Bits and Pieces of Spin Glass Physics

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Abstract

Spin glasses, an emergent phase of matter in random magnetic systems, is briefly discussed with an aim of understanding Replica Field theory, a tool to tackle quenched disorder in general. The physics of spin-glasses plays a pivotal role in our understanding of quenched disordered sytems. The Replica symmetric solution of the SK model is discussed in affordable detail and the necessity of Replica Symmetry Breaking (RSB) is argued. The TAP equation is introduced as an alternative to RSB. The experimental scenario is briefly dicussed, where *aging*, a peculiarity of spin-glass is introduced.

1 Introduction: The spin-glass episode

Spin glasses are random magnetic systems in which the interaction between the spins are in conflict to each other; a phenomenon known as *frustration*. It is a result of strong frozen-in structural disorder called *quenched randomness*. I will explain these two terms shortly. An experimentalist identifies a magnetic material as a spin glass if it exibits the following characteristic properties:

1. The frequency dependent susceptibility (ac susceptibility) exibits a cusp in at a certain temperature T_f (known as the freezing temperature). The cusp is very sesitive to magnetic field.

2. Below T_f the magnetic response is history dependent, ie. the magnetic susceptibility measured in a field-cooled sample is higher than one cooled in zero-field. A hysteresis curve is observed.

3. Below T_f, decay of remanent magnetization (magnetic relaxation) is on macroscopic time scale.

4. No sharp anomaly appears in the specific heat around T_f.

5. Below T_f, no signature of Long Range Order (LRO) is observed (for example, in magnetic Bragg scattering experiments with neutrons).

For a rather detailed description of experimental results on spin-glasses please refer to the excellect review by Binder and Young [2] and the book by Debashish Choudhury [4](page 278).

1.1 Model of spin-glass: The Sherrington Kirkpatric model

Spin glass is a vast topic and I will make no attempt to be comprehensive nor accurate in the many subtle aspects of its physical characteristics and numerous theoretical models. Instead, in this section, I will describe *one model* of spin-glasses, which may have the drawbacks of being an oversimplified one. The Sherrington Kirkpatric model of spin-glasses is defined by the usual Ising spin Hamiltonian (SK Hamiltonian) :

$$\mathcal{H} = -\sum_{i<j}^{N} J_{ij}\ \sigma_i \sigma_j$$

where the spin spin interactions J_{ij} are randomly quenched variables and the σ_i s are classical spins taking values ± 1. This simple model tries to capture the basic ingredients of spin-glass physics ie, quenched randomness and frustration, and it can be solved exactly at the mean-field level. *Quenched randomness* means that there is no equlibrium statistical mechanical fluctuation associated with it, in contrast to *annealed randomness* which is due to random thermal fluctuations. The annealed variables are averaged over thermodynamic ensembles when we extract macroscopically observable quantities. The set of quenched random variables differ quantitatively in different realization of the random system and is frozen in one particular realization for the system and the time span over which it is observed. In the Sherrington-Kirpatric model, for example, the spins are annealed variables and the random bonds are quenched variables. Next we need to understand a crucial ingredient of spin-glass physics, namely frustration. Consider three Ising spins on a triangle with veritces $1, 2, 3$. The three interactions between the spins J_{12}, J_{13}, J_{23} taking values ± 1. If the product of the J's for any particular realization is negative (ie. one of the interactions is negative or all three of them are) then the ground state of the sytem is degenerate, one of the spins has mutually conflicting 'order' from its neighbor as to which way to point minimize mutual interaction energy and the system is said to be 'frustrated'. It should be noted however that not all disorder induces frustration. For example if in the SK Hamiltonian the J_{ij}'s are taken to be $J_{ij} = \eta_i \eta_j$ where the quenched variable η_i takes values ± 1 with equal probability, then no frustration appears in the system. This is because, under a trivial redefinition of the spin variables, $\sigma_i \rightarrow \eta_i \sigma_i$ an ordinary ferromagnetic Ising model is recovered.

Before analyzing the SK model with the Replica Field Theory toolbox, we need to understand the nature of ergodicity breaking, choice of order parameter and the nature of phase transition in spin-glasses.

