Vortex Matter Driven by Alternating Current

Y. Liu
Department of Physics, University of Illinois at Urbana-Champaign

Experimentally, it has been found that in a well-defined range of fields, temperatures and driving amplitudes, the vortex matter displays novel types of nonlinear response. As is the case with ordinary solids, it appears that unidirectional drives tend to disorder the system, while shaking tends to order it. A numerical result states that the oscillatory motion of vortices, provided that its amplitude is of the order of the lattice constant, can favor an ordered structure, even when the motion of the vortices is plastic when the same force is applied in a constant way.

I. INTRODUCTION

Vortex matter (VM) in the presence of disorder is a paradigm to study the general problem of elastic manifolds in random media. The competing roles of order due to vortex-vortex interactions ($\mathbf{F}^{\mathrm{vv}}$) and disorder due to pinning ($\mathbf{F}^{\mathrm{vp}}$) together with thermal fluctuation ($\mathbf{F}^{\mathrm{T}}$) in VM account for its complex phases and dynamics (*Blatter 1994, Brandt 1995*).

One of the most interesting dynamic phenomena of VM is the depinning process. In the past several decades, it has been studied deeply. Both theoretical and experimental results show that this is a very complex process. For a steady driven VM, two limiting cases have been distinguished. (i) With a high density of weak short-range pinning centers, the motion of the VM is inhomogeneous only in a narrow region near the critical force F_{c}, which is determined by the collective pinning theory (*Larkin 1979*). (ii) With strong pinning centers (or a small concentration of them) plastic deformations become important and the motion is disordered (*Koshelev 1992*). For the second case, there is mounting evidence that plastic flow in VM involves the formation of channels in which vortices are more weakly pinned than in the surrounding areas (*Jensen 1988, Grønbech-Jensen 1996, Olson 1998*). Furthermore, three characteristic driven force (current): F_{c}, F_{p}, and F_{t}, are determined by calculating the current dependent differential resistance and structure factor. Consequently, three different dynamic regimes ("plastic flow", "smectic flow" and "frozen transverse solid") are distinguished (*Kolton 1999*). Typical vortex trajectories of the three regimes are shown in Fig.1.

