# Vortex Glass Transition in High $T_c$ Superconductor

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## Abstract

This paper tries to give a clear description of the motion of vortex and the vortex glass transitions in high $T_c$ superconductors from both theoretical analysis and experimental observation. Then a controversy of the location of the vortex glass transition in high $T_c$ superconductors is brought forward by comparing different datum from different groups. A possible resolution is discussed thereafter.

## I. INTRODUCTION

Superconductivity was discovered by Kamerlingh Onnes [1]in mercury at 4K in 1911. Its perfect conductivity and perfect diamagnetism absorbed many scientists who dedicated themselves to this field and obtained many great achievements. After a slow increase in the highest known transition temperature $T_c$ over the decades, reaching a plateau at $23K$, the discovery of high-temperature superconductivity by Bednorz and Muller in 1986 opened a new chapter in the field of solid-state physics in general and in superconductivity in particular. Currently, the highest $T_c$ achieved in the YBCO, BSCCO, and TBCCO systems are 93K, 110K, and 130K, respectively. These very high transition temperatures were of obvious technical interest because they opened the way to applications which required only liquid $N_2$ cooling (77K), rather than liquid helium.

The new high-temperature superconductors are strongly type II and their phenomenology is dominated by the presence of vortices over most of the phase diagram; see Fig.1. This phase diagram comprises a Meissner phase characterized by complete flux expulsion at low magnetic fields $H < H_{c_1}$, where $H_{c_1}$ is the lower critical field which is mainly determined by the London penetration depth $\lambda$,which is the length scale determining the electromagnetic response of the superconductor. The vortex glass phase lies in the field range between $H_{c_1}$ and irreversibility line ($T_g(H)$), where the magnetic field penetrates the superconductor in the form of flux lines (or vortices), which form a triangular lattice. The magnetic flux enclosed in a vortex is quantized in units of $\Phi_0 = hc/2e \approx 2 \times 10^{-7} Gcm^2$, the flux quantum. With increasing field the density of flux lines increases. When the vortex cores overlap another phase, vortex liquid, appears in the range between the irreversibility line and the upper critical field $H_{c_2}$, which is determined by the coherence length $\xi$ of the superconductor. Beyond this field we recover the normal metallic state. The upper critical field $H_{c_2}$ indicates the appearance of diamagnetism, but it is not a definite boundary.

It is necessary to discuss the irreversibility line in detail. Muller et al. [2] first found that there existed a wide region of reversible magnetization below $H_{c_2}$, and a boundary, below which the magnetization was hysteretic. This boundary is the so-called irreversibility line which can be scaled as $H_{irr} \propto (T_C - T)^{1.5}$ approximately. D.R. Nelson et al. [3] claimed that the irreversibility line marks the transition of ordered vortex lattice to the vortex liquid due to the great thermal fluctuations.

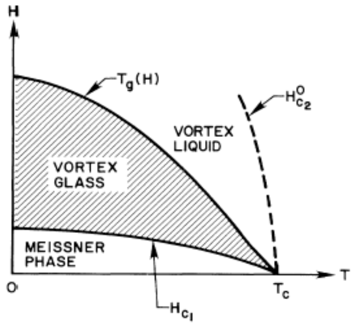



FIG. 1: Schematic phase diagram in high $T_c$ superconductors

## II. MOTION OF FLUX LINES

Let's look flux lines in an ideal homogeneous material first. When $H_{c_1} < H < H_{c_2}$, the flux lines penetrate the material in a regular triangular array of flux tubes, each carrying a quantum of flux $\Phi_0$. This vortex array was first demonstrated experimentally by a magnetic decoration technique coupled with electron microscopy. [4] The force between flux tubes is repulsive. In the presence of a transport current $J$,the flux tubes experience a *Lorentz force*