1.1.1 Ergodicity Breaking and nature of phase transition

I am assuming familiarity with ergodicity breaking in classical Ising model of ferromagnets. It manifests itself as Spontaneous Symmetry Breaking (SSB) in the ground state which ends up having a reduced symmetry compared to the defining Hamiltonian. Recall that SSB results from the breakdown of the ergodic hypothesis when system size is taken to be infinity *before* the observation time is taken to be infinity and the system gets trapped in one of the ground states. In spin glasses the nature of SSB is quite complex. Since the ground state is highly degenerate because of frustration, and since the (coarse-grained) free energy landscape itself depends on the exact realization of the quenched disorder and is very complicated and jaggered, it is quite probable that the system may be trapped ('frozen') in one of the many metastable states seperated by high energy barriers and hence a very long relaxation time. The SK model (to be precise, its mean-field solution) captures some of these features, as we will see in the discussion of Replica Symmetry Breaking. An interesting consequence is SSB takes place not just at a certain critical temperature T_c but at *any* tempertaure below T_c! **This is because below T_c the free energy landscape gets divided into macroscopically large number of valleys seperated by possibly infinite barrier and as one lowers the temperature each of these valleys split up into more valleys and barriers.** Contrast this to the free energy landscape of a Ising ferromagnet for example, which has only two global minima at zero temperature. This process of fragmentation of the space of states continue and therefore at any temperature below T_c the system is in a critical state (in the sense that ergodicity breaking continues to occur).

1.1.2 Choice of order parameter

In order to motivate the physical order parameter let us consider the following thought experiment. We take a specific realization (ie. our experimental sample of spin system) of the quenched random variables (the set of J_{ij} in SK model) and allow the spins to thermally equlibriate. At each site i the thermal average of the spin has some value, let it be $\langle\sigma_i\rangle_\alpha$, where α is the experiment number. We repeat this process (we needn't change the set of J_{ij}), and since the ground state is highly degenerate the system will in general thermally relax to completely different states. Now let us define the object

$$q_{\alpha\beta} = \frac{1}{N}\sum_i^N \langle\sigma_i\rangle_\alpha \langle\sigma_i\rangle_\beta$$

which we will call the *overlap*. The maximum value of one is achieved by it when the two final states in the experiment coincide (self-overlap). To describe the statistical property of the overlap the following probability distribution is invoked:

$$P(q) = \sum_{\alpha\beta} \delta(q_{\alpha\beta} - q)$$

Suppose the spin system is a paramagnet. Then there is only one global minimum of the the free energy; the one in which all the site magnetization are equal to zero. $P(Q)$ in this case is a delta function at zero. If we were looking at a ferromagnet the minima of the free energy occurs at values of site magnetization $\pm m$. $P(Q)$ in this case has delta-function peaks at $\pm m^2$. According to the mean-field theory of spin-glasses, this distribution function for the spin glass phase looks essentially different. There are two delta-function peaks at $q - \pm q_{max}(T)$ corresponding to the self-overlap (remember that the degeneracy of the ground state is macroscopically large). But between these delta-function peaks there is a continuous curve. This corresponds to the finite overlap between the hierarchy of fragemented valleys described earlier. The distribution function $P(Q)$ can be considered to be the physical order parameter because it adequately describes the peculiarities of the spin-glass phase. It can be calculated exactly within the mean-field theory of SK spin glass model.

1.2 Mean field solution of the SK model

The mean-field description of the SK model is exceptionally non-trivial as we will see. Each spin of the SK model interacts with all other spins, in a sense all of them can be considered to be nearest neighbours and the dimensionality of space and the type of lattice is irrelevant simply because in the thermodynamic limit one can interpret it as an infinite-dimensional lattice. It is this property that makes the mean-field approach exact. The SK Hamiltonian, as already introduced is:

$$\mathcal{H} = -\sum_{i<j}^N J_{ij}\ \sigma_i\sigma_j$$

where the probability distribution of the spin-spin interaction J_{ij} is taken to be Gaussian distribution for *all* pair of sites (i,j) :

$$P[J_{ij}] = \prod_{i<j}\left[\sqrt{\frac{n}{2\pi}}\ \mathbf{exp}\left(\frac{-NJ_{ij}^2}{2}\right)\right]$$

where

$$\overline{J_{ij}} = 0 \qquad : \qquad \overline{J_{ij}^2} = \frac{1}{N}$$

and

$$\overline{(...)} \equiv \int \mathcal{D}\mathrm{JP[J]}(...)$$

Note that I will always use $\langle .... \rangle$ to denote thermal averaging and $\overline{(...)}$ to denote disorder averaging.

