Can we find metal-insulator transitions in 2-dimensional systems?

Marcelo Kuroda

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It has been almost a quarter of a century since the belief of the non–existence metallic state or the non-occurrence of metal-insulator transition for a two–dimensional system in the zero magnetic field was accepted. Nevertheless, during the last decade atypical behavior in semiconductor samples suggested the possibility of those transitions in two dimensions. In this essay we will review some of the experiments that can evidence such transtions and the theoretical models proposed to explain the observed phenomena.

1 Introduction

The study of 2-dimensional (2D) systems was initiated several decades ago. In 1979, a simple scaling argument of localization proposed by Abrahams *et al* [1] discussed the impossibility of metal–insulator transitions in those systems. The predictions made by this theory for 2D systems were either that the resistance should grow logarithmically ("weak localization") with decreasing temperature or exponentially ("strong localization"), becoming infinite as $T \to 0$. Supported by different experiments on silicon metal-oxide-semiconductor field-effect transistors (MOSFETs) performed in the early 80's and some numerical simulations, this belief came to be accepted. Although this prediction was made for 2D systems of noninteracting particles, subsequent theoretical work showed that weak interactions between the electrons increase the localization even further (Altshuler, Aronov, and Lee, 1980). In the opposite limit of very strong interactions between particles, numerical simulations predict a 2D electron system to become a Wigner crystal.

Nevertheless, in the 1990's the development of technology for semiconductor fabrication permitted the manufacture of high quality low dimensional samples and, thus, low disordered systems were experimentally accessible. Kravchenko *et al* [2] showed, in a series of experiments performed on thin metallic films in the absence of magnetic field ($B = 0$) suggested, that the conventional picture might be incomplete. Due to the absence of a convincing theory, the metal-insulator transition (MIT) has become one of the main issues in the understanding of 2D systems. The further comprehension of these structures seeks not only the basic understanding of the phenomenon but also the possibility of technological developments. In this paper, mainly based on some recent valuable reviews (ref. [4]), we focused on the MIT transition in 2D systems for $B = 0$ at $T \to 0$. In the following sections we will review the questioned theory in section 2. In section 3 we will describe some experiments violating the predictions from this theory. Finally, we will discuss some attempts to explain such behavior in section 4.

2 Theory

The phenomenon of localization is a property of the quantum states in systems. The main idea is that for strong disorder, the one–particle wavefunctions in macroscopic, random, quantum systems can be exponentially localized at $T = 0$. In other words, the amplitudes decay exponentially with the distance and thus, particles are confined to finite portions of space. This behavior is expected for strong disorder or "sufficiently small" density of states. For weak disorder the amplitude and phases of the wavefunctions will extend through the system randomly varying in space (extended states). Disorder can be imagined as a set of randomly distributed atoms or as the insertion of dislocations, vacancies or impurities in an ideal crystal lattice. On the one hand, in strong disorder (localized states), particles cannot contribute to transport at $T = 0$ when coupling to other degrees of freedom and particle-particle interactions have become negligible. Therefore, if there are only localized states near the Fermi energy, the system will be an insulator at $T = 0$. On the other hand, extended states can contribute to transport, leading to

a metallic behavior if those states lie close to the Fermi level. In the insulating regime, for $T > 0$, conductivity σ depends exponentially on the inverse of the temperature, since the transport is a result of thermal activation.

Abrahams *et al* [1], by using scaling arguments, demonstrated that for non-interacting electrons in a disordered system, even weak disorder was sufficient to localize electrons and, hence, it could not be possible to have metallic states in 2D systems at $T = 0$. The basic assumption of this theory was that the function $\beta(g) = d[\log(g)]/d[\log(L)]$ is a function of the conductance g itself but not an explicit function of the length scale L. The behavior of $\beta(g)$ is well known in both large and small disorder:

$$\beta(g) = (d-2) - 1/g + ... \qquad \text{for } g \to \infty \tag{1}$$

$$\beta(g) \to -\infty \qquad \text{for } g \ll 1 \tag{2}$$

Moreover, it seems reasonable to assume that in between those two limits the function is continuous. In figure 1 we can see the results from this theory for the univversal curve $\beta_d(g)$ in $d = 1, 2, 3$ dimensions. According to this theory, in one–dimensional systems the charges are always strongly localized while in three–dimensional systems the electronic states can be either extended or localized. The authors took for granted that $\beta(g)$ should be not only smooth but monotonic. In 2D the conductivity could never be positive as $T \to 0$ because the metallic behavior happens when $\beta(g) > 0$. The system can be conductor at room temperature but, as the temperature decreases, the resistance is expected to have a weak logaritmic increase. This leads to a localization of the electrons, despite its large scale, usually known as "weak localization".

