Bose-Einstein condensation in optical lattices

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Abstract:

The advent of coherent matter waves in the form of Bose-Einstein condensation, coupled with periodic potentials in the form of optical lattices, has established a new area of research on the boundary between atomic and condensed matter physics. This essay is a brief review of the recent theoretical and experimental progress in the area of degenerate Bose gases loaded into optical lattices.

1. Introduction

It has been several years since I developed the interest in optical lattices. After taking the class this semester on Emergent States of Matter, I really have a desire to learn more things about relationship between the Bose-Einstein condensates (BECs) and optical lattices. After reading some publications in this field, I decided to write the review article to improve my knowledge on the exciting and thriving subject.

Observation of the quantization of the energy levels of laser-cooled atoms trapped in optical standing waves in 1992 can be viewed as the start of the research in optical lattices [1]. In 1995 we saw the first creation of dilute gas BECs [2], which provide a source of deBroglie waves with a coherence length equal to the size of the condensate, typically 10-100μm. Shortly after the first realization of BECs, a number of research groups started investigating the properties of BECs in periodic potentials, often preceded and sometimes followed by theoretical efforts. Now BECs in optical lattices have matured into an active field of research in its own right [3].

It is natural to want to know why we study BECs in optical lattices. In general, optical lattices offer several advantages: a vast number of potentials can be created with almost complete control over the parameters, and the potential can be altered or switched off entirely during the experiment. On the other hand, BECs typical values are on the order of tens to hundreds of nano-Kelvins for the temperature and up to 10^{14} cm^{-3} or more for the densities. This order of magnitude difference has several advantages: First lower temperatures means that a BEC will be in the lowest energy levels of the lattice wells without the need for further cooling. Secondly the higher densities lead to an increased filling factor of the lattice, which can exceed unity for BECs.

In this paper, we will discuss the theoretical description of a Bose-Einstein condensate in periodic potentials. We mainly focus on the physical situation in which we deal with a very large number of atoms where we can ignored atom number fluctuations and use mean-field method. We also will give a brief description on the experimental studies on BECs in optical lattices.

Because the amount of theoretical and experimental work on this topic is so large that I only can give a description on some important ones that I think. For more detailed and specialized reviews, please refer to the publications: Morsch et. al., Rev. Mod. Phys., **78**, 179 (2006) and Bloch et. al., Phys. B **38**, S629. (2005) etc.

2. Theory

The general mathematical description of BEC of a weakly interacting gas has already been addressed in different review articles such as Dalfovo *et al.*, Rev. Mod. Phys. **71**, 463. (1999). in this review we, therefore, concentrate on the results obtained for a BEC in periodic potentials.

The many-body Hamiltonian describing N interacting bosons in an external trapping potential V_{ext},

$$\hat{H} = \int d^3x \hat{\psi}^\dagger(\mathbf{x}) \left[-\frac{\hbar^2}{2m}\nabla^2 + V_{\text{ext}} \right] \hat{\psi}(\mathbf{x}) + \frac{1}{2}\frac{4\pi a_s \hbar^2}{m} \int d^3x \hat{\psi}^\dagger(\mathbf{x}) \hat{\psi}^\dagger(\mathbf{x}) \hat{\psi}(\mathbf{x}) \hat{\psi}(\mathbf{x}), \tag{1}$$

Where $\hat{y}(x)$ is a boson field operator for atoms in a given internal atomic state. The ground state of the system and its thermodynamic properties can be calculated from this Hamiltonian. In general these calculations can get very complicated and, in most cases, impracticable. In order to overcome the problem of solving exactly the full - 3 -many-body Schrödinger equation, mean-field approaches are commonly developed. A detailed derivation can be found in Dalfovo's article. The generalization of the original Bogoliubov description to the physical situation in real experiments is given by describing the field operators in the Heisenberg:

$$\hat{\Psi}(\mathbf{x},t) = \psi(\mathbf{x},t) + \delta\hat{\Psi}(\mathbf{x},t) \tag{2}$$

Where $y(x,t)$ is a complex function defined as the expectation value of the field operator and its modulus represents the condensate density.

If we ignored the depletion of the condensate, the time evolution of the condensate wave function at temperature T=0 is obtained by taking the ansatz for the field operator and using the Heisenberg equation. Then we can get the Gross-Pitaevskii equation for the mean field (see some condensed matter physics text books such as by Chaikin),

$$i\hbar\frac{\partial}{\partial t}\psi(\mathbf{x},t) = \left(-\frac{\hbar^2\nabla^2}{2m} + V_{\text{ext}}(\mathbf{x}) + g|\psi(\mathbf{x},t)|^2 \right)\psi(\mathbf{x},t)$$

$$g = \frac{4\pi\hbar^2 a_s}{m}. \tag{3}$$

The description of the propagation of noninteracting matter waves in periodic potentials is straightforward once one has found the eigenstates and corresponding eigenenergies of the system. If we consider a one dimensional *sinusoidal* periodic potential of the form,

$$V_{\text{ext}} = V_0 \cos^2(kx) = sE_R \cos^2(kx) \tag{4}$$

With $k=\pi/d$, where d is the periodicity of the potential. In the context of ultracold atoms in standing light waves, this connection was discussed in the early days of atom optics by Wilkens et. al., Phys. Rev. A, **44**, 3130 (1991).