1.2.1 Replica symmetric solution

The method of dealing with quenched disorder is to introduce the concept of *self-averaging*. The free energy of a system is an extensive quantity. In the thermodynamic limit any macroscopic system can be subdivided into a very large number of macroscopic systems. The total free energy of the system will be the sum of the free energies of the subsystems plus contribution from the interaction of the subsystems at the boundaries. But the effect of the boundaries is negligible in the thermodynamic limit. Each of these subsystems can be considered to be a realization of the quenched set of variables. Therefore the sum of the free energies of these subsystems can be considered to be the average over the quenched disorder, ie.

$$\mathcal{F} \equiv \overline{F_J} = -\frac{1}{\beta}\overline{\ln(\mathrm{Z_J})}$$

Doing the average on the logarithm is very difficult, since the partition function in general has a compliocated dependence on the spin-spin couplings. Here comes in the replica trick. We use the identity

$$\overline{\ln(\mathrm{Z_J})} = \lim_{n \to 0} \frac{\overline{Z^n} - 1}{n}$$

We will introduce n replicas (copies) of the system (n is an integer). These replicas are non-interacting, though after performing the disorder average the n-replica partition function will not be separable into the n replicas, in general. This will be our $\overline{Z^n}$ and finally we take the $n \to 0$ limit. The final step seems to be dubious, but replica field theory results have been confirmed wherever other methods of calculation exists. Using greek letters to denote replica index, the n-replica partition function is

$$\overline{Z^n} = \sum_{\sigma_i^\alpha} \int \mathcal{D}\mathrm{J_{ij}} \, \exp\left(\frac{1}{K_B T} \sum_{\alpha=1}^{n} \sum_{i<j}^{N} J_{ij}\sigma_i^\alpha \sigma_j^\alpha - \frac{1}{N}\sum_{i<j} J_{ij}^2 \right)$$

On integrating over J_{ij} we get

$$\overline{Z^n} = \sum_{\sigma_i^\alpha} \exp\frac{1}{(K_B T)^2}\left[\frac{N}{2}\sum_{\alpha<\beta}^{n}\left(\frac{1}{N}\sum_i^N \sigma_i^\alpha \sigma_i^\beta\right)^2 + \frac{Nn}{4}\right]$$

(**Caveat**: I will be forced to skip most algebraic steps, to maintain brevity and simplicity, possibly at the cost of mathematical clarity, refer [5], [2]).

The above equation can be linearised (in the spins) by introducing replica matrix (dummy integration variables) $Q_{\alpha\beta}$ by, what is known as, a Hubbard Stratonovich transformation :

$$\overline{Z^n} = \left(\prod_{\alpha<\beta}^{n} \int dQ_{\alpha\beta} \right) \sum_{\sigma_i^\alpha \sigma_i^\beta} \exp \frac{1}{(K_BT)^2} \left[\frac{Nn}{4} - \frac{N}{2} \sum_{\alpha<\beta}^{n} Q_{\alpha\beta}^2 + \sum_{\alpha<\beta}^{n} \sum_{i}^{N} Q_{\alpha\beta} \sigma_i^\alpha \sigma_i^\beta \right]$$

We will soon find a physical interpretation of the quantity $Q_{\alpha\beta}$. Note that we can write :

$$\overline{Z^n} = \int \mathcal{D}\hat{Q} exp \left(-\frac{1}{K_BT} nN f_n[\hat{Q}] \right)$$

where we have defined the effective energy to be

$$f_n[\hat{Q}] = -\frac{1}{K_BT} + \frac{1}{2nK_BT} \sum_{\alpha<\beta}^{n} Q_{\alpha\beta}^2 - \frac{K_BT}{n} log \left[\sum_{\sigma_\alpha \sigma_\beta} exp \left(\underbrace{\frac{1}{(K_BT)^2} \sum_{\alpha<\beta}^{n} Q_{\alpha\beta} \sigma^\alpha \sigma^\beta}_{defined\ to\ be\ L} \right) \right]$$