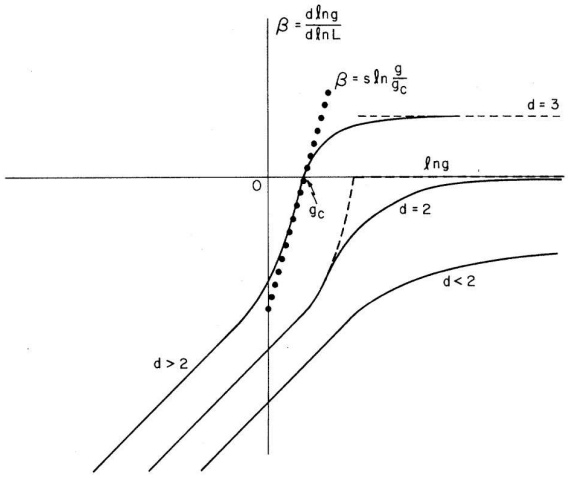

Figure 1: Temperature dependence of $\beta(g)$ as a function of g for different dimensions $d = 1$, 2, 3 [1]. For $d = 2$ the function $\beta(g) < 0$ so no metallic phase is expected.

In the early 1980's experiments on different relatively low–mobility 2D samples and numerical simulations supported the predictions of this theory. In 1981 some computa-

tional results found by A. MacKinnon *et al* [3] using a recursive method were consistent with those expected by the scaling theory. With no *a priori* assumption about the shapes of the scaling function $\beta(g)$, their results not only supported the existence but also the continuity and monotonicity of the $\beta(g)$. However, some people argued that systematic errors in these calculations could not confirm the issue of presence of power–law localized states in the 2D Anderson model. The agreements between theoretical predictions and experiments answered to the question of whether a metal-insulator transitions in 2D is possible.

3 Experimental evidence

Fifteen years later than the scaling theory was proposed the development of semiconductor fabrication techniques allowed the creation of thin layers in which the disorder parameter could be manipulated. The first series of experiments reporting a strong metallic temperature dependence were performed by Kravchenko *et al* [2] in 1994 and 1995. Such "unusual" behavior, associated with the existence of a metallic state and a metal-insulator transition in 2D was observed on very low disordered Si MOSFET's. These high quality samples allowed studies of the 2D system in very dilute regime (i.e. electronic densities n_s below 10^{11}cm^{-2}). Under these conditions, the electron-electron Coulomb interaction energy E_{e-e} is larger than the electronic Fermi energy E_F. Estimates for $n_s = 10^{11}\text{cm}^{-2}$ yield:

$$E_{e-e} = \frac{e^2}{\epsilon}(\pi n_s)^{1/2} \approx 10\text{meV} \qquad E_F = \frac{\pi\hbar^2 n_s}{2m^*} \approx 0.58\text{meV} \tag{3}$$

Therefore, the dimensionless parameter measuring the strength of the interaction $r_s = E_{e-e}/E_F$ takes values of above 10 in these samples. Numerical simulations predict 2D electrons to form Wigner crystals when r_s is a few times larger. Thus, the system is expected to behave as a strongly correlated liquid in these samples.

The temperature dependence of the resistivity of a low-disordered MOSFET is shown in figure 2 for 30 different electronic densities varying from 7.12×10^{10} to $13.7 \times 10^{10}\text{cm}^{-2}$ for which the corresponding value of r_s ranges between approximately 15 and 20. On the one hand, the characteristic behavior of an insulator, i.e. the resistivity growing monotonically as the temperature decreases, is found for low densities (upper curves). As the electron density increases, the observed behavior is that of an insulator for $T \gtrsim 2\text{K}$ but the resistivity decreases quickly as the temperature is lowered, exhibiting a strong metallic dependence on the temperature.