$$\vec{f} = \vec{J} \times \frac{\Phi_0}{c} \tag{1}$$

which is the force on a single vortex and is analogous to the macroscopic force density $\vec{J} \times \vec{B}/c$. It tends to drive the flux tubes move sideways. If they move with a velocity $\vec{v}$, they essentially induce an electric field of magnitude

$$\vec{E} = \vec{B} \times \frac{\vec{v}}{c} \tag{2}$$

This acts like a resistive voltage, and power is dissipated. Bardeen and Stephen showed that this flux motion is resisted only by a viscous drag. [5] Assume a viscous drag coefficient $\eta$ such that the viscous force per unit length of a vortex line moving with velocity $\vec{v}$ is $-\eta\vec{v}$. Equating this to the driving force (1), we find the magnitudes related by

$$J\frac{\Phi_0}{c} = \eta v \tag{3}$$

Combing this with (2), we find that a type II superconductors show a resistance

$$\rho_f = \frac{E}{J} = B\frac{\Phi_0}{\eta c^2} \tag{4}$$

which is comparable to that in the normal state, only reduced by a factor $B/H_{c2}$.This means that a pure type II superconductor is unable to sustain a persistent current unless some mechanism exists which prevents the Lorentz force from moving the vortices. Such a mechanism is called a *pinning* force since it "pins" the vortices to fixed locations in the material. Pinning results from any spatial inhomogeneity of the material since local variations of $\xi$,$\lambda$, or $H_c$ due to impurities, point defects, twin planes, grain boundaries, voids, etc., will cause some locations of the vortices to be favored over others. It is the pinning which allows the system to sustain the Lorentz force without flux motion and dissipation, thus giving the material a nonzero critical current.

In 1962, Anderson claimed that the pinned vortex is not absolute static. [6] Two years later, Anderson and Kim proposed a flux creep theory. [7] At finite temperature, the thermally activated motion causes a bundle of flux lines to jump between adjacent pinning centers. They assumed the bundle to jump as a unit because the range $\lambda$ of the repulsive interaction between flux lines is typically large compared to the distance between lines; this encourages cooperative motion. Assume that the bundle lies in the bottom of the pinning potential well with depth $U_0$, where $U_0$ is the difference of the Gibbs free energy of the bundles bounded in the well and that of the bundles moving freely. When the driving force is zero, the jump rate of the thermally activated bundle is given by

$$\nu = \nu_0 e^{-U_0/k_B T} \tag{5}$$

where $\nu_0$ is some characteristic frequency of flux-line vibration. Since the jump rates along both sides of the well are the same, there is no net motion of flux lines in the material.

When a flux-density gradient is introduced, the bundle experiences a Lorentz force $\vec{f_L} = (1/c)\vec{J} \times \vec{B}$. We can construct a energy $U_L$ given by

$$U_L = \frac{1}{c} JBV_c r_p \tag{6}$$

using the Lorentz force $\vec{f_L}$, the volume of the bundle $V_c$ and the effective distance of the pinning force. With the increasing of $U_L$, the pinning potential tilts like a washboard; see Fig.2. This will lead to a net hopping rate in the direction of the Lorentz force of

$$\nu_{net} = \nu_+ - \nu_- = 2\nu_0 e^{-U_0/k_B T} sinh(U_L/k_B T) \tag{7}$$

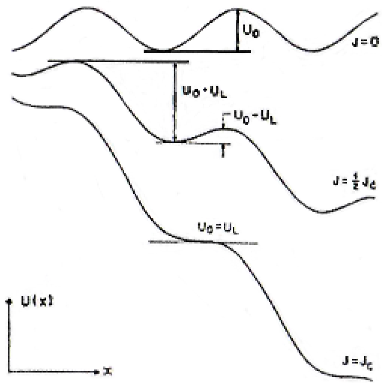



FIG. 2: Schematic change of potential along the direction of the Lorentz force. The curve on the top corresponds to the zero Lorentz force; the one on the bottom corresponds to depinning($U_0 = U_L$).