The site indices are absent on the spins since all of the sites are equivalent and N can be extracted out. In the thermodynamic limit $N \to \infty$ we can use the method of steepest descent to evaluate the integral. The saddle point equation is:

$$\frac{\delta f}{\delta Q_{\alpha\beta}} = 0$$

Note that we are taking the infinite system limit $N \to \infty$ *before* taking the replica limit $n \to 0$. The saddle point condition gives us :

$$Q_{\alpha\beta} = (K_BT)^2 \frac{\partial}{\partial Q_{\alpha\beta}} Tre^L = \frac{Tr \sigma^\alpha \sigma^\beta e^L}{Tre^L}$$

So $Q_\alpha \beta$ is average of the spin-spin correlation with respect to the 'Lagrangian' L. It can be shown that this expression for $Q_{\alpha\beta}$ is identical to the expression

$$Q_{\alpha\beta} = \overline{\langle \sigma^\alpha \sigma^\beta \rangle}$$

Compare with the physical order parameter discussed before. We need to add a site index, to be precise (same index for both the spins).

The strategy of solving the problem, ie. finding out the order parameter $Q_{\alpha\beta}$ should ideally be to take an arbitrary matrix Q, find the solution for the saddle point equation and the corresponding value of the replica free enrgy. This will provide us with the partition function. But it is impossible to carry out this strategy for an arbitrary matrix. So we have to assume the form of Q. Naively, since all replicas in the system is equivalent it is tempting to assume a replica symmetric form of $Q_{\alpha\beta}$. Let

$$Q_{\alpha\beta} = q \; ; for\ all\ \alpha \neq \beta$$

(There are no terms on the expression of the effective energy in which $\alpha = \beta$). This is the replica symmetric (RS) approximation. If one works to some algebra one should get the final form of the final form of the effective free energy in the limit $n \to 0$ (replica limit) to be:

$$f(q) = \frac{1}{4K_BT}(1-q)^2 - K_BT\int_{-\infty}^{+\infty}\frac{dz}{\sqrt{2\pi}}exp\left(\frac{-z^2}{2}\right) ln\left[2cosh\left(\frac{z\sqrt{q}}{K_BT}\right)\right]$$

and from the condition $\partial f/\partial q = 0$ one gets the following saddle-point equation for the parameter q :

$$q = \int_{-\infty}^{+\infty}\frac{dz}{\sqrt{2\pi}}exp\left(\frac{-z^2}{2}\right) tanh^2\left(\frac{z\sqrt{q}}{K_BT}\right)$$

The self-consistent equation for q has the solution $q = 0$ for $T \geq T_c = 1$ (actually $T_c = J/K_B$; the mean value of the random quenched variable J_{ij} which we chose to be equal to one for simplicity). When $T < T_c$ there exits a non-trivial solution $q(T) \neq 0$. Near T_c $q(T) \approx T_c - T + O(T_c - T)^2$. Note that for $T \approx 0,\ q = 1$.

The most striking aspect of the RS solution is that *it is wrong!* Amongst other things, for example, the RS solution gives a negative ground state entropy at low temperature. It can be shown that for very small temperature $q(T) \approx 1 - \sqrt{2/\pi}/J$ (recall that we have taken $J = 1$ throughout). The energy at low temperature therefore behaves as

$$f(q)|_{T\approx 0} = -\sqrt{\frac{2}{\pi}}J + \frac{T}{2\pi}$$

which gives a negative ground state energy and negative entropy equal to $-1/2\pi$.

A clearer way to convince oneself that the RS solution cannot be correct is to check its stability. One needs to check whether the RS solution defines the correct extremum of the free energy by considering small deviations about the RS form of the order parameter matrix $Q_{\alpha\beta}$ ie. substituting $Q_{\alpha\beta}$ by $q + \eta_{\alpha\beta}$ and expanding the free energy up to second order in $\eta_{\alpha\beta}$ which is the small parameter. Formally ;

$$f(q, \hat{\eta}) = f_{RS}(q) - \frac{\beta}{2n}\sum_{(\alpha\beta)(\gamma\delta)} G_{(\alpha\beta)(\gamma\delta)}\eta_{\alpha\beta}\eta_{\gamma\delta}$$

