

# Is Gravity Emergent?

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*The purpose of this paper is to discuss the consequences of regarding gravity as an emergent phenomena and how this approach is beneficial in solving long-standing problems like information loss in black holes, the entropy paradox, and the cosmological constant problem. Contrary to common belief, there are observational consequences for regarding gravity as an effective theory, and one can perform experiments to test such a proposal. The microscopics of the effective theory will be briefly discussed because till the present moment there isn't a complete theory in which space-time is emergent.*

## 1 Introduction

The incompatibility of quantum mechanics with classical general relativity is one of the major problems in physics. One possible solution to this problem is to regard general relativity as an emergent property of an inherently quantum mechanical theory. General relativity is by construction a classical theory, and probably Nature behaves differently when  $\hbar \neq 0$ . In 1968, Sakharov was the first to note that space-time in general relativity was similar to the elasticity of matter<sup>i</sup>. He showed that the quantum fluctuations in the matter field induced the curvature term in the action. Still, his analysis doesn't contain an order parameter like that of superfluids. After nearly thirteen years Unruh noticed an analogy between the propagation of sound in a background hydrodynamic flow and the propagation of scalar fields in curved space-time, whereby one can define an effective acoustic metric<sup>ii</sup>. For example, the motion of acoustic phonons is distorted by a crystal lattice, or in background flow field of a superfluid condensate. Based on this analogy many models for "black holes" in superfluids were proposed. This paper will not deal with this analogy, which is strictly kinematic. Instead, it will be concerned with gravity as an emergent property of some sort of collective behavior of a quantum theory, and the observable consequences of such a point of view. In section two, alternatives to classical black holes will be discussed. These models are based on the idea that space-time is some sort of a gravitational Bose-Einstein condensate. The alternatives to black holes are free from long lasting problems of relativity like quantum information loss, thermodynamic instability of black holes, and the entropy paradox. In section three, a possible solution to the cosmological constant problem is suggested based on an analogy with quantum liquids. Contrary to general belief, these ideas are subject to experiments and can be falsified.

## 2 Alternatives to Classical Black Holes

Classical general relativity predicts that massive stars evolve toward the end of their life to black holes. First we will review what classical and semi-classical general relativity say about black holes.

The static and spherically symmetric line element for an isolated mass  $M$  is,

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{h(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (2.1)$$

where the functions  $f(r)$  and  $h(r)$  are given by

$$f(r) = h(r) = 1 - \frac{2GM}{r}. \quad (2.2)$$

This is the well-known Schwarzschild solution.

At  $r=0$  there is a dynamic singularity, and Einstein's equations cease to exist. At  $r = R_s = 2GM$ , which corresponds to the surface of the black hole or the event horizon, the singularity is only kinematic and can be removed by a suitable coordinate transformation. Gravity is well behaved on this surface and a free-falling observer passes through the surface with a finite proper time. Still light falling onto the surface will experience an infinite blue shift. A photon with frequency  $\omega$  far away from the black

hole will acquire a local energy  $E = \hbar \omega f^{-\frac{1}{2}}$  that diverges at  $r = R_s$ . Put in other words, at the surface of black holes, time stops relative to a clock at infinity. This leads to an ambiguity in the definition of a universal time.

This ambiguity is one of the main reasons why quantum mechanics and classical general relativity are incompatible. In order for the Schrodinger equation

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi \quad (2.3)$$

to hold, a universal time needs to be defined. This has lead to revising the laws of quantum mechanics<sup>iii</sup> and to claims that there is loss of quantum information in black holes<sup>iv</sup>.

There is a second problem with the semi-classical approximation used to calculate the entropy of black holes. When a massless field like that of photons is quantized with respect to the fixed Schwarzschild background, one finds that the black hole radiates

quanta with thermal spectrum at Hawking temperature  $T_H = \frac{\hbar}{8\pi k_B GM}$ <sup>v</sup>. (2.4)

However, for  $\hbar \neq 0$ , the quantum effects can't be neglected. Although it is assumed that the backreaction effect of the emitted radiation by the black hole is small, calculations show that the energy momentum components  $\langle T_t^t \rangle$  and  $\langle T_r^r \rangle$  of the radiation have an  $f^{-1}$  divergence at the horizon, and that wavelengths of order  $R_s$  contribute to this infinite blue shift factor. This implies non-locality at the scale of the black hole, and it can't be removed by a suitable change of coordinates<sup>vi</sup>. Moreover, from (2.4), the Hawking temperature  $T_H$  is inversely proportional to the mass of the black hole  $M$ . Hence a black hole in thermal equilibrium with its Hawking radiation has negative specific heat, and it is unstable to thermodynamic fluctuations<sup>vii</sup>. The thermal radiation spectrum and energy conservation imply that black holes have entropy that far exceeds that of the progenitor star. Again there is a paradox in this case. The source of the excess entropy isn't known.

