = 1.73$ . Again the problem with this experiment may lie in the imperfect absorbing state.

### *Discussion*

From the above discussion and other literature, one can draw a few broad conclusions about attempts at experimentally realizing a system with DP class critical behavior. It is clear that the most common problem is finding a sufficient absorbing state. In the model the absorbing state is perfect, but in nature fluctuations play a significant role and may mask or change the critical behavior of a system. This influence is most readily apparent in the catalysis reaction experiments. So clearly a very stable absorbing state is important for realizing DP behavior. Additionally, both spatially and temporally quenched disorder will change the critical dynamics of the system. Experiments that occur over long time scales, inhomogeneous media or large length scales are susceptible to these effects. On the other hand this must be balanced with the need to have sufficient averaging time to get precise results and sufficiently many degrees of freedom to avoid finite size effects. These two competing conditions mean that one desires a system with fast dynamics to allow for shorter averaging times and one with many degrees of freedom per unit size.

For various reasons, all the experiments described in this work and the wider literature up until now have not satisfied the above criteria well enough to represent a truly robust realization of directed percolation critical behavior. Though their failings have given great insight into what is necessary. In the final section I will describe an experiment crafted using the lessons learned so far and which unambiguously displays DP critical behavior both through agreement with calculated critical exponent values and with data collapse to estimated scaling functions.

### **Directed Percolation Criticality between Turbulent States in Liquid Crystals**

In 2007 Takeuchi, Kuroda, Chate, and Sano reported the observation of directed percolation critical behavior in the phase transition between two different turbulent states of a driven liquid crystal system [12]. They published a comprehensive follow-up article expanding on this work in 2009 [13]. Following the conjecture of Pomeau, the group studied an electrohydrodynamic phase transition in a thin layer of nematic liquid crystals. Nematic liquid crystals are made up of long rod-like molecules which spatially order such that the long axes generally point in the same direction, giving them a characteristic anisotropy. The direction of average molecular orientation defines what is called the director field which can be subject to various defects. When an increasing external voltage is applied, the quasi-2-D system moves from laminar through a succession of turbulent phases with spatio-temporally intermittent regimes in between. The difference between the inactive phase (called DSM1) and active phase (DSM2) that Takeuchi *et al.* studied is the density of topological disclination defects in the

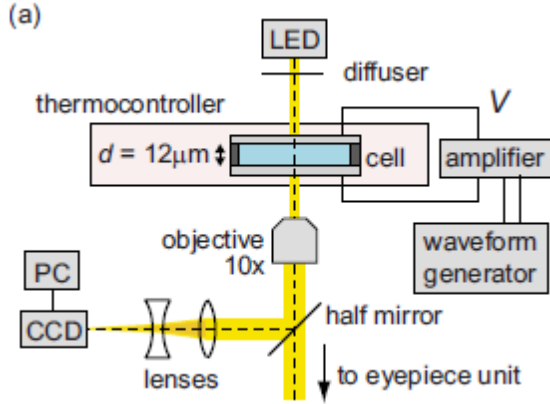


Figure 5: Experimental setup for observing STI transition between turbulent phases of nematic liquid crystals [13].

director field. In DSM1 the density is low and the defects are small; thus they disappear almost immediately. In DSM2 the disclinations are denser and grow large enough that they are split by the turbulent flow of the liquid. When many topological defects are present as in DSM2 macroscopic anisotropy is lost and light shone through the liquid has a lower transmittance. Thus the two phases can be distinguished experimentally simply by shining a light through the system and keeping track of dark and light patches which was done with a CCD.

With this setup the researchers were able to perform a wide variety of experiments. Steady state experiments were performed by putting the system into the critical regime and measuring quantities such as correlation length and order parameter scaling. Critical-quench experiments characterize the system as it decays from fully active initial conditions. Critical-spreading experiments measure the opposite behavior, that is, propagation of active regions from a single activated seed. In all, the group measured 12 critical exponents, 5 scaling functions, and 8 scaling relations all of which agreed to good precision to the best theoretical estimates outlined in Figure 2 and in the wider literature for (2+1)-dimensions.

The results of Takeuchi *et al.* are equally remarkable in their thoroughness and in their agreement with theoretical predictions. I will not attempt to discuss them in detail, merely call attention to some of the more salient points. Figure 7 summarizes values the group measured for the various critical exponents and compares them to theoretical calculations. As one can see, all are well within the range one might expect for an imperfect experimental system and all but a few are strikingly close.

The group's data were also robust and numerous enough to plot tests of data collapse to expected scaling functions, a hallmark of critical behavior. These are less striking mostly because they all show divergence from the calculated scaling functions at small deviations from criticality. However for a real

Exponent	DSM1-DSM2		DP <sup>a</sup>
Density order parameter	$\beta$	0.59(4)	0.583(3)
Correlation length <sup>b</sup>	$\nu_{\perp}$	0.75(6) 0.78(9)	0.733(3)
Correlation time	$\nu_{\parallel}$	1.29(11)	1.295(6)
Inactive interval in space <sup>b</sup>	$\mu_{\perp}$	1.08(18) 1.19(12)	1.204(2) <sup>c</sup>
Inactive interval in time	$\mu_{\parallel}$	1.60(5)	1.5495(10) <sup>c</sup>
Density decay	$\alpha$	0.48(5)	0.4505(10)
Local persistence	$\theta_1$	1.55(7)	1.611(7) <sup>d</sup>
Aging in autocorrelator	$b$	0.9(1)	0.901(2)
	$\lambda_C/z$	2.5(3)	2.583(14)
Survival probability	$\delta$	0.46(5)	0.4505(10)
Cluster volume	$\theta$	0.22(5)	0.2295(10)
Cluster mean square radius	$\zeta$	1.15(9)	1.1325(10)

Figure 6: Results of the measurements of various critical exponents [13].