(recall that all Greek indices are replica indices). G is known as the **Hessian matrix** and its eigenvalues has to be positive definite for stability. (Caveat: there are a few subtleties here which I was forced to gloss over. It turns out that the solution $q > 0$ for $T < T_c$ *maximises* the the free energy instead of *minimising* it. This is an artifact of replica field theory; the form of the free energy changes sign at $n = 1$ and we have negative number of replica pairs in the limit $n \to 0$ which causes maiximisation. The number of replica pairs appear in the expression of the free energy before taking the replica limit) It turns out that if we include an external magnetic field in the problem we get a line of stability of the RS solution in the $h-T$ plane known as the *Almeida-thouless line*. For the curious mind, this line is defined by two equations; the equation for q given above (modified in an obvious way to include magnetic field h) *and* the following

$$\int_{-\infty}^{+\infty}\frac{dz}{\sqrt{2\pi}}cosh^{-4}\left(\frac{1}{K_BT}(z\sqrt{q}+h)\right) exp\left(-\frac{1}{2z^2}\right) = T^2$$

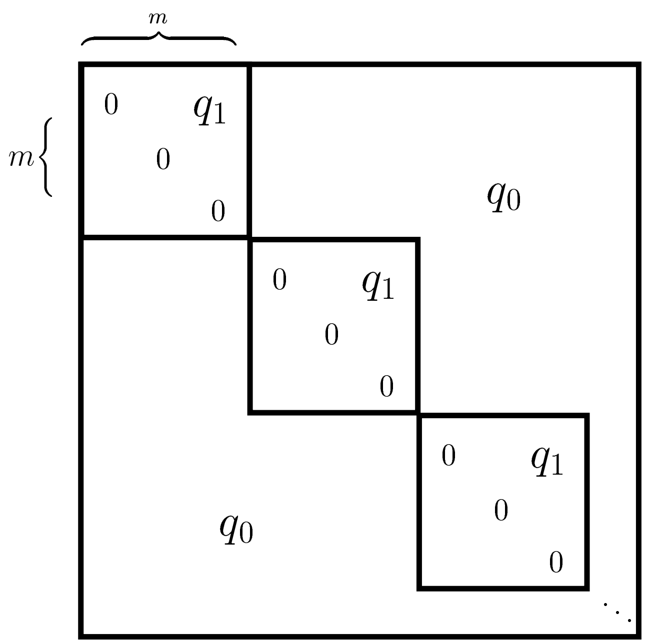

Figure 1: Parisi Matrix: One-step RSB

Below this line RS solution is unstable (a subclass of all the eigenvalues of the Hessian is negative) and above it the RS solution is stable. In the limit $h \to 0$ the line meets the point T_c in the $h - T$ plane, hence *RS solution is unstable at zero external magnetic field in the entire spin-glass region of the phase diagram.*

The above discussion convinces us that we need to look for more sophisticated form of the matrix $\hat{Q}$ with more parameters; calculate the Hessian for the ansatz and check whether it is positive definite, and repeat this process till we get the right matrix. *Surprisingly, the right solution is much more sophisticated than that; no ansatz that contains a finite number of parameters can provide a stable solution.* A strategy for finding the true solution is the famous *Parisi Replica Symmetry Breaking (RSB) Scheme*. This is an infinite sequence of ansatzs that approximate the true solution better and better and in the limit of infinite sequence, the true solution can be formulated in terms of continuous function. I will not go into the details of RSB; it is too technical and long to fit in here. But just to give a flavor of RSB; the form of $Q_{\alpha\beta}$ one needs to assume for one-step RSB is

$$Q_{\alpha\beta} = \begin{cases} q_1 \text{ if } & I\left(\frac{\alpha}{m}\right) = I\left(\frac{\beta}{m}\right) \\ q_0 \text{ if } & I\left(\frac{\alpha}{m}\right) \neq I\left(\frac{\beta}{m}\right) \end{cases}$$

where I is a integer valued function which which is equal to the smallest integer larger than or equal to its argument, and we have divided n replicas into m groups each containing m replicas. we have assumed that n and n/m are integers. All the above words are supposed to describe the following matrix! (all the off diagonal elements in each block is non-zero and equal to the element placed off-diagonlly in the figure) :

The form of the Parisi matrix is similar for higher-step RSB, only more complicated. The matrices fragment into smaller and smaller sub-blocks within each block. There are a few funny things going on here, for example, we have chosen $1 \leq m \leq n$ where m is a integer (number of replicas in each group); but in the replica limit $n \to 0$ the interval turns into $0 \leq m \leq 1$ and m becomes a continuous parameter! Ignoring all these mathematical details [5][3], the nitty-gritty of one-step RSB is the following:

- Although at low temperature the entropy of the system in one-step RSB solution becomes negative as in the RS solution, its absolute values in much smaller: $S(T = 0) \approx -0.01(for\ one - step\ RSB) : \ S(T = 0) \approx -0.17(for\ RS)$
- The absolute value of the most negative eigenvalue of the Hessian near T_c in the one-step RSB solution is smaller than the RS solution by a factor of nine.