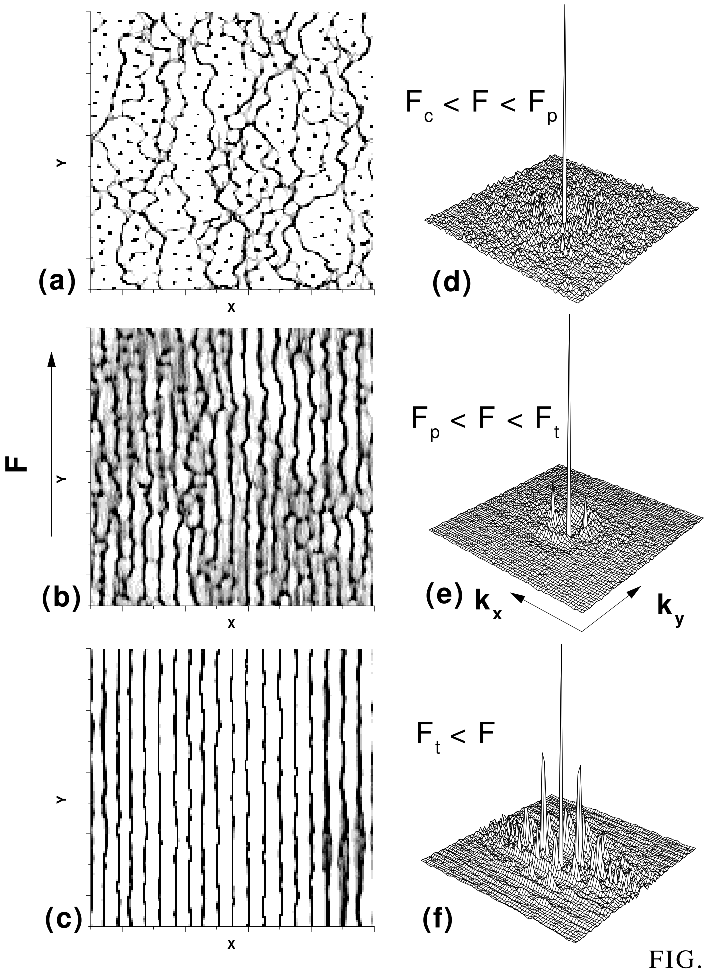

FIG.1

FIG. 1 Vortex trajectories and structure factors. (a),(d): $F_{\rm c} < F^{\rm L} < F_{\rm p}$, plastic flow; (b),(e): $F_{\rm p} < F^{\rm L} < F_{\rm t}$, smectic flow; (c),(f): $F^{\rm L} > F_{\rm t}$, frozen transverse solid.

This is the basic picture of the depinning process in dc drive case, which is already complex enough. Recently, the depinning process of vortex matter driven by alternating current has attracted much attention. Some new physics emerges, which has been proved to be intrinsic to the alternating dynamics. In this paper, I will present some remarkable results about this interesting phenomenon. In the following sections, I discuss experimental results first (*Henderson 1998*). Then I introduce a numerical simulation (*Valenzuela 2002*). Finally, an overview of the alternating dynamics in vortex matter is given.

II. EXPERIMENTAL RESULTS

Results shown here are for a single crystal sample of the low T_c superconductor 2H-NbSe$_2$ with the zero-field transition temperature $T_c = 5.8K$. Measurements were done by the standard four lead technique. Three types of driving currents were used: (a) a current is switched between $-I_0$ and $+I_0$ (bidirectional pulses); (b) switched between 0 and $+I_0$ (positive unidirectional pulses); (c) between 0 and $-I_0$ (negative unidirectional pulses).

A. Driving type Dependent Threshold Current

Driving type dependent threshold current is shown in the main panel of Fig.2. The current voltage characteristics for unidirectional and bidirectional pulses at 1KHz for three values of temperature T are shown in the insets. Note that while the response to dc and unidirectional drives is almost the same, the response to bidirectional drives differs significantly, especially at 4.55 K. For the other two temperatures shown in the insets, there is little or no difference in the response to the two kinds of drives. The region where there is finite response to bidirectional pulses but no response to unidirectional ones is indicated by the shaded area in the main panel of Fig.2. Comparison with the dc critical currents shows that a large difference between the unidirectional and bidirectional thresholds only occurs between $T_m(H)$ and $T_p(H)$.

Note that there is a peak in the $I_c(T)$ curve, which is known as the peak effect. In this case, increasing T at fixed H leads to a sharp rise in I_c which sets in at $T = T_m(H)$, reaches its maximum value at $T_p(H)$, and finally goes to zero at the transition temperature $T_c(H)$. Further studies of the dc current-voltage characteristics (*Bhattacharya 1993, Henderson 1996*) have revealed that the variation of I_c is connected to the existence of three states of the VM which exhibit distinct dynamic properties. For $T < T_m(H)$, vortex-vortex interactions dominate and VM form an ordered lattice that responds elastically when driven by a current $I > I_c$. Above $T_p(H)$, the system is in a glassy state where VM is highly disordered. Between $T_m(H)$ and $T_p(H)$, VM is in an intermediate state in which it behaves like a soft solid that tears when it is depinned. The vortex motion in this state is thought to involve the flow of channels (*Jensen 1988, Grønbech-Jensen 1996, Kolton 1999*) of relatively weakly pinned vortices past more strongly pinned neighbors, corresponding to a plastic flow.