where $\nu_0$ would be the creep velocity if there were no barrier. The average velocity of the creep of the bundle is $\bar{v} = \nu_{net} l$, where $l$ is the average distance of the jumping of bundles.Then the induced electric field of the creep of the bundle is given by

$$E = \nu B/c = \frac{2\nu_0 Bl}{c} e^{-U_0/k_B T} sinh(U_L/k_B T) \tag{8}$$

In the limit of small transport current, (8) becomes

$$E = \frac{2\nu_0 B^2 l J V_c r_p}{c^2} e^{-U_0/k_B T} \tag{9}$$

We observed this Ohmic dissipation in the high temperature oxide superconductor. When the Lorentz force exceeds the pinning force, the vortices move in a rather steady motion and give a flow resistivity $\rho_f$, which is comparable with $\rho_n$.

Bardeen-Stephen model discusses the effect of the viscous drag, while Anderson-Kim model emphasizes the effect of thermal fluctuations on the system; to the extent, both models are reasonable and show that motion of the flux lines leads to dissipation. Then an important question is whether the flux lines can be collectively pinned by defects sufficiently strongly that they have no linear response to the Lorentz force. This zero resistance state was first proposed for bulk superconductors by Fisher [8] and called the vortex glass.


FIG. 3: An excitation of a single pinned vortex line. The dashed line represents the initial configuration, the dotted line is the final configuration, and the solid line is the vortex loop. The transport current is assumed to be normal to the paper.

## III. VORTEX GLASS MODEL

In the Bardeen-Stephen model [5] and Anderson-Kim theory of flux creep, [6] each vortex line or bundle of lines is effectively modeled as a single, approximately independent, zero-dimensional "particle"; the pinning potential depends on the applied current $J$. However, the elastic deformation of flux line has an important effect on the pinning potential. M.P.A.Fisher [8] proposed a theory of the possible existence of a vortex glass phase transition based on the effect of the random point disorder, which underlies the collective pinning and vortex glass model. Fisher et al. [9, 10] has discussed it extensively. This theory claims that a vortex liquid phase, with linear resistance, experiences a second-order transition to a vortex glass phase, with zero linear resistance, in bulk disordered superconductors at a well-defined glass-melting temperature $T_g$. Therefore, the vortex glass phase is a true superconductor. Let's illustrate this point by first considering a single, infinitely long, vortex line in the presence of random pinning.

In the presence of a transport current $\vec{J}$, the vortex line experiences a Lorentz force which deforms it. Since different parts of the vortex line experience different pinning forces, the entire line can not move with the same velocity, but move with an excitation of vortex loop; see Fig.3. The superposition of the initial configuration of the vortex line with the excitation induces the motion of the part of it. The transverse displacements of the vortex-line segment of length $L$ is $L_\perp \sim L^\zeta$ with an exponent $\zeta \simeq 0.6$. Then the length of a typical loop is $\sim L^\theta$, and its area is $\sim L^k$, where $\theta$ is some exponent and $k = 1 + (1/\zeta)$. The vortex loop will have free energy given by

$$F_{tension} \sim \gamma L^\theta \tag{10}$$

where $\gamma$ is a stiffness coefficient. The transport current exerts a free energy

$$F_{current} \sim -JL^k \tag{11}$$

on the vortex loop. So the total free energy change of the vortex loop is given by

$$F_{total} = F_{tension} + F_{current} \sim \gamma L^\theta - JL^k \tag{12}$$

The extremum of $F_{total}$ gives the size of the vortex loop given by

$$L_{opt} \sim (\frac{\gamma\theta}{kJ})^{\frac{1}{k-\theta}} \tag{13}$$

To obtain the excitation from the ground state by continuous deformation, the vortex line will typically have to pass over a free-energy barrier

$$U \sim \Delta L_{opt}^{\Psi} \sim J^{-\mu} \tag{14}$$

where exponent $\Psi \geq \theta$ and $\mu = \Psi(k - \theta) \leq 1$. Motion of the vortex line will proceed via thermal nucleation of loops of size $L_{opt}$ at a rate $\upsilon \sim exp(-U/T)$. This motion causes a steady-state dissipative electric field