This seems to hint that one-step RSB is a better approximation than the RS solution, since the latter's disease seems to have been alleviated, though not cured, and higher-RSB may do better; which is correct. RSB brings along with it some really neat mathematical structure in the space of the probability distribution $P(Q)$ of overlap we defined earlier, which is known as *ultrametricity*. I refer the avid reader to an excellent article on ultrametricity[1].

The obvious question one can ask is whether the RSB happens in real spin-glasses or is it a peculiar feature of the infinite-dimensional SK model. I address this question when I discuss experimental results later. An interesting question to ask would be; what can be said about spin glasses from mere symmetry arguments? Before addressing this question I will give a brief overview of a different point of view on spin glasses; an equation of state for the local magnetisation within the framework of the SK model, known as the TAP equation after Thouless, Anderson and Palmer [2]. Owing to the lack of space the next few sections will be very sketchy. I will provide ample references to bolster up the discussion.

2 The TAP equation

The TAP equation is a solution of the SK model without having to use the Replica trick, ie. without having to take the configuration average over the distribution of J; the random interaction. The TAP equation for the local magnetisation m_i of the SK model in the absence of external magnetic field is :

$$m_i = tanh \frac{1}{K_B T} \left[\sum_j J_{ij} m_j - \frac{1}{K_B T} \sum_j J_{ij}^2 (1 - m_j^2) m_i \right]$$

Compared to the well-known mean-field solution of the Ising ferromagnet with *no disorder*, ie the solution

$$m_i = tanh \left[\frac{1}{K_B T} \sum_j J_{ij} m_j \right]$$

the TAP equation has an extra term known as the *reaction field of Onsager*. The explaination for the presence of this term is the following. The magnetization m_i on site i is affected by the magnetization of the neighbouring sites m_j. But m_i itself affects the magnetization on sites j and that contribution (self-response) should be removed. Let us calculate what should be the reduction. In the presence of a local magnetic field h_i the self-consistent solution looks like

$$m_i = tanh\left[\frac{1}{K_BT}\sum_j J_{ij}m_j + h_i\right]$$

Therefore the succeptibility at site j from the above equation is

$$\chi_{jj} = \left.\frac{\partial m_j}{\partial h_j}\right|_{h_j\to 0} = \frac{1}{K_BT}(1-m_j^2)$$

m_i affects m_j through the internal field $J_{ij}m_i$ which changes m_j by the amount $\chi_{jj}J_{ij}m_i$. This in turn *increases the internal field* at site i by $J_ij\chi_{jj}J_{ij}m_i = 1/K_BT\ J_{ij}^2(1-m_j^2)m_i$. Obviously we need to remove this self response which explains the origin of the second term. Note that in the SK model J_ij^2 term is *not* negligible. There is a much more convincing way of deriving this result by extremising the TAP free energy. The derivation of this free energy on the other hand is quite neat, see for example page 40 of reference [3].

Solution to the TAP equation can be studied around the spin-glass transition point, where it is valid to assume that both the local magnetization and the local field (if any) is small. The linearized form is (introducing a local field for convinience)

$$m_i = \frac{1}{K_BT}\sum_j J_{ij}m_j + \frac{1}{K_BT}h_i - \left(\frac{1}{K_BT}\right)^2 m_i\sum_j J_{ij}^2$$

If we expand the matrix J_{ij} in terms of its eigenvectors

$$J_{ij} = \sum_\lambda < i|\lambda >< \lambda|j >$$

and defining λ-magnetization λ-field by

$$m_\lambda = \sum_i < \lambda|i > m_i \quad ; \quad h_\lambda = \sum_i < \lambda|i > h_i$$

we find the λ-susceptibility to be

$$\chi_\lambda = \frac{\partial m_\lambda}{\partial h_\lambda} = \frac{1}{K_BT - J_\lambda + K_BTJ^2}$$

where $J^2 = \sum_j J_{ij}^2$. For the Gaussian distribution of J_{ij} in the SK model, it is known that the eigenvalues are distributed between $-2J$ and $2J$ (recall that J is the standard deviation of J_{ij} is $J/\sqrt{N}$, we took it to be in the previous section).For the largest eigenvalue, $2J$ the susceptibility diverges for temperature $T = J/K_BT$ which coforms to the result from previous section. The funny thing to note is that, since the spectrum of J_{ij} is continuous, susceptibility *continues to diverge* below T_c. What about the stability of the TAP solution? As the RS solution is is unstable (or marginally stable). Nevertheless, TAP has been quite useful to the subject, for details, see pg 877 of [2] and pg 52 of [4].