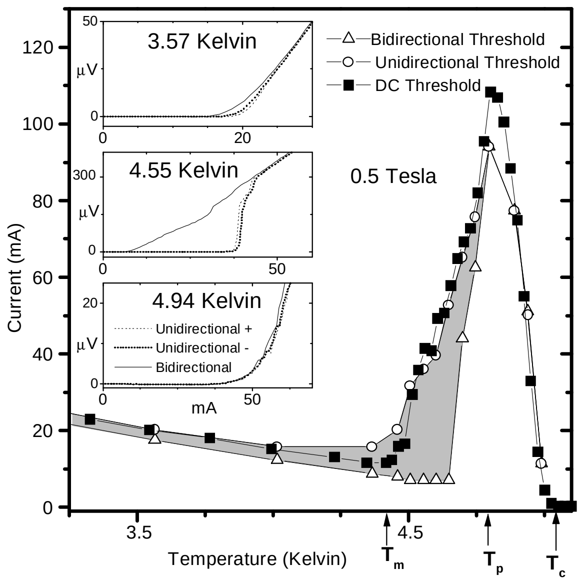

FIG. 2 Temperature dependent threshold current for unidirectional, bidirectional pulses and dc currents. Insets show the V-I characteristic at three temperatures.

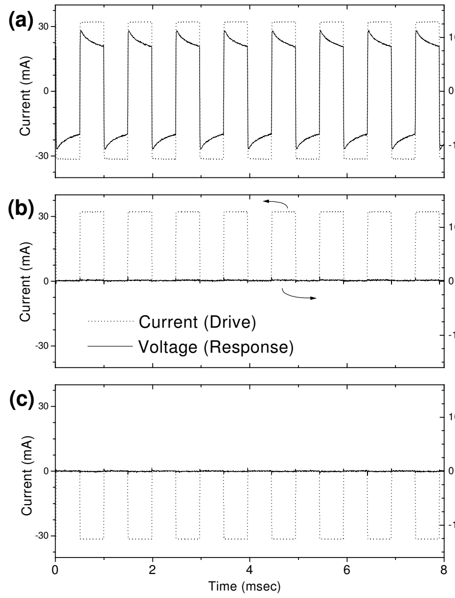

FIG. 3 The response of the flux lattice at H=0.5 T and T=4.59 K to bidirectional pulses (a), positive unidirectional pulses (b), and negative unidirectional pulses (c).

B. Steady State Response to Different types of Drives

Fig.3 shows the response of a steady state of VM to each types of driving currents at H=0.5 T and T=4.59 K. When driven with the bidirectional pulses (see Fig.3(a), the VM were shaken back and forth by a current that switched between -30mA and +30mA, which is lower than the dc critical current, i.e. $I_0 < I_{\rm c}$. The measured voltage, which is proportional to the averaged velocity of the vortices over the entire system, is about 30% of what it would be in the free vortex flow limit (i.e. in the complete absence of pinning). This result tells us that vortices are shaken loose to be in a “easy to move” state. When the driving current is switched between 0 and +30mA or 0 and -30mA (unidirectional pulses, see Fig.3(b)-(c)), the response is essentially zero. This means the mobility of the vortices do not change when driven by a unidirectional pulse. The vortex matter is in a “hard to move” state.

C. Steady State Response to Asymmetric Ac Drives

How will the system response to asymmetric ac drives? Intuition implies that something different must occur. The effects of varying the symmetry of the driving current are studied in two cases: amplitude asymmetry and temporal asymmetry at T= 4.59 K and H= 0.5 T. As shown in Fig.4(a)-(c), an asymmetry in the amplitude, even when it is small, causes a significant reduction in the response compared with that of the symmetric case. Remarkably, increasing the drive in either direction leads to a sharp decrease in the response (provided the drive is not too large). Actually, we have discussed the limiting case of the amplitude asymmetry, which is the unidirectional pulses. In that case, the voltage response is essentially zero. In Fig.4(d)-(f), the effects of varying the temporal symmetry of the pulse are studied while keeping the pulse amplitude symmetric. The pulse duration in each direction is varied from symmetric (equal pulse lengths) to asymmetric while keeping the repeat frequency of pulses fixed. The response is only weakly affected: there is still a substantial voltage when the drive is positive (or negative) 95% of the time. In this case, there is a net flow of vortices, not just back and forth motion.