$$E(J) \sim \upsilon \sim e^{-(J_T/J)^\mu} \tag{15}$$

where $J_T \sim \gamma(\Delta/T)^{1/\mu}$. Eq.(14) and Eq.(15) show that $U \to \infty$, for$J \to 0$, thus the linear resistance disappearances. So the vortex glass state is a true superconductor. Fisher claims that the point pinning induces divergent potential and vortex glass is the consequence of the divergence of potential. The concept of the excitation of vortex loop can be applied to the interaction of vortex lines in vortex glass phase. Any local excitation of the vortex lines can be described in terms of one or more vortex loops relative to the initial state; see Fig.4. The transverse size of the vortex loop is one vortex spacing.

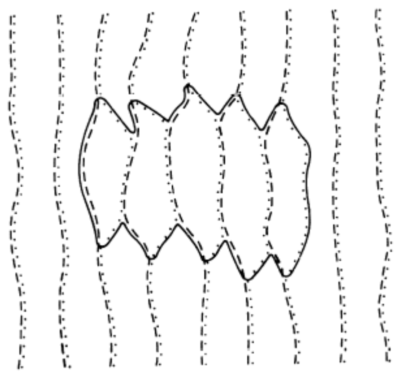


FIG. 4: Excitation of a multiple vortex-line array. The dashed lines represent the initial configuration, the dotted lines are the final configuration, and the solid line is the relative vortex loop. The transport current is assumed to be normal to the paper.

## IV. VORTEX GLASS TRANSITION AND EXPERIMENTAL EVIDENCE

The above section gives a detail description of the vortex glass phase.Note that disorder destroys the crystalline long-range order of the flux-line lattice [11]beyond a correlation volume $V_c$,within which there is short-range order, predicted by mean field theory. Then a question is: Is there a well-defined glass-melting temperature $T_g$ which marks the second order transition from the vortex-liquid phase($\rho_f \neq 0$) to the vortex-glass phase($\rho_g = 0$)? Fisher et al. [8–10] approach the vortex-glass critical temperature $T_g$ by means of scaling analysis. They claim that the vortex-glass phase correlation length $\xi_g$ diverges at $T_g$ as

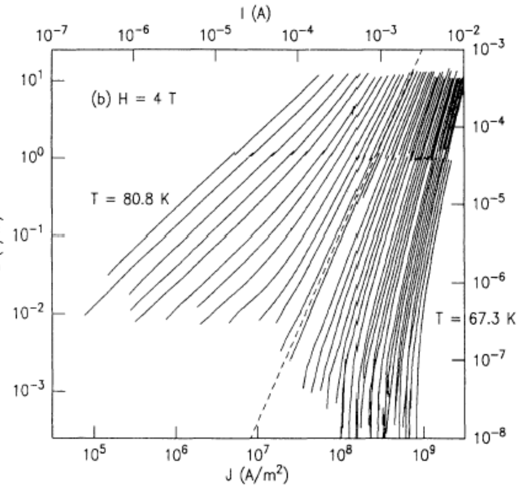
$$\xi_g \propto |T - T_g|^{-\nu} \tag{16}$$

where $\nu$ is a single exponent defined. The critical slowing down of the characteristic relaxation time $\tau_g$ is given by

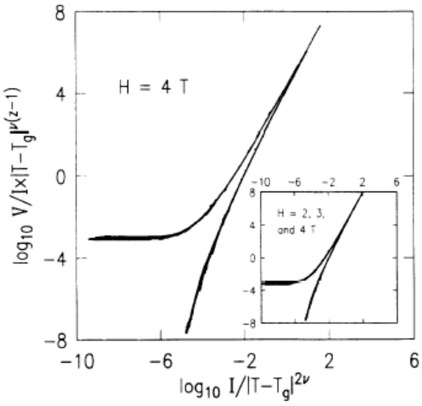
$$\tau_g \propto \xi_g^z \tag{17}$$

where $z$ is another exponent defined. They argue that the electric field should scale as $1/(length \times time)$ and that $J$ should scale as $1/(length)^{D-1}$. Thus, they obtain the scaling hypothesis

$$E\xi_g^{z+1} \approx \mathcal{E}_\pm(J\xi_g^{D-1}) \tag{18}$$




(a)