This behavior strongly suggests the existence of a mobility edge (i.e. a metal-insulator transition) in 2D system at zero magnetic field, contradicting the theoretical predictions by Abrahams [1]. However, on the metallic side of the transition, for $T \lesssim 2K$ the conductivity ρ drops by an order of magnitude while on ordinary metals it saturates when the energy of the phonons is of the order of the thermal fluctuations ($\sim$10K in the regime of interest). This behavior cannot be explained by the electron-phonon scattering or by the temperature-dependent screening.

A striking feature observed by Kravchenko [2] in these experiments is the fact that resistivity can be represented as a function of T/T_0 where the characteristic T_0 depends only on n_s. As shown in figure 3, the data collapse into two different curves, one representing the insulating side (open symbols) and the other curve the metallic one (closed symbols). Moreover, there is a reflection symmetry in the temperature range T/T_0 between $3.10^{-}1$ and 10^1 approximately as we can see in the inset of figure 3. Another important aspect to note is the power law behavior of both the insulating and conducting curves given by:

$$T_0 \sim |n_s - n_c|^b \tag{4}$$

with approximately the same value for the power ($b = 1.6 \pm 0.1$ for the insulating side and $b = 1.62 \pm 0.1$ for the metallic side of the transition).

In further experiments the same behavior was found using not only dilute silicon MOSFET's with different geometries but a variety of 2D systems. However we should emphasize that the temperature dependence of resistivity is not the same in very disordered systems as in low disordered systems. Experimental results show that only in the vicinity of the MIT and in systems with weakly disorder potential the the resistivity curves are nearly "universal".

In the experiment described above, the criterion used to determine the critical density n_c is the change in sign of the time derivative of the resistivity ($d\rho/dT$). At the lowest achieved temperatures, a negative sign of the derivative corresponds to an insulating phase, while the positive sign is empirically associated with the metal state of the system. Nevertheless, this criterion has the weakness that it requires extrapolation to $T = 0$. Another criterion, based on the activation energy E_a was proposed by Shashkin *et al* [5]. The MIT point has been determined by analyzing the *I–V* characteristics when the system behaves as insulator. Far from the critical density ($n_s < n_c$), the *I–V* curve is almost a step function. For small currents the voltage rises suddenly and then it saturates, as shown in the inset of figure 4. The magnitude of that step is $2V_c$. As we get closer to the critical density, the function becomes nonlinear but still presents a steplike behavior. In the metallic phase, the *I–V* characteristics become linear. The activation energy E_a is the energy needed by the carriers to reach the mobility edge E_c and thus $E_a = eV_c - E_F$. This is related to the localization length L by:

$$eV_cL/d = E_c - E_F \tag{5}$$

where d is the distance between the electrodes. Since the localization lenth (temperature independent) diverges near the MIT as $L(E_F) \sim (E_c - E_F)^{-s}$ with $s \approx 1$, we can find the relation $V_c^{1/2} \sim n_s$ as we can observe in figure 4.

The critical densities obtained using the activation energy criterion were the same as those determined by using the derivative criterion. Since the localization length method is independent of temperature, the existence of a MIT for low–disordered MOSFET's at $T = 0$ is supported by this equivalence. Nevertheless, the authors of the latter criterion [5] emphasize that this equivalence was found only in the zero magnetic field MIT. It should also be noted that the critical densities found with the latter method in highly disordered samples differ to those obtained using the derivative criterion. Therefore, this