In the following section we will discuss a fully quantum mechanical resolution to the first problem proposed in [x] whereby the event horizon is regarded as a quantum phase transition akin to liquid-vapor transition. This suggestion might hold a solution to the problem of thermal instability of black holes depending on the order of the transition. Another solution to the entropy paradox and the above two problems was proposed in [xiv] such that an ultra cold gravitational condensate star replaces the black hole. Both of these suggestions have observable consequences and are amenable to experimental tests.

## 2.1 Continuous Quantum Phase Transition in Space-time Vacuum

There aren't many theories that regard space-time as emergent. G. Chapline proposes that macroscopic space-time emerges from microscopic fluctuations in topology<sup>viii</sup>. The latter might correspond to gravitons. He suggests that the condensate wave function for the three dimensional anyonic superfluid<sup>ix</sup> can be reinterpreted as a quantum wave function for space-time. Just like electrodynamics is represented on the quantum level by coherent states of photons, space-time can be represented as a coherent sum of nonlinear gravitons. The details supporting this claim are rather technical and will not be presented. It suffices to note that the constructed space-time wave function has a

long range off-diagonal order, and hence one expects a universe described by it to have quantum effects on the macroscopic scale. One such possible effect is that the event horizon of a black hole differs from the one predicted by classical general relativity whereby space-time vacuum rearranges itself in such a manner that time is universally defined<sup>x</sup>.

In [x], Laughlin *et al.* suggest a resolution to the universal time problem by proposing that the event horizon is a continuous phase transition of the vacuum of space-time analogous to the quantum liquid-vapor critical point of an interacting Bose gas. The classical description of the vacuum no more holds for scales smaller than a characteristic length  $\xi$ , which is a quantum-mechanical quantity that diverges at the event horizon.

Before establishing the analogy, we need to discuss modeling the zero temperature liquid- vapor transition in quantum fluids as a quantum phase transition.

<sup>4</sup>He, which is a simple example of bosonic matter, is a solid at pressure above 25 bar and at zero temperature. As pressure decreases beyond 25 bar it melts into a liquid<sup>1</sup> with a small amount of quantum vapor pressure<sup>xi</sup>. It will not evaporate at zero temperature. Since the temperature is much lower than the condensation temperature, it is regarded as a pure superfluid.

Moreover, the vapor phase of bosonic matter was experimentally observed in BEC. Cornish *et al.* performed experiments with magnetically trapped <sup>85</sup>Rb BEC where condensate self-interaction energy is magnetically tuned from strong repulsion to large attraction<sup>xii</sup>. When the magnetic field is increased to the limit in which the interaction is expected to change sign, the <sup>85</sup>Rb ball of vapor first contracts, which is something expected if the pressure decreases. However, after nearly 5 ms there is a sudden explosion that ejects a large fraction of the condensate. This leaves a small yet observable remnant of the condensate surrounded by a relatively hot cloud at about 100nK. Although interpreting this result is complicated by the metastability of the condensate and the increase in the rate of recombination into the ground state that occurs at high densities, it occurs abruptly where such a transition is expected, and softening of the compressibility heralds it. One may regard this behavior as something similar to a quantum liquid-vapor transition.

The simplest model for such a transition in classical fluids is the Van der Waal equation of state

$$(V - b)(P + \frac{a}{V^2}) = Nk_B T. \quad (2.5)$$

Laughlin *et al.* consider the phenomenological quantum equation of state<sup>2</sup>

$$(V^2 - b)(P + \frac{a}{V^4}) = c. \quad (2.6)$$

The effective Lagrangian  $L$  generates such an equation

$$L = \psi^* (i\hbar \frac{\partial}{\partial t} + \mu) \psi - \frac{\hbar^2}{2m} |\nabla \psi|^2 - U(|\psi|^2), \quad (2.7)$$

where  $|\psi|^2$  is the density of states  $\rho = \frac{N}{V}$ , and  $U = \frac{c}{2\sqrt{bV}} \ln[\frac{V + \sqrt{b}}{V - \sqrt{b}}] - \frac{a}{3V^4}$ . (2.8)

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<sup>1</sup> See fig. 1.

<sup>2</sup> See fig. 2.