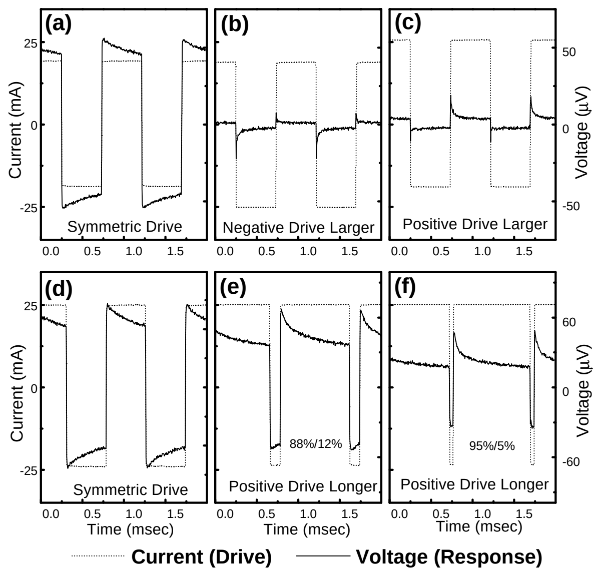

FIG. 4 Effect of changing the symmetry of the driving current. (a),(d) symmetric bidirectional pulses; (b),(c) drive amplitude asymmetry; (e),(f) temporal asymmetry.

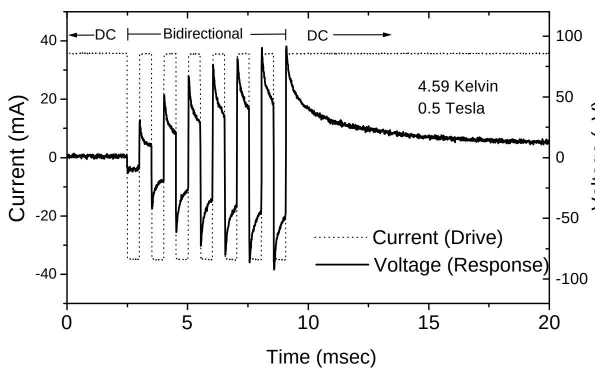

FIG. 5 In this measurement a dc current was first applied for several seconds. Then several bidirectional pulses were applied after which the current was set back to dc.

D. Evolution of Response when Changing Drives

The data shown so far were taken after the response settled to a steady state. If the drive is switched from the bidirectional pulses shown in Fig.3(a) to the unidirectional pulses in Fig.3(b), the response takes some time to decay to zero. Such transient effects can be studied by observing how the response evolves when the drive is changed. As shown in Fig.5, a +35mA dc current was first applied for several seconds. Subsequently the current was switched back and forth between +35 and -35mA several times, after which it was left at +35mA. We see that initially the response is zero, consistent with the fact that 35mA is less than the dc critical current. But a small voltage appears as soon as the direction of the current is reversed, and this voltage jumps up to a larger value on each subsequent reversal. If the bidirectional drive persists the response eventually saturates to the steady state shown in Fig.3(a). The voltage decays somewhat between each reversal. After the drive is switched back to dc the decay continues until the response goes back to zero. The entire pattern shown in the figure repeats exactly, if the current is cycled repeatedly through the sequence: dc, bidirectional pulses, dc.

III. NUMERICAL SIMULATION

A key issue that naturally surges is the microscopic dynamics of vortices and the relation between VM's order and mobility with different types of drives. Moreover, the frequency dependent response of the system arouses our curiosity. Can similar phenomena always be observed no matter how fast we apply the alternating drives? A delicate numerical study(*Valenzuela 2002*), which addresses simultaneously the order and mobility of the VM in ac and dc drives, can answer our question.