3 Does RSB occur in spin-glass materials?

There are two types of experiment that seem to support the existence of a hierarchial tree of spin-glass states as in RSB solution. But first, I need to explain a well known phenomenon in spin glasses; *aging*. It demonstrates the absence of true thermodynamic equilibrium in spin glasses. A sample of spin-glass (experiments were done on $\mathbf{CdCr_{1.7}In_{0.3}S_4}$ [6]) in cooled down below T_c in the presence of weak uniform magentic field, and after a waiting time t_w (during which the temperature is kept constant) the field is switched off. The magnetization (called *thermo-remanent magnetization* TRM, as opposed to *isothermal remanent magnetization*IRM, see pg 287 of [4]) is measured as a function of time. The main observations are

- relaxation is extremely slow, may take hours or minutes.
- relaxation is non-stationary, ie. the bigger t_w is, the slower the relaxation. There are two time scales, the aging time t_w and the observation time.

One can also perform the 'mirror' experiment, where system is cooled down in zero magnetic field to spin glass phase, and then maintained at a constant temperature for the waiting time t_w. The magnetic field is turned on next and the magnetization (IRM) is measured. Similar effects are observed. The dependece on t_w is called aging.

Coming back, the two types of experiments are:

- The sample, in the presence of a weak magnetic field is cooled down to the spin-glass phase ($T < T_c$), and after a waiting time t_{w1} the temperature is slightly lowered ($-\Delta T$). After another waiting time t_{w2} it is heated up to the original temperature, the magnetic filed is switched off and the relaxation of the magnetization is measured. If ΔT is not too small the magnetization vs time curves are identical to aging experiments with aging time $t_{w1} + t_{w2}$. This means that during t_{w3} the system remained completely frozen.

  On the other hand if the temperature was raised ($+\Delta T$), insted of being lowered in the previous experiment (before the origin temperature being re-established) the magnetization vs time curves are indentical to aging experiments *with aging time* t_{w2} *instead of* $t_{w1} + t_{w2}$. This means even slight heating wipes out all aging that has been achieved and equilibrium process starts all over again. Such a assymetric response can be explained in terms of the qualitative picture of a continuous hierarchy of phase transition and the tree-like structure of spin-glass state; a result of RSB which I didn't get a chance to discuss thoroughly in this article. Please refer for further details in [6] and [5].
- the other experiment is on the temperature dependence of energy barriers, refer [5].

4 Conclusion (or termination and regrets?)

Let me first give a list of topics I wished to discuss but couldn't because of the size limit on this article:

- Gauge theory of spin glasses [3], [2]. Of particular interest was Nishimori line on which, merely from the gauge symmetry $\sigma_i \to S_i\sigma_i$ and $J_{ij} \to J_{ij}\sigma_i\sigma_j$ for the SK type models, one can find the exact internal energy. One can also find bounds on free energy and specific heat, and find entropy of frustration from gauge symmetry arguments.
- Hydrodynamic theory of spin waves developed by Halperin and Saslow in 1977 which is good for esplaining resonant frequencies observed in metallic spin glasses.[2]
- A detailed discussion of RSB.
- Substances which show spin glass behavior and why. Realistic models of such substances.
- Relationship with normal glass.
- Disordered sytems in general, and amorphous sytems in particular [7]. I would love to discuss the application of Replica filed theory and related physics to vulcanization transition and the theory of randomly cross-linked macromolecules (like rubber), which is an active field of research that I partake in. [8]

To conclude, I have tried to motivate the concept of Replica Symmetry Breaking and the underlying physics it emphasizes; the extremely complicated ground state of a spin-glass. Replica field theory is a wonderful tool in dealing with quenched randomness, though RSB doesn't necessarily happen in random system with quenched randomness. The RG theory of disordered system should follow this discussion, but I am forced to relegate it to the references, hoping that the interested reader can make good use of them!

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