At  $T = 0$ , the system has an order parameter  $\psi$  that satisfies the Gross-Pataevski equation

$$i\hbar \frac{\partial}{\partial t} \psi = -\frac{\hbar^2}{2m} \nabla^2 \psi + [U'(|\psi|^2) - \mu] \psi. \quad (2.9)$$

The current conservation and momentum conservation are

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \bar{v}) = 0 \quad \text{and} \quad M \frac{\partial}{\partial t} (\rho \bar{v}) + \nabla P = 0, \quad (2.10)$$

$$\text{where the current density is } \rho \bar{v} = \frac{\hbar}{2Mi} (\psi^* \nabla \psi - \psi \nabla \psi^*). \quad (2.11)$$

The ground state of the fluid is described by the uniform solution  $\psi_0$  that satisfies

$$U'(|\psi_0|^2) - \mu \psi_0 = 0 \quad \text{and} \quad \mu = E_0 + PV. \quad (2.12)$$

Perturbing the solution  $\psi = \psi_0 + \delta\psi_R + i\delta\psi_I$ , and substituting back, one obtains

$$\begin{aligned} \hbar \frac{\partial \delta\psi_R}{\partial t} &= -\frac{\hbar^2}{2m} \nabla^2 (\delta\psi_I) \\ -\hbar \frac{\partial \delta\psi_I}{\partial t} &= -\frac{\hbar^2}{2m} \nabla^2 (\delta\psi_R) + \frac{2B}{\rho} (\delta\psi_R) \end{aligned} \quad (2.13)$$

to first order.

$$\text{The dispersion relation is then } \hbar\omega_k = \sqrt{(\hbar v_s k)^2 + \left(\frac{\hbar^2 k^2}{2m}\right)^2}. \quad (2.14)$$

One identifies  $\xi = \frac{\hbar}{Mv_s}$  as the length scale beyond which hydrodynamics fails<sup>3</sup>.

Since a system with negative compressibility is unstable and hence unphysical, the latter is replaced by Maxwell's construction. Thus, there is only one point in the PV- diagram where the bulk modulus  $B = -V \left(\frac{\partial P}{\partial V}\right)$  is zero. This point corresponds to the critical point<sup>2</sup>.

In order to investigate the analogy between the critical surface and the event horizon, consider the following thought experiment. Suppose that a tank filled with a quantum fluid that satisfies (2.6) is put on the surface of the earth. The pressure inside the tank increases toward the bottom of the tank due to gravity. At some point, the pressure reaches and exceeds the critical pressure. Sound waves incident on this critical surface will refract toward it just as light is refracted toward a black hole horizon. The reason is that the propagation speed measured by a clock at infinity vanishes.

Consider the particular case when  $c = \frac{8a}{27b}$  in (2.6). At the critical point,

$$\rho_c = \frac{1}{\sqrt{3b}}, \quad P_c = \frac{a}{27b^2}, \quad v_c = \sqrt{\frac{P_c}{M\rho_c}}, \quad \text{and} \quad \frac{P}{P_c} - 1 \approx 12 \left(\frac{\rho}{\rho_c} - 1\right)^3. \quad (2.15)$$

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<sup>3</sup>The same length scale can be found from the Bogoliubov solution.

Near the critical surface,  $\frac{P}{P_c} \approx 1 - \frac{gz}{v_0^2}$  and  $\frac{v_s}{v_0} \approx 6 \left| \frac{gz}{12v_0^2} \right|^{\frac{1}{3}}$ . (2.16)

In order to find how sound propagates, consider small density fluctuations  $\rho \rightarrow \rho + \delta\rho$ , then  $\nabla \cdot [v_s^2 \nabla(\delta\rho)] = \frac{\partial^2 \delta\rho}{\partial t^2}$ . (2.17)

This is similar to the form of the scalar wave equation

$$\nabla \cdot [v_s \nabla \phi] = \frac{1}{v_s} \frac{\partial^2 \phi}{\partial t^2}, \text{ which is obtained from } \frac{\partial}{\partial x^\mu} (\sqrt{-g} g^{\mu\nu} \frac{\partial \phi}{\partial x^\nu}) = 0, \quad (2.18)$$

where the gravitational metric  $ds^2 = g_{\mu\nu} dx^\mu dx^\nu = dx^2 + dy^2 + dz^2 - v_s^2 dt^2$ . (2.19)

In order to retain the metric just outside the event horizon, weaken the gravity

$$g = g_0 (1 - e^{-\frac{z^2}{l^2}}), \quad (2.20)$$

then  $\frac{P}{P_c} \approx 1 - \frac{g_0 z^3}{3l^2 v_0^2}$  and  $\frac{v_s}{v_0} \approx 6 \left| \frac{g_0 z^3}{36l^2 v_0^2} \right|^{\frac{1}{3}}$ . (2.21)

The metric just outside the event horizon can be written in this form with  $v_s = \frac{c^3 z}{4GM}$ .