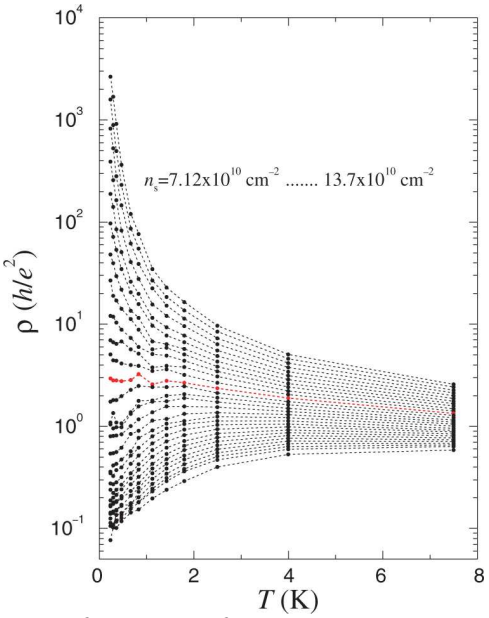

Figure 2: Resistivity as a function of temperature in a dilute low-disordered Si MOSFET [2]. The curve in red is the separatrix of the metallic and the insulating phases corresponding to a change in sign of the derivative at the lowest achievable temperature.

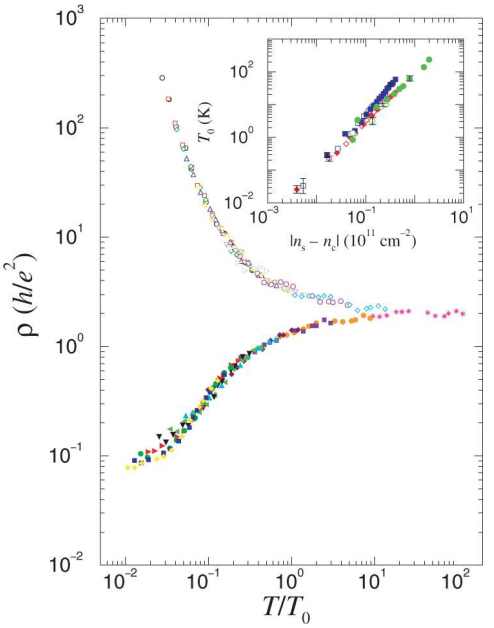

Figure 3: Resistivity as a function of T/T_0 where T_0 is chosen to yield scaling with temperature [2] from three different Si MOSFET's. Open (close) symbols correspond to the insulating (metallic) side of the transtion. The inset shows the scaling temperature T_0 versus the deviation from the critical density.

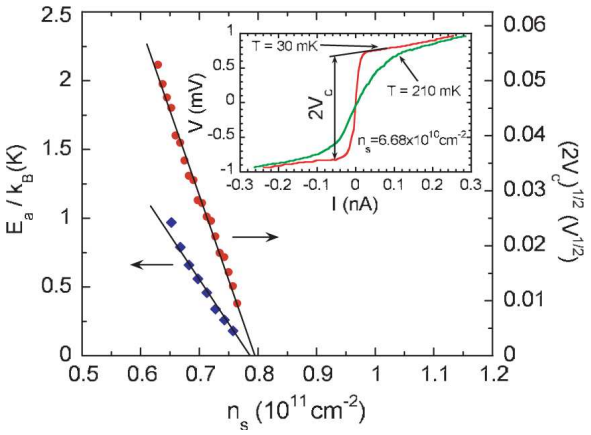

Figure 4: Activation energy and square root of the threshold voltage as a function of electron density in $B = 0$. The inset shows current–voltage characteristics for $T = 0$mK.

suggests that the mechanisms leading to the MIT are different depending on the order of the system.

Before describing the theoretical approaches to describing the MIT in low–dimensional 2D systems, we might note some relevant aspects of these transitions. First of all, if localization at and just above the critical density were present, the temperature dependence of the conductivity due to the localization effects should be exactly cancelled by the (opposite sign) temperature dependence caused by interactions in order to get the temperature independent flat separatrix curve. This coincidence does not seem probable for these two unrelated mechanisms. Second, in 1992 some computational work done by Godin *et al* [6] indicated the presence of a pseudo–mobility–edge for electrons in disordered layers. Using the recursion method, they calculated the transmittance in 2D potentials and observed a fall off similar to that displayed at a 3 dimensional mobility edge. Nevertheless, the authors emphasized that this could be an effect due to the finite size of the sample (and thus the prefix "pseudo").