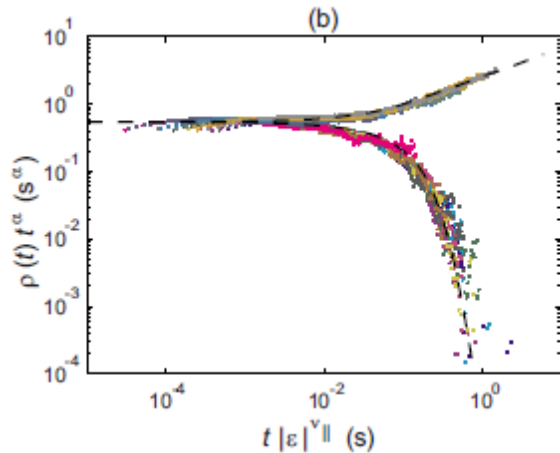


Figure 7: Data collapse to universal scaling functions. Here  $\rho$  is the proportion of active sites,  $t$  is time, and  $\epsilon$  is the deviation from criticality. The exponents  $\alpha$  and  $\nu_{||}$  are given in Figure 6. [13]

system that is finite and imperfect these deviations are expected. One example is shown in Figure 7 where data for the critical decay of active patches is plotted alongside to the expected scaling function. In the figure  $\rho$  is the proportion of active sites,  $t$  is time, and  $\epsilon$  is the deviation from criticality. The two dashed curves are the universal scaling functions calculated numerically for (2+1)-dimensions. The observed data collapse is in good agreement both for above the transition threshold (upper curve) and below (lower curve).

### Success of This Experiment

A natural question now is why was DP critical behavior so easily observed in this experiment, but not in the previous ones? As the authors indicate, the keys lie finding a suitable absorbing state, taming quenched disorder, and sufficient averaging. The most important element is clearly the absorbing state. In previous experiments the absorbing states are theoretically laminar or fluctuation free. But in the real world long-range effects due to laminar rigidity or propagating fluctuations can break DP scaling or reduce the number of degrees of freedom [13]. In the experiment of Takeuchi *et al.* the absorbing state is itself turbulent and fluctuating, thus reducing long-range effects and ensuring that fluctuations do not destroy the absorbing state.

Additionally, in previous work absorbing states have not been perfect. Molecules may desorb from a surface spontaneously or turbulent zones may nucleate from laminar ones. However in this last experiment the active state is characterized by topological defects which are in principle forbidden from forming spontaneously. Even with fluctuations, spontaneous nucleation of a topological defect in this system is an extremely rare event and in order to drive an inactive site active a significant enough number would have to form simultaneously so as to detectably decrease light transmittance. Finally, this experiment realized a system that was orders of magnitude larger than previous ones and had very short dynamics allowing for the nullification of quenched disorder and the ability to have precise averaging on a reasonable time scale.

### Conclusion

The directed percolation model is one of the simplest and most important classes of non-equilibrium phase transitions, much akin to the Ising model in its pedagogical and conceptual role. Furthermore a wide variety of physical models are expected to fall into the DP universality

class, making for a rich set of possible experimental realizations. Indeed proposed experiments range from epidemic or fire spreading to catalysis to turbulence. Unfortunately past experiments have been unconvincing in their demonstration of DP critical behavior due to the presence of quenched disorder, imperfect absorbing states, and limited averaging time and system size. Beginning in 2007, Takeuchi *et al.* conducted an extended series of experiments on a turbulent nematic liquid crystal system which circumvents all these problems. They measured 12 critical exponents, saw data collapse to 5 scaling functions, and tested 8 scaling relations all of which agreed very well with theoretical calculations and simulations. Moving forward, experimenters hoping to study DP in other systems now have a prototypical experiment which conquers all the previous experimental challenges and which may serve to inspire and inform future work.

## References

- [1] G. Odor, *Rev. Mod. Phys.* **76**, 663 (2004).
- [2] H.K. Janssen, *Z. Phys. B* **42**, 151 (1981).
- [3] P. Grassberger, *Z. Phys. B* **47**, 365 (1982).
- [4] H. Hinrichsen, *Adv. Phys.* **49**, 815 (2000).
- [5] P. Grassberger and A. de la Torre, *Ann. Phys. (N.Y.)* **122**, 373 (1979).
- [6] H. Hinrichsen, *Braz. J. Phys.* **30**, 69 (2000).
- [7] R.M. Ziff, E. Gulari and Y. Barshad, *Phys. Rev. Lett.* **56**, 2553 (1986).
- [8] Y. Pomeau, *Physica D* **23**, 3 (1986).
- [9] M. Sipos and N. Goldenfeld, *Phys. Rev. E* **84**, 035304(R) (2011).
- [10] M. Ehsasi, M. Matloch, O. Frank, J.H. Block, K. Christmann, F.S. Rys and W. Hirschwald, *J. Chem. Phys.* **91**, 4949 (1989).
- [11] P. Rupp, R. Richter and I. Rehberg, *Phys. Rev. E* **67**, 036209 (2003).
- [12] K.A. Takeuchi, M. Kuroda, H. Chate and M. Sano, *Phys. Rev. Lett.* **99**, 234503 (2007).
- [13] *Ibid.*, *Phys. Rev. E* **80**, 051116 (2009).