Since the principles of hydrodynamics no more hold for length scales smaller than the correlation length  $\xi = \frac{\hbar}{Mv_s}$  and time scales longer than  $\frac{\xi}{v_s}$ , sound ceases to make sense at the critical surface. In analogy to light, a sound quantum with fixed frequency  $\omega$  propagating toward the critical surface reaches a point at which  $\omega \geq \frac{v_s}{\xi}$  in finite time and decays there into soft excitations of the critical point. Hence most of the energy is thermalized if such excitations are dense. Although this effect hasn't been observed experimentally, its classical analogue, critical opalescence, has been observed by light scattering<sup>xiii</sup>.

By writing down the effective Lagrangian at criticality and the corresponding quantum mechanical Hamiltonian,

$$L_{eff} = \psi^* (i\hbar \frac{\partial}{\partial t} + \mu) \psi - \frac{\hbar^2}{2m} |\nabla \psi|^2 - \frac{3P_c}{\rho_c^2} (|\psi| - \psi_0)^4, \\ H_{eff} = \sum_k \frac{\hbar^2 k^2}{2M} a^\dagger_k a_k + \frac{3P_c \psi_0^4}{V \rho_c^2} \sum_{k_1, k_2, k_3, k_4} \delta(k_1 + k_2 + k_3 + k_4) \prod_{i=1}^4 (a_{k_i} + a^\dagger_{-k_i}), \quad (2.22)$$

one may investigate the experimental signatures of this effect.

One can show that the decay rate for a phonon with energy  $\hbar\omega = \frac{\hbar^2 k^2}{2M}$  is

$$\frac{\hbar}{\tau} = \frac{1}{3\pi^2} \left(\frac{M}{\hbar^2}\right)^3 \left(\frac{P_c}{\rho_c^2}\right)^2 (\hbar\omega)^2. \quad (2.23)$$

Moreover, one regains the ideal Bose gas behaviour for low energy from (2.23) because the phonon (boson) becomes more sharply defined as the energy decreases.

#### *Reflectivity*

Low frequency phonons incident on the critical surface are transmitted or reflected depending on their energy.

$$\text{Let } \delta\psi_R = \frac{\partial\phi}{\partial z}, \text{ then } \hbar^2 \frac{\partial^2\phi}{\partial t^2} = \left(\frac{\hbar}{\tau_0}\right)^2 \nabla \cdot (z^2 \nabla\phi) - \left(\frac{\hbar^2}{2M}\right)^2 \nabla^4\phi, \quad (2.24)$$

$$\text{where } \frac{1}{\tau_0} = \frac{\partial v_s}{\partial z}.$$

There are transition resonances that depend on the momentum component  $Q$  in the plane of incidence. When  $Q$  is large, the transmission resonances occur at harmonic values

$$\hbar\omega_n \approx \frac{\hbar^2 Q^2}{2M} + \left(n + \frac{1}{2}\right) \sqrt{2} \frac{\hbar}{\tau_0}. \quad (2.25)$$

When the incident phonon is normal ( $Q \rightarrow 0$ ), there is a singularity, and the reflectivity becomes continuous<sup>4</sup> with

$$|R|^2 = \begin{cases} 1 & \text{if } \omega\tau_0 < \frac{1}{2} \\ \cosh^{-2}\left(\pi\sqrt{(\omega\tau_0)^2 - \frac{1}{4}}\right) & \text{if } \omega\tau_0 > \frac{1}{2} \end{cases}. \quad (2.26)$$

#### *Inelastic Scattering*

The critical surface is opaque to high frequency sound waves and scatters about  $\frac{1}{8}$  of

them back with a strong red shift. When  $\frac{\tau_0}{\tau} > 1$ , the incident phonon decays with a

hundred percent probability to three phonons, and only one of the three can escape the surface. This gives a differential cross-section<sup>5</sup> per unit area  $A$  to scatter sound of frequency  $\omega$  back in a solid angle  $d\Omega$  and frequency  $\omega' < \omega$  as

$$\frac{d\sigma}{d\Omega d\omega'} = \frac{27A}{16\pi^2\omega} \sqrt{3x[1-3x-2\sqrt{x}\cos(\theta)]}, \quad (2.27)$$

where  $x = \frac{\omega'}{\omega}$ . Thus, the critical surface looks red, while being an energy thermalizer.

#### *Energy Density*

One may even calculate the energy density when  $z \rightarrow 0$ . The energy density at a distance  $z$  from the critical surface is

$$\frac{E}{V} = \frac{1}{2\pi^2} \int_0^\infty \frac{\hbar\omega_k k^2 dk}{e^{\beta\hbar\omega_k} - 1}, \quad (2.28)$$

where  $\omega_k$  is given by (2.14).