A. Basic Equations and parameters

The simulation is based on the standard Langevin dynamics. The overdamped equation of motion of a vortex in position $\mathbf{r}_i$ is given by $\mathbf{F}_i = \sum_{j\neq i}^{N_\mathrm{v}} \mathbf{F}^{\mathrm{vv}}(\mathbf{r}_i - \mathbf{r}_j) + \sum_{k}^{N_\mathrm{p}} \mathbf{F}^{\mathrm{vp}}(\mathbf{r}_i - \mathbf{r}_k^{\mathrm{p}}) + \mathbf{F}^{\mathrm{L}} + \mathbf{F}_i^{\mathrm{T}} = \eta\ \mathbf{v}_i$, where $\mathbf{F}_i$ is the total force on vortex i due to vortex-vortex interactions ($\mathbf{F}^{\mathrm{vv}}$), pinning centers ($\mathbf{F}^{\mathrm{vp}}$), the driving current $\mathbf{J}$ ($\mathbf{F}^{\mathrm{L}} \sim \phi_0 \mathbf{J} \times \hat{\mathbf{z}}$) and thermal fluctuations ($\mathbf{F}_i^{T}$). Here, η is the Bardeen-Stephen friction coefficient, N_v the number of vortices, N_p the number of pinning sites and $\mathbf{r}_k^{\mathrm{p}}$ the location of the kth pinning center. Normalized scales of length and force are λ and $f_0 = \phi_0^2/(8\pi^2\lambda^3)$, respectively. Periodic boundary conditions are used.

The governing equation of motion automatically provides positions and velocities of all vortices as a function of time. Therefore, the average velocities of vortices is obtained $\langle \mathbf{v} \rangle = \frac{1}{N_{\mathrm{v}}} \sum_{i=1}^{N_{\mathrm{v}}} \mathbf{v}_i$, which is proportional to the mobility of the vortex matter and the resulting voltage. As for structural information, it can be collected by working in reciprocal space. The time averaged static structure factor is constructed as $S(\mathbf{k}) = \frac{1}{N_{\mathrm{v}}} \langle | \sum_{j=1}^{N_{\mathrm{v}}} e^{i\mathbf{k}\cdot\mathbf{r}_j(t)}|^2 \rangle$. To quantify the degree of order of the VM, one can determine the average concentration of vortices with coordination number not equal to 6, n_{def}, using the Delaunay triangulation procedure.

B. Vortex Configurations

To study the alternating dynamics, square alternating currents (bidirectional pulses) are chosen, with strength F^{L} ($\mathbf{F}^{\mathrm{L}} = F^{\mathrm{L}}\hat{\mathbf{x}}$) and frequency $\omega = 2\pi/P$. For steady driving forces between $F_{\mathrm{c}} \sim 0.0085 f_0$ and $F_{\mathrm{T}} = 0.012 f_0$ the vortex flow is always disordered with a high density of defects, n_{def} (of the order of 20%). For higher driving forces, n_{def} diminishes and the movement is ordered with all of the vortices moving at the same average velocity. Experimental results (*Valenzuela 2001*) suggest that when vortices perform an oscillatory motion whose amplitude is of the order of the lattice constant, the healing of defects should be important. For this reason, F^{L} and P are initially chosen to satisfy $F^{\mathrm{L}}\, P/4 \sim 1$ (in normalized units)[1].

The starting state is a perfectly ordered vortex array. Then a steady current with $F^{\mathrm{L}} = 0.0118 f_0$ in the horizontal direction is applied[2]. The up panel of Fig.6 shows a snapshot of the configuration of the VM after reaching a stationary state in which the average velocity of vortices and the average concentration of defects remained constant. This state presents a relatively high concentration of defects. Then an alternating square driving

[1] A rough estimation is given here. Suppose the equation of motion of a single vortex is $F^{\mathrm{L}} = \eta v$. Then the amplitude of the oscillatory motion is $vP/4$, which is of the order of the lattice constant a_0, i.e. $(F^{\mathrm{L}}/\eta) \times P/4 \sim a_0$. For a hexagonal lattice, a_0 is determined by $n_{\mathrm{v}} = (3 \times 1/6)/(\sqrt{3}a_0^2/4) = 1$. So $a_0 \approx 1.07$. Finally, one gets $F^{\mathrm{L}} \times P/4 \sim \eta a_0 \sim 1.07$, in the normalized units.