4 Proposed Theories

As we have mentioned previously, the dimensionless measure of the interaction strength r_s is of the order of 10 or even higher. Therefore, this is a strong-interacting many-body problem, for which theoretical methods have not gone too far. Nowadays, the metallic behavior (i.e. the presence of extended states) at $B = 0$ is believed to arise because the electron-electron interactions overwhelm the quantum localization.

The existence of a metallic state in two dimensions for $B = 0$ was first proposed in 1983 by Finkelstein [7] and Castellani *et al* [8]. Using perturbative renormalization group (RG), they studied the effects of both interactions and disorder. In their theory,

as temperature is decreased, resistivity is increased but starts to decrease for low temperatures, insinuating the possibility of having a low-temperature metallic state. The results were in partial agreement with the experiments. However, for weakly disordered 2D systems, as temperature is decreased but before reaching zero, the interaction parameter scales to infinity. Consequently, the applicability of this perturbative theory has not been accepted. The renormalization group could describe the scenario in two different regimes: the diffusive and the ballistic regimes. The first corresponds to the condition $k_B T \ll \hbar/\tau$, where τ is the elastic mean–free time extracted from the Drude conductivity. This condition is equivalent to $T \ll T_F \rho/(h/e^2)$ being satisfied only close to the transtion (because T_F is small in the low density systems considered). In 2D disordered systems, electron–electron interactions lead to corrections of the Drude conductivity σ_0 in the diffusive limit given by:

$$\Delta\sigma = -\frac{e^2}{2\pi^2\hbar}\log\left(\frac{\hbar}{k_B T\tau}\right)\left[1+3\left(1-\frac{\log(1+F_0^\sigma)}{F_0^\sigma}\right)\right] \tag{6}$$

where F_0^σ is the Fermi liquid interaction constant in the triplet channel, and is responsible for the sign of the divergent correction (positive for metallic state and negative for insulating state). Quantitative agreement with the experiment was found in ultra clean Si MOSFET's when considering the interplay between electron–electron interactions and disorder by the RG.

Far from the transition, $k_B T \ll \hbar/\tau$ in the temperature range of the experiments, and hence it corresponds to the ballistic regime. In this case, a theory proposed by Zala *et al* takes into account the coherent electron scattering off the Friedel oscillations. Thus, the quantum correction to the conductivity now becomes:

$$\Delta\sigma = -\frac{e^2}{\pi\hbar}\frac{k_B T\tau}{\hbar}\left(1+\frac{3F_0^\sigma}{1+F_0^\sigma}\right) \tag{7}$$

Once again the sign of the correction depends on the interaction parameter F_0^σ. Even though some quantitative difference was found between theory and experiment, this RG theory reasonably describes this scenario in the ballistic regime.

However, the theories discussed before take as starting point a Fermi liquid model, but their applicability is suspecious because of the strength of interactions. For strongly interacting systems only a few theoretical approaches has been developed without successfully describing the whole scenario. The different models lead to a variety of states representing the ground state in strongly correlated systems. Some of them are the Wigner crystal, the itinerant ferromagnet with spontaneous ordering and the paramagnetic Fermi liquid. In recent numerical simulations, Attacalite *et al* [9] claimed that the ground state in low disordered systems could be described as Wigner crystals and paramagnetic Fermi liquid for low and high electron densities, respectively. But between those two phases they also found a ferromagnetic Fermi liquid, as represented in the figure 5.

Spivak [10] predicted a first order phase transition between the Fermi liquid and the Wigner crystal. This model discusses the existence between those two phases of some more complicated intermediate phases such as bubbles of Fermi liquid, stripes and bubbles

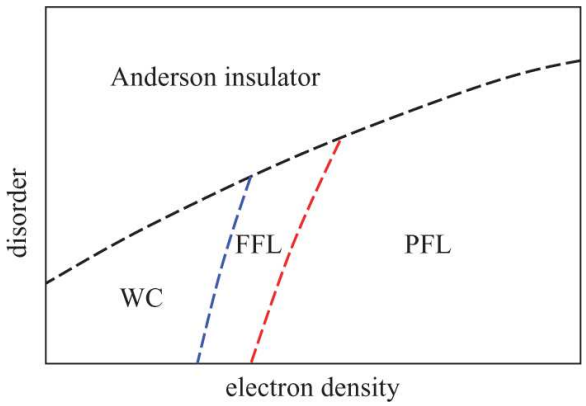

Figure 5: Phase diagram scheme accorrding to the simulations done by Attaccalite *et al* [9]. The phases correspond to Anderson insulator, Wigner Crystal (WC), ferromagnetic Fermi liquid (FFL) and paramagnetic Fermi liquid (PFL).