<sup>4</sup> See fig. 4.

<sup>5</sup> See fig. 5.

$$\text{As } z \rightarrow 0, \frac{E_0}{V} = 0.128 \left( \frac{M}{\hbar^2} \right)^{\frac{3}{2}} (k_B T)^{\frac{5}{2}}. \quad (2.29)$$

The divergence predicted by Planck's law  $\frac{E}{V} = \left( \frac{\pi^2}{30} \right) (k_B T)^4 \left( \frac{\tau_0}{\hbar z} \right)^3$  is removed by criticality, and the specific heat is large but finite.

Now how does all of that relate to black holes? Assume that the event horizon is a quantum liquid-vapour transition. The equations of classical general relativity outside the black hole are obeyed except at the event horizon. At the latter, space-time reorganizes itself in such a way that a universal time is still defined. Locally, the properties of the vacuum are the same inside and outside of the vacuum. We will investigate the consequences on black holes based on these assumptions.

The most general spherically symmetric metric in four dimensions is given by (2.1),  $ds^2 = -f(r)dt^2 + \frac{dr^2}{h(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$ . By imposing that Einstein's field equations are obeyed outside the event horizon, one retrieves the Schwarzschild solution (2.2) with  $f(r) = h(r) = 1 - \frac{2M}{r}$ . Requiring that the local properties across the horizon are similar,  $f(r) = h(r)$  just inside the horizon; however, it converges to zero with opposite sign. This implies that there is negative pressure inside the black hole. Moreover, matter should resist falling into the horizon, which is the minimum of the gravitational potential. Therefore, one needs a nonzero cosmological constant inside the black hole, and Einstein's equations are modified inside the black hole such that

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} = \frac{3}{4M^2} g_{\mu\nu}, \quad (2.30)$$

which corresponds to vacuum with positive cosmological constant (de Sitter space is the more technical term). The total energy inside the black hole is  $M$ , and the metric<sup>6</sup>

$$f(r) = \begin{cases} 1 - \frac{2M}{r} & \text{if } r > R_s \\ 1 - \left( \frac{r}{2M} \right)^2 & \text{if } r < R_s \end{cases}. \quad (2.31)$$

The singularity at the horizon is equivalent to a negative surface tension or stress to accommodate the negative pressure inside the black hole. Similar to a balloon, a black hole with pressure  $P = -\frac{3c^8}{32\pi G^4 M^2}$  will have a surface tension  $T = -\frac{3c^2}{32\pi G^2 M}$ <sup>7</sup>. The tension is generated by space-time itself as it undergoes the transition. Still, it is small. Since we are regarding the event horizon as the critical surface, a distant observer can measure a finite heat capacity of the black hole. The expression is similar to (2.29),

<sup>6</sup> See fig. 3.

<sup>7</sup> The above expressions are in proper coordinates.

$$\begin{aligned}
E &= \int_0^{\frac{2GM}{c^2}} \left\{ \frac{f^{-\frac{1}{2}}(r)}{2\pi^2} \int_0^\infty k^2 \frac{\hbar\omega_k}{e^{\beta\hbar\omega_k} - 1} dk \right\} r^2 dr, \\
&\approx 1.1 \times \left\{ \left[ \frac{4\pi}{3} \left( \frac{2GM}{c^2} \right)^3 \right] \left[ \frac{\pi^2}{30} \frac{(k_B T)^4}{(\hbar c)^3} \right] \right\} \frac{Mc^2}{k_B T}
\end{aligned} \tag{2.32}$$

where  $\omega_k$  is given by (2.14).

### *Experimental signature for black holes*

If the above analogy is true, a photon incident on the critical surface of a black hole will decay into three photons. No such effect has been observed experimentally. Still, it is a prediction that can be falsified by experiments. In this picture, the break down of relativity must be observed experimentally as the spontaneous decay of bosons such as photons. Probably this can be observed in cosmic rays.

Moreover, black holes have a specific spectroscopic signature that can be observed outside the event horizon. The horizon reflects light with frequency higher than  $c$  time the black hole radius, while being transparent to light with frequencies slightly above this frequency in resonances that depend on the incidence angle. For one solar mass black hole the reflection and transmission resonances are in the radio frequency range  $\approx 10^5$  Hz .

Just like the case of the critical surface, high frequency electromagnetic radiation is inelastically scattered from the black hole, and it's normal component is red shifted. Furthermore, the black hole thermalizes radiation and acts as a thermal body with a finite heat capacity.