[2] Note that F^{L} is slightly larger than F_{c}, which is different from Henderson's experiments

current with the same strength ($0.0118 f_0$) is turned on to the disordered state shown in the up panel. After 500 cycles, the obtained configuration of the vortex array is shown in the down panel. The corresponding structure factors of both configurations are also shown. The Delaunay triangulation and the structure factor reveal an important reduction in $n_{\rm def}$ when the alternating current is applied. It is fundamental to emphasize that the initial state was obtained with a steady current with the same strength starting with a perfectly ordered vortex array. This implies that, for certain values of the applied current, the flow of vortices can be plastic and disordered introducing defects in the system but, if the same current is applied in an alternating way, the defects heal and the vortex matter reorders.

C. Evolution of the Reordering

To show the the evolution of this reordering, the concentration of defects $n_{\rm def}$, the average value of the absolute velocity $\langle v \rangle$, and the average quadratic displacements of vortices per cycle, $\langle \Delta^2 x_N \rangle$ and $\langle \Delta^2 y_N \rangle$, as a function of the number of cycles N, are calculated. See Fig.7(a). The results shown correspond to the average over 4 different random distributions of the pinning centers. It is seen that, for an increasing number of cycles, $n_{\rm def}$ clearly decreases and the vortex mobility is enhanced (as reflected by a 40% increase in the average vortex velocity). It is also noted that $\langle \Delta^2 x_N, y_N \rangle$ diminish and that all magnitudes vary in an approximately logarithmic way as observed experimentally (*Henderson 1998, Valenzuela 2001*). The combination of these observations indicates that, as the number of defects decreases, vortices are clearly more mobile and that, after completing one oscillation, they return closer to their original positions ($\langle \Delta^2 x_N \rangle$ and $\langle \Delta^2 y_N \rangle$ decrease). This suggests that vortices organize and move more coherently as they are forced to oscillate.

In Fig.7(b), it is shown what happens when varying $P(F^{\rm L})$ while keeping $F^{\rm L}(P)$ fixed. $n_{\rm def}$ is plotted as a function of the parameter $F^{\rm L} P/4$, after applying 100 cycles of the square current to an initial disordered state as in Fig.6(up). An important result is that, if the period P is increased so that the excursion of vortices greatly exceeds the lattice constant, the vortex matter distorts and does not reorder. This is not surprising because if the amplitude of the oscillation is high enough, the system should behave in the same way as when driven by steady forces. For low enough P, vortices oscillate in their pinning sites and the VM does not reorder either. Notably, if there is a tiny asymmetry in the amplitude of

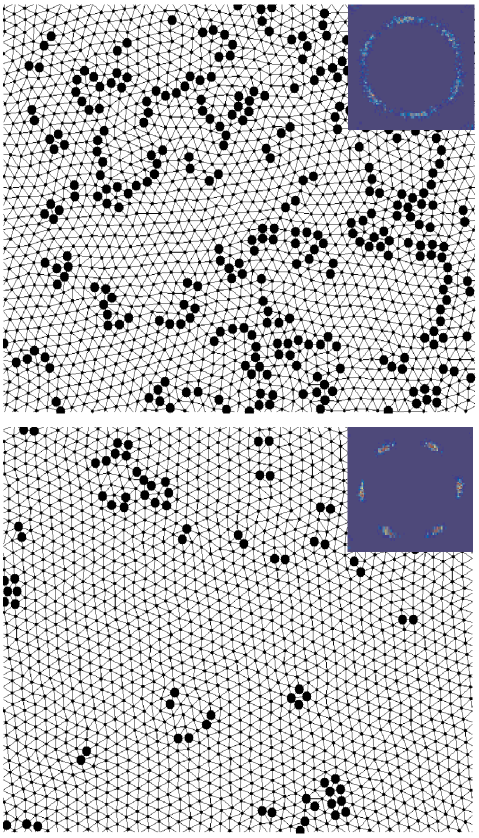

FIG. 6 Delaunay triangulation and structure factor, $S(\mathbf{k})$ for steady forces (up) and oscillatory square-forces (down). The more-ordered vortex configuration at the down panel was obtained after 500 cycles of the oscillatory force applied to the disordered state in the up panel. See text. $F^{\mathrm{L}} = 0.0118 f_0$ (in the horizontal direction).