(b)

FIG. 5: Excitation of a multiple vortex-line array. The dashed lines represent the initial configuration, the dotted lines are the final configuration, and the solid line is the relative vortex loop. The transport current is assumed to be normal to the paper.

where $\mathcal{E}_{\pm}$ are scaling functions for temperatures above $T_g$ and below $T_g$ respectively and D is the spatial dimension.

According to this scaling hypothesis, they predict that in the vortex-glass phase below $T_g$, the nonlinear electric-field response to a current density $J$ is of the form

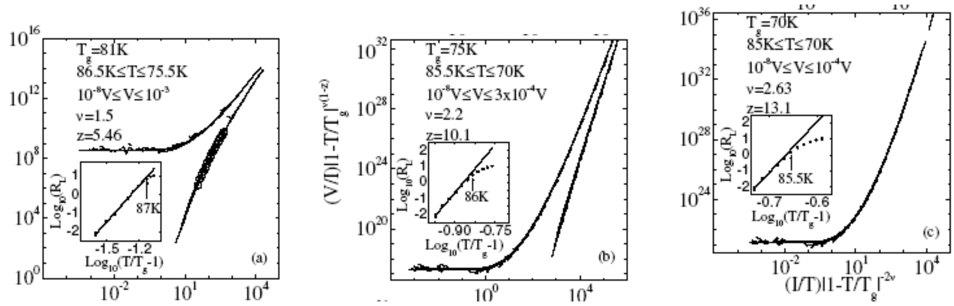
$$E(J) \propto e^{-(J_T/J)^{\mu}} \tag{19}$$

where $J_T \sim \gamma(\Delta/T)^{1/\mu}$ and $\mu \leq 1$. At the vortex-glass temperature $T_g$, they predict a power-law I-V characteristic curve with

$$E(J) \propto J^{(z+1)/(D-1)} \tag{20}$$

Finally, they predict that for $T > T_g$, the I-V characteristic at very low current levels should behave linearly near $T_g$. This is a critical issue in testing the model.

Koch et al. [12, 13] confirmed these predicts by measuring the I-V characteristic of the YBCO epitaxial thin film at different temperature in a strong magnetic field ($H \gg H_{c1}$); see Fig.5a. At a single well defined temperature T, which is defined as $T_g$, the I-V curve is a straight line which is consistent with the prediction of the power law at $T = T_g$. At $T > T_g$,the I-V curves have positive curvature. At $T < T_g$, the curvature of the I-V curves




(a) (b) (c)

FIG. 6: Collapses of the I-V isotherms for a 2200ÅYBCO film in 4T using various critical parameters with experimental windows denoted. [16]

is negative which is consistent with the prediction Eq.(19). We can see this below. Frow Eq.(19), we get

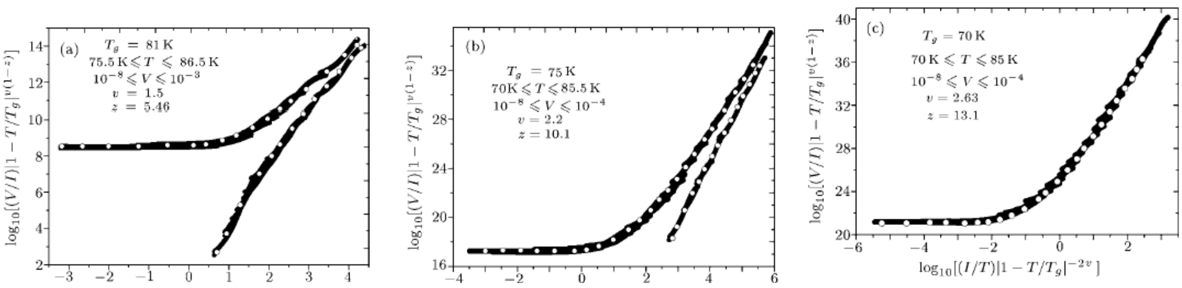
$$\frac{\partial^2 lnE}{(\partial lnJ)^2} = -\mu^2 (J_T/J)^\mu \tag{21}$$