The possibility of the transition to be first order is unlikely because it is unstable to density perturbations. From fig. 2 perturbing the equation of state downwards causes the susceptibility of phonons to become negative, and the density perturbations of the uniform state grow. For gravity, the excitations are gravitational waves

$\delta g_{xx} = -\delta g_{yy} \approx e^{i(kz - \omega t)}$ . If the phase transition is first order, the excitations are unstable, and they will generate a non-uniform metric with sharp jumps. Irrespective whether it is a first or second phase order transition, quantum information isn't destroyed by the black hole. Instead, energy is scattered as if by dark paint.

## **2.2 Gravitational Condensate Stars as Alternatives to Black Holes**

P.O. Mazur and E. Mottola extend further the idea of gravitational Bose-Einstein condensation of space-time, and they propose a singularity free alternative to black holes<sup>xiv</sup>. Like the black hole proposed by Laughlin *et al.*, their gravitational condensate star is a compact object with de Sitter interior and exterior Schwarzschild geometry. However, instead of the critical surface, a thin layer of ultra-relativistic fluid whose equation of state is  $P = \rho c^2$  separates the two regions. Their solution is singularity free without an event horizon and with a universal time<sup>8</sup>. Moreover, it is stable to thermodynamic fluctuations and its entropy is far less than the progenitor star. The same

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<sup>8</sup> For a review of nonsingular quasi-black-holes see I. Dymnikova, [gr/0010016]

authors in another paper suggest the process by which the separating layer comes about<sup>9</sup>. It is based on quantum anomalies that arise in conformal theories.

According to their solution, there are three regions with respect to the gravitational condensate star,

- (1) Interior:  $0 \leq r \leq r_1$ ,  $\rho = -P$ , and  $f(r) = Ch(r) = C(1 - H_0 r^2)$
- (2) Shell:  $r_1 < r < r_2$  and  $\rho = P$
- (3) Exterior:  $r > r_2$ ,  $\rho = P = 0$ , and  $f(r) = h(r) = 1 - \frac{2GM}{r}$ . (2.33)

All the parameters entering the Einstein field equations are required to be continuous, but not necessarily differentiable. The derivation is quite simple. The only components that enter are the Einstein field equations for a perfect fluid, the conservation of the stress tensor, and the condition of continuity across the boundaries. In principle, one can solve the equations numerically, but an analytic solution exists in the thin shell limit. In the analytic solution, one will find an integration constant  $\varepsilon \ll 1$  related to the thickness of the shell  $l \approx \varepsilon^2 R_s$ .

The entropy of the fluid within the shell is obtained from the equation of state  $\rho = P = \left(\frac{a^2}{8\pi G}\right)\left(\frac{k_B T}{\hbar}\right)^2$ . The entropy  $S \approx ak_B \frac{M}{\hbar} R_s \varepsilon^{\frac{3}{2}} \approx ak_B \frac{Ml}{\hbar} \ll S_{BH}$ , (2.34)

where  $S_{BH} \approx 10^{77} k_B \left(\frac{M}{M_{Sun}}\right)$  the Bekenstein-Hawking entropy. In region (1),  $\rho = -P$  and the entropy  $S$  vanishes. This is expected for a gravitational Bose-Einstein condensate described by a single macroscopic wave function.

Furthermore, the local temperature of the shell is of the order  $\frac{\hbar}{k_B GM}$ , and the gravitational condensate star doesn't emit Hawking radiation because it doesn't have an event horizon. With no such emission, it is ultra-cold and completely dark<sup>10</sup>.

The cold fluid in region (2) is confined by the surface tensions at  $r_1$  and  $r_2$ . The discussion is based on the equation of state in different regions and is similar to the one in the previous section.

Although the shell is responsible for the entropy of the condensate star, it doesn't contribute much to the mass of the object. The energy within the shell as measured from infinity is  $E_{(2)} \approx \varepsilon^2 M \ll M$ . Hence, the energy density of the vacuum condensate in the interior region contributes to the mass of the condensate star.

Moreover, since the fluid in the shell has a positive heat capacity and all regions other than the shell are vacuum, the condensate star is thermodynamically stable. One can verify this stability by working in the microcanonical ensemble and maximizing the entropy of the shell with respect to small fluctuations.

#### *Observable Consequences of the gravitational condensate star*

The gravitational compact star proposed by P.O. Mazur and E. Mottola is a stable compact non-singular spherically symmetric solution of Einstein's equations. Its entropy

<sup>9</sup> See P.O. Mazur and E. Mottola, *Phys Rev. D* 64, 104022 (2001)

<sup>10</sup> This doesn't preclude the fact that it might "reflect" energy, as will be discussed later.

is around twenty orders of magnitude less than the entropy of the progenitor. Hence, unlike the semi-classical case, there is no entropy paradox. The supernovae explosion of the progenitor star might be so violent that it sheds enough entropy to produce the relatively cold gravitational condensate star<sup>11</sup>. This might be a possible explanation of why there are gamma-ray bursts from distant stars, which is something unexplained until the present moment.