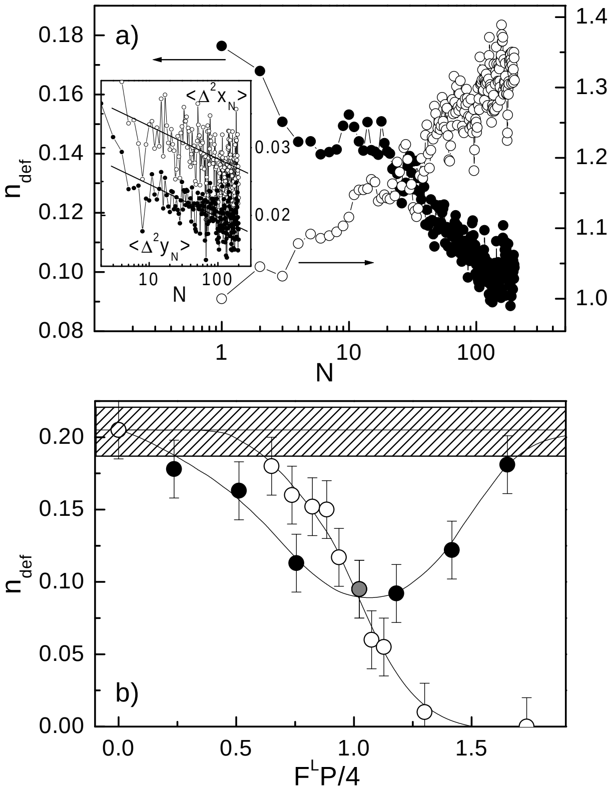

FIG. 7 Dynamical ordering. (a) Defect concentration, n_{def}, average absolute-velocity, $\langle v \rangle$, and average quadratic displacements of vortices per cycle, $\langle \Delta^2 x_N \rangle$ and $\langle \Delta^2 y_N \rangle$ vs. the number of cycles, N. (b) n_{def} as a function of $F^L P/4$ after 100 cycles (filled circles: F^L fixed, open circles: P fixed). The point $F^{\mathrm{L}} P/4 = 1$ (gray) is common to both curves.

the square current ($\sim 5\%$) the VM quickly disorders and, after a few cycles, both n_{def} and the vortex mobility reach values close to those found with steady forces. This observation is reminiscent to the experimental results (*Henderson 1998, Valenzuela 2001*).

IV. SUMMARY AND OPEN QUESTIONS

Results shown above look very similar with plasticity effects in ordinary solids (*Sandor 1972, Hertzberg 1989*). When we apply stresses greater than the “yield point” to a solid, regions of the material will slide past one another, which creates permanent plastic deformations. As the plastic flow continues, an initially soft material, such as annealed copper, becomes more and more difficult to deform and the motion eventually stops, provided the applied stress is not too large. This phenomenon is known as “strain hardening”. The material remains in a hardened state after the applied stress is removed. (The vortex version of “strain hardening” can be seen in Fig.5 and the final vortex configuration is shown in Fig.6(up).) However, after strain hardening, the yield point is typically somewhat anisotropic, i.e. less stress is required to cause plastic flow in the direction opposite to which the object was deformed during the hardening process. When the stress on a heavily strain hardened material is repeatedly reversed, the material becomes progressively easier to deform: this process is known as “cyclic softening”. (The vortex version of “cyclic softening” can be seen in Fig.5 and the final vortex configuration is shown in Fig.6(down).) Strain hardening is usually associated with an increase in the density of dislocations, whereas cyclic softening involves healing of dislocations and reordering of the lattice.