The wave function and the potential in a Fourier series with the reciprocal-lattice vector defined:

$$\Phi_{n,q}(x) = e^{iqx}\sum_m c_m^n e^{imGx} \qquad V(x) = \sum_m V_m e^{imGx} \tag{5}$$

The stationary solutions can be found in a simple way by applying Bloch's theorem. Putting this ansatz for the eigenfunctions into the Schrödinger equation and truncating the sum at m=N, then the 2(2N+1)-dimensional system of linear equations, (G=2Π/d).

$$\left\{\frac{\hbar^2}{2m}(q-mG)^2+V_0\right\}c_{q-mG}+V_G c_{q-(m+1)G} + V_{-G}c_{q-(m-1)G}=Ec_{q-mG}, \tag{6}$$

With $m=-N, -N+1, \ldots, N-1, N$. The eigenenergies and eigenstates depend on the potential depth V_0 and the quasimomentum q. In the *weak potential* limit, the eigenenergies depend critically on the quasimomentum q. Since the so-called gap energy between the nth and (n+1)th band scales with V_0^{n+1} in the weak potential limit (Giltner *et al.*, phys. Rev. A **52**, 3966, (1995)), it only has appreciable magnitude between the lowest and first excited band. In the limit of *deep periodic potentials*, also referred to as the tight-binding limit, the eigenenergies of the low-lying bands are only weakly dependent on the quasimomentum. The quasimomentum dependence of the lowest band energy was given analytically by Zwerger (J. Opt. B: quantum Semiclassical Opt.**5**,9, (2003)).Typical phenomena studied in this regime only involve the lowest band, which is well described by localized wave functions at each site.

When the equation of motion of the condensate wave function is defined via a nonlinear Schrödinger equation due to the interaction between the particles, this introduces a new energy scale and thus, in contrast to the linear propagation, new parameter regimes with associated new phenomena and dynamics for special potential parameters are expected. One of the most striking of these is the appearance of solitonic propagation and instabilities (i.e., small perturbations of the condensate wave function can grow exponentially in time).

The mean-field energy per atom corresponding to a given condensate wave function is defined as

$$U=g\int d^3x|\psi(\mathbf{x})|^4 \tag{7}$$

In the case of periodic potentials, it is more sensible to calculate the on-site interaction energy, which measures the strength of the interaction within one period of the lattice. The theoretical descriptions for these regimes have several cases: Nonlinear energy scale is the smallest; Nonlinear energy scale in the intermediate range; Nonlinear energy scale is dominant. For more detail consideration about the three cases, please refer to the publications such as Konotop and Salerno Phys. Rev. A, **65**, 021602, (2002); smerzi and Trombettoni, Phys. Rev. A **68**, 023613, (2003) and choi and Niu, Phys. Rev. Lett. **82**, 2022, (1999).

3. Experiments

The first BEC/optical lattice experiment to begin to probe beyond single-particle physics was carried out at Yale in 2001 [4]. A BEC was adiabatically located into a one dimensional optical standing wave and the coherence from site to site was analyzed by releasing the atoms from the lattice and looking at the interference pattern formed from the array of overlapping wavepackets. A loss of interference contrast was observed as the adiabatic loading time was increased. In a 1999 paper, the group of Peter Zoller made the suggestion [5] that Bose-Hubbard could be realized in 2 and 3D optical lattices. In 2002 in Munich a group [6] reported the observation of the superfluid-Mott insulator phase transition in a 3D optical lattice, observing the loss of interference as the system reached the insulating state as well as gap in the spectrum associated with the insulating state in transport with the application of a large gradient.

Optical lattices can easily be constructed in 1-, 2- or 3D geometries. A series of recent experiments [7] have been studying 1D Bose gases by confirming them in a 2D optical lattice. An experiment at NIST [8] observed a factor of 7 reductions in three-body loss in the tubes over that for a 3D gas. Recent work at Penn state [9] looking at dipole oscillations of a 1D gas as an optical lattice is turned on.

The most striking effects of the band structure of periodic potentials are the occurrence of Bloch oscillations and Landau-Zener tunneling which have been observed in ultracold atoms before condensates entered the scene (Dahan et. al., Phys. Rev.Lett. 76, 4508 (1996) and Niu *et al.*, Phys. Rev. Lett. **76**, 4504. (1996)). Because they have shown that Bose-Einstein condensates offered the possibility to investigate them more systematically and in different regimes. The first experiment along these lines with Bose condensates in optical lattices was carried out by Anderson and Kasevich in 1998, sparking considerable interest in both the theoretical and experimental communities.