GCS might shine more brightly because matter falling onto it is transformed into radiation that can escape, while a black hole “gulps” all the matter that falls onto it. As such, GCS might be a candidate for an energetic astrophysical source, although it doesn’t radiate on its own.

One may consider that we are living in such a GCS. This might explain the problem of the cosmological constant. By analyzing the data from 42 type Ia supernovae discovered by the Supernovae Cosmology Project, S. Perlmutter *et al.* found that the red-shifted light from these supernovae is dimmer than expected if the universe were flat<sup>xv</sup>. Until recently, the only conceived solution to this problem was that there is some sort of dark energy in the universe that gives an outward pressure in space, and that our universe is expanding<sup>12</sup>. As such, we live in a universe with positive cosmological constant. If one regards the whole universe to be the condensate interior of a GCS whose radius is that of the universe, with the quantum phase interface replacing the horizon of the universe, then the estimated cosmological constant is close to the observed one, which is of the order of the matter density present in the universe<sup>13</sup>. Although this idea seems contentious, in the following section we will discuss how one can resolve the cosmological constant problem by regarding the vacuum of space-time as analogous to that of quantum liquids.

### 3 Quantum vacuum and the cosmological constant problem

Experimental evidence [xv] supports the existence of a positive cosmological constant that is of the order of the present matter density. If one naively applies the approximations of quantum field theories, one will find an estimate of the cosmological constant that is 120 orders of magnitude more than the observed one. This estimate is based on calculating the vacuum energy density as zero point energy from bosonic fields and negative from the occupied fermionic levels of the Dirac sea. Even if supersymmetry is accounted for in order to tune this estimate, the discrepancy between the theoretical value and the observed value is enormous. This disparity means that self-energy of the vacuum isn’t gravitating, which is in contradiction with the equivalence principle. By regarding the vacuum of quantum liquids as analogous to the vacuum in general relativity, G.E. Volovik proposes a solution to the cosmological constant problem<sup>xvi</sup>.

As discussed before, the motivation to regard space-time vacuum as similar to the vacuum of quantum liquids is that there is an effective Lagrangian metric in the latter, and hence some sort of gravity arises in the low energy limit. Analogy with quantum

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<sup>11</sup> GCS for short.

<sup>12</sup> Quite recently, C. Csaki *et al.* proposed an alternative solution [C. Csaki, N. Kaloper, and J. Terning, Phys. Rev. Lett. **88**, 161302 (2002)]. They claim that photons emitted by supernovae turn into axions while propagating in the intergalactic magnetic field, and this is why the supernovae appear dimmer than what is expected for a flat universe. Axions are hypothetical particles that account for the left-handedness and right-handedness asymmetry. The CERN Axion Solar Telescope will search for them in the future.

<sup>13</sup> See fig. 6.

liquids is especially beneficial because both the effective theory and the microscopies are known, and hence it is a possible route from effective theories to microscopies.

For  $^4\text{He}$  and  $^3\text{He}$ , when temperature is reduced below 1K, one can no more resolve the motion of isolated atoms in the liquid. The lower the energy, the better the description in terms of collective mode and dilute gas of quasi-particles. This is the Landau picture of two-fluid model that incorporates the collective motion of the ground state and the dynamics of the quasi-particles in the background of the moving fluid. In the general relativity picture, quasi-particles may represent matter that moves in the space-time background<sup>14</sup>. Moreover, interaction and zero point motion of the atoms in the condensate compete and provide an equilibrium ground state of the ensemble of atoms. This equilibrium exists even without external pressure. In the language of relativity, the cosmological constant in the effective theory of gravity is exactly zero without any tuning, and the equilibrium vacuum isn't self-gravitating. This isn't the whole story. The perturbations due to quasi-particles (or analogously matter) generate the remaining vacuum energy, and this is why the cosmological constant is of the order of the mass density present in the universe. Other possible sources of vacuum energy are fluctuations in the topology and the discreteness of the number of particles in a condensate. However, the latter might not hold for space-time vacuum. The analogy is incomplete because in quantum liquids there is a conservation of number of particles, while in space-time vacuum no equivalent conservation number is known. This indicates the need to know the microscopies.

Notice that unlike [viii], G.E. Volovik regards gravitons as low-lying excitations like phonons in a crystal, and not as the building blocks of space-time. However, in either case there is collective behavior.