of liquid crystal. Crakravarty *et al* (1999) suggested a model where the transtion phase between metallic and insullating states is the melting of Wigner glass (phase characterized by the competition of the ferrromagnetic and antiferromagenetic exchange interactions and its quasi–long–range translational order). Due to disorder, the phase transition is supposed to be second order.

The fact that the metallic and insultating curves are reflection symmetric for temperatures between 300mK and 10K looks very similar to the behavior found for the resistivity near the transition between the quantum Hall liquid and insulator. In 1997, Dobrosavljević *et al* [11] claimed that for interacting electrons there is no scaling law violation in the metal-insulator transition in 2D. In their analysis both the scaling and the reflection symmetry emerge as consequences of a quantum critical point describing the phase transition at $T = 0$. Moreover, the authors doubted that the metallic state could be a Fermi liquid because, in the absence of interactions, the system would not behave as a metal but as an Anderson insulator.

5 Conclusions

Despite the fact that the problem has been studied for decades, the theory behind the metal-insulator transition is not yet completely understood. Even though the renormalization group theory seems to qualitatively describe the phenomenon in the vecinity of the transition, its applicability is still suspicious. The temperature dependence of the resistivity is found to be "universal" among samples with weak disorder potentials. Nevertheless, this dependence on the order suggests that the mechanisms producing the transition should also depend on the disorder. Up to now, other theories for strong

electron–electron interaction are poorly developed.

To date, the experiments performed to study these systems are mainly transport measurements which are relatively straightforward. There are also some other experiments studying the Hall coefficient, measurements of noise and compressibility (a broad review on these experiments can be found in ref. [4]). Tunnelling measurents could provide useful information to determine if these are indeed Fermi liquids at the metal insulator transition. Due to the small number of carriers in these systems, some thermodynamic properties (specific heat or magnetization measurements) are very still difficult to perform. Direct comparisons between different systems (depending on the material and type of carriers) are still difficult to make because of the different temperature scales that they present. The discussion could go even further if we take into account some peculiar effects observed with non–zero (parallel and perpendicular) magnetic field.

Therefore, despite the advances achieved in the last decade, the phenomenon still has to be studied both theoretically and experimentally.

References

[1] E. Abrahams, P. W. Anderson, D. C. Licciardello, and T. V. Ramakrishnan, Phys. Rev. Lett. **42**, 673 (1979).

[2] S. V. Kravchenko, W. E. Mason, G. E. Bowker, J. E. Furneaux, V. M. Pudalov, and M. DIorio, Phys. Rev. B **51**, 7038 (1995).

[3] A. MacKinnon and B. Kramer, Phys. Rev. Lett., **47**, 1546 (1981)

[4] E. Abrahams, S.V. Kravchenko, and M.P. Sarachik, Rev. Mod. Phys. **73**, 251 (2001); S. V. Kravchenko and M. P. Sarachik, Rep. Prog. Phys., **67**, 1 (2004).

[5] A. A. Shashkin and S. V. Kravchenko, Phys. Rev. Lett. **87**, 266402 (2001).

[6] T.J. Godin and R. Haydock, Phys. Rev. B **46**, 1528 (1992)

[7] A. M. Finkelstein, Sov. Phys.JETP **57**, 97 (1983).

[8] C. Castellani, C. Di Castro, P.A. Lee and M. Ma, Phys. Rev. B **30**, 527 (1984).

[9] C. Attaccalite, S. Moroni , P. Gori-Giorgi and G. B. Bachelet, Phys. Rev. Lett. **88** 256601 (2002).

[10] B. Spivak, Phys. Rev. B **67**, 125205 (2002).

[11] V. Dobrosavljević, E. Abrahams, E. Miranda and S. Chakravarty, Phys. Rev. Lett. **79**, 455 (1997).