In terms of these plasticity effects in ordinary solids, we can interpret the similar behaviors in vortex matter. We conclude that in a well-defined regime of the H-T plane (see Fig.2), where the vortex matter behaviors like a soft solid. The vortex motion in this state involves the flow of channels of relatively weakly pinned vortices past more strongly pinned neighbors. The symmetric back-and-forth shaking of vortices facilitates the formation and growth of these channels. Note that the channel growth is strong sensitive to the symmetry of the driving amplitudes. When the amplitude of the drive is not the same in both directions, the larger amplitude pulses lead to jamming of the channels which cannot be completely unjammed by the smaller amplitude reverse drive. For the limiting case, i.e. applying a unidirectional drive, the easy-flow channels will become blocked quickly. Consequently, the voltage response diminishes quickly, resembling the phenomenon of strain hardening in ordinary solids.

Numerical simulation has shown some basic features of the alternating dynamics of the vortex matter. However, there are still some questions:

- First, as mentioned above, the amplitude of square driving current is slightly larger than the dc critical current. That is the reason why the average velocity of vortices in the final stationary state remains constant, rather than zero. But in Henderson's experiments, the amplitude of the alternating current is lower than the dc critical current. Therefore, the simulation doesn't reflect the experimental results exactly.

- Second, the time evolution of the voltage response is not shown in the simulation. In fact, up to now there is no relative numerical work reported. Technical difficulty lies in the time scale problem in the alternating dynamics. In the Langevin dynamic simulation, proper time step has to be taken with care to obtain accurate and stable solutions of the equations of motion.

- Third, recalling that the yield point in ordinary solid is typically somewhat anisotropic after strain hardening, we wonder whether the similar phenomenon occurs in the vortex matter. More specifically, does a square drive perpendicular to the previous steady drive reorder the vortices more easily than a square drive parallel to the previous steady drive? As far as I know, this has not been simulated at all.

All in all, in order to completely understand the alternating dynamics of the vortex matter, more work is needed, especially theoretical analysis and numerical simulation.

References

[1] Bhattacharya, S. and M. J. Higgins, Phys. Rev. Lett. **70**, 2617 (1993).

[2] Blatter, G., M. V. Feigel'man, V. B. Geshkenbein, A. I. Larkin, V. M. Vinokur. Rev. Mod. Phys. **66**, 1125(1994).

[3] Brandt, E. H. Rep. Prog. Phys. **58**, 1465(1995).

[4] Henderson, W., E. Y. Andrei , M. J. Higgins and S. Bhattacharya. Phys. Rev. Lett. **77**, 2077(1996).

[5] Henderson, W., E. Y. Andrei and M. J. Higgins. Phys. Rev. Lett. **81**, 2352(1998).

[6] Hertzberg, R. W. *Deformation and Fracture Mechanics of Engineering Materials* John Wiley & Sons, New York, (1989).

[7] Jensen, H. J., A. Brass and A.J. Berlinsky. Phys. Rev. Lett. **60**, 1676(1988).

[8] Kolton, A. B., D. Domínguez, N. Grønbech-Jensen. Phys. Rev. Lett. **83**, 3061(1999).

[9] Koshelev, A. E. Physica C **198**, 371 (1992).

[10] Larkin, A. I. and Y. N. Ovchinnikov. J. Low. Temp. Phys. **34**, 409(1979).

[11] Olson, C. J., C. Reichhardt and F. Nori. Phys. Rev. Lett. **81**, 3757(1998).

[12] Pippard, A. B., Phil. Mag. **19**, 217(1969).

[13] Sandor, B. I., *Fundamentals of Cyclic Stress and Strain* Univ. of Wisconsin Press, Madison, (1972).

[14] Valenzuela, S. O. and V. Bekeris. Phys. Rev. Lett. **86**, 504(2001).

[15] Valenzuela, S. O. Phys. Rev. Lett. **88**, 247003(2002).