## 4 Conclusion

In this paper we discussed the observable consequences of regarding classical gravity as an effective theory that emerges from a quantum mechanical one. Just like a tranquil fish that lives in a “calm” pond unaware of the incessant motion of water particles around it, we might be living in an intrinsically quantum mechanical space-time. Although there isn't any complete microscopic theory, this approach suggests answers to problems in classical and semi-classical general relativity, like information loss in black holes, the entropy paradox and the cosmological constant problem. The proposed ideas can be subject to experimental tests that may falsify them. Quantizing gravity *per se* in this picture doesn't make sense, and one needs instead to search for a microscopic quantum theory of the vacuum of space-time<sup>15</sup>.

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<sup>14</sup> G.E. Volovik pushes the analogy with quantum liquids further and proposes that the whole of relativistic quantum field theory is an emergent phenomena arising from the low energy corner of fermionic vacuum. Collective fermionic and bosonic modes in  $^3\text{He-A}$  give chiral fermions, gauge fields, and in some respect gravitational fields. See G.E. Volovik, Phys. Rep. **351**, 195 (2001);[gr-qc/0005091].

<sup>15</sup> At the present moment String theory isn't a good candidate because it assumes a background metric. However, with the application of non-commutative geometry in String theory, this might change in the future.

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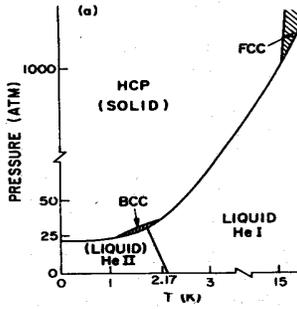


Fig. 1. Schematic phase diagram of condensed He

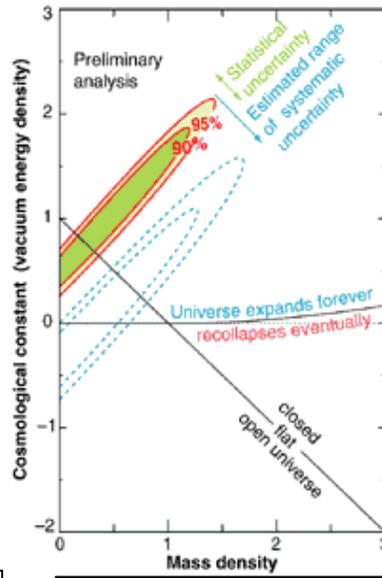


Fig. 6. Results from SCP

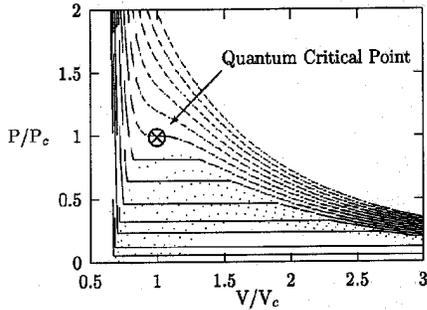


Fig. 2. Phenomenological equation of state defined by (2.6). The Maxwell loops are indicated by dotted lines.

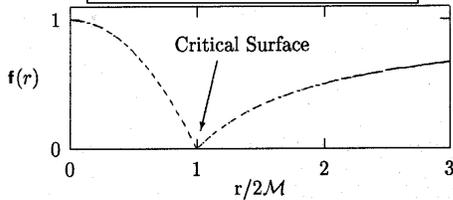


Fig. 3. Prototype time dilation factor  $f(r)$  in the vicinity of a black hole event horizon.

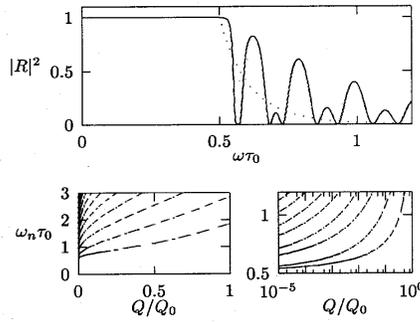


Fig. 4. Top: Reflectivity as a function of  $\omega\tau_0$  for case  $\frac{Q}{Q_0} = 10^{-5}$ ;  $Q_0 = \left(\frac{2M}{\hbar\tau_0}\right)^{1/2}$ . Bottom: Dispersion relation of interface bound states plotted linearly (left) and logarithmically (right).

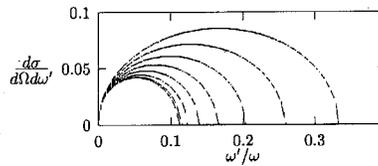


Fig. 5. Differential cross-section given by (2.27).