which shows that the curvature of the $lnE - lnJ$ curve is always negative at $T < T_g$. The striking dc I-V data collapse shown in Fig.5b strongly supports the theory of vortex glass transition of Fisher [8–10]. Gammel [14] et al. showed the same experimental evidence latter.

The same experimental law was observed by many different groups. Fisher claims that this is a strong evidence of the vortex glass state and calls the phase below $T < T_g$ vortex glass. The phenomena of the change from the positive curvature to the negative curvature can't be explained by the Anderson-Kim flux creep theory and the logarithmic potential model [15]. The nonlinear response function of the Anderson-Kim theory is $E \sim sinh(J/J_0)$ which shows that the $lnE - lnJ$ curves have a positive curvature, and the one of the logarithmic potential gives $lnE \sim (U_0/k_BT)ln(J/J_c)$, thus the $lnE - lnJ$ curves have no curvature.

## V. AN CONTROVERSY AND POSSIBLE RESOLUTION

Though it has so many experiments supporting, the vortex glass transition theory has met serious doubt recently. Strachan et al. [16] show wide range accurate isothermal I-V measurements over 5 or 6 decades and find although the I-V isotherms measured in a strong magnetic field can be collapsed onto scaling functions proposed by Fisher et al. [9],




(a) (b) (c)

FIG. 7: Collapse of the simulated I-V isotherms using various cirtical parameters compared with the experimental I-V isotherms for a 2200ÅYBCO film in 4T measured by Strachan et al. [17]

these excellent data collapse can also be achieved for a wide range of exponents and glass temperature $T_g$ as demonstrated in their Fig.2(a),2(b)and 2(c); see Fig.6. Thus they argue that the scaling criterion in the literature [12] can't determine the critical temperature $T_g$ uniquely and especially the scaling function of the I-V characteristic $V_{-} \sim exp(-1/x^{\mu})$ of the vortex solid may not be right.

X. Hu et al. [17] show that dc current-voltage characteristic of mixed state superconductors has the general form of extend power law given by

$$y = xexp[-\gamma(1 + y - x)^p] \tag{22}$$

where

$$\gamma = \frac{U_c}{k_B T}(\frac{J_c}{J_L}), \quad x = \frac{J}{J_L}, \quad y = \frac{E(J)}{\rho_f J_L}, \quad p = \mu$$

The numerical solution of (22) has a fair agreement with the wide-range experimental data of Ref.[16]; see Fig.7. They argue that the general extended power law (22) is helpful in settling the controversy. However, they also don't give a criterion to decide the vortex glass transition temperature $T_g$. Thus, one expects a reasonable theory to describe the vortex glass phase completely.

## VI. CONSLUSION

We have briefly surveyed the motion of vortices in a mixed state of superconductor and the vortex glass transition. Bardeen-Stephen model discusses the effect of the viscous drag on the vortices. Anderson-Kim model emphasizes the effect of thermal fluctuations on the system. Vortex glass model claims that there is a second order transition from the vortex-liquid phase($\rho_f \neq 0$) to the vortex-glass phase($\rho_g = 0$). The interest point is the controversy on the vortex glass transition between two groups. X.Hu et al. [17] proposed a possible resolution to the controversy, but it is not complete. A correctness of the vortex glass picture may be required.

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