Morsch *et al.* (Phys. Rev. Lett. **87**, 140402. (2001)) carried out experiments in linear regime, loading BECs of rubidium atoms into a shallow optical lattice that was subsequently accelerated with acceleration a by chirping the frequency difference between the lattice beams (Cristiani *et al.*, Phys. Rev. A **65**, 063612, 2002). From the resulting interference pattern, the condensate group velocity in the frame of reference of the lattice could be calculated and plotted against the lattice velocity clearly showing the Bloch oscillations.

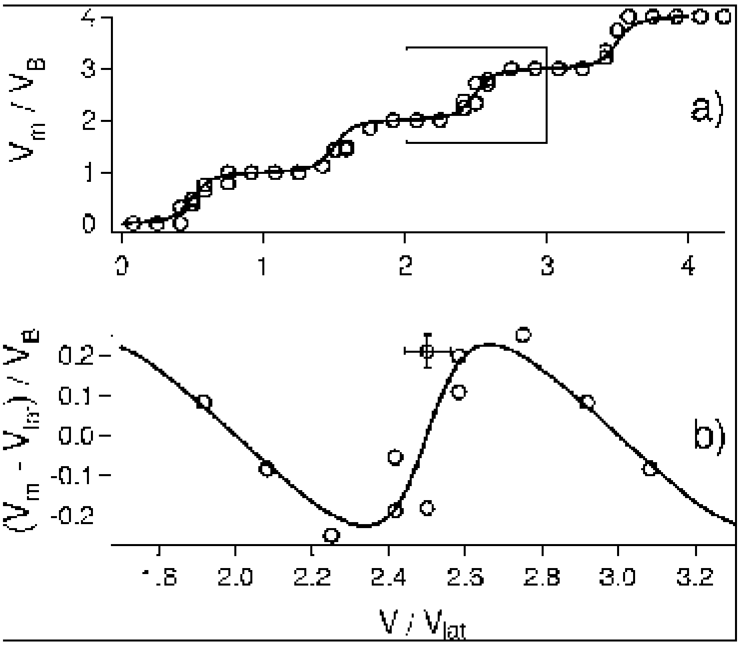

Fig. 1. Bloch oscillations of a condensate in an optical lattice. When the instantaneous lattice velocity (indicated on the horizontal axis) is subtracted from the mean velocity of the condensate measured in the laboratory frame of reference (a), one clearly sees Bloch oscillations in the lattice frame (b). From Cristiani *et al.*, 2002.

Another phenomenon occurring in an accelerated lattice is Landau-Zener tunneling which was observed for ultracold atoms in a lattice (Niu *et al.*, 1996). In the experiment by Anderson and Kasevich (1998), a vertically oriented lattice was used, with the Earth's acceleration *g* driving the atoms. The Landau- Zener tunneling events led to atomic "droplets" falling out of the lattice (see Figure 2).

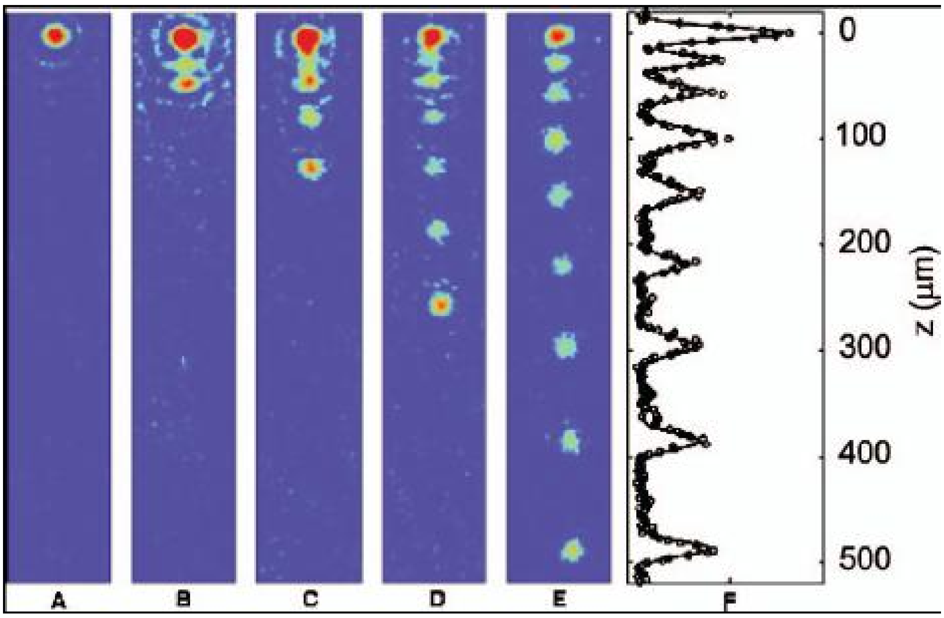

Fig. 2. (Color) Coherent "droplets" tunneling out of a condensate held in a vertical 1D optical lattice. This effect can be interpreted in terms of the condensate undergoing Bloch oscillations under the influence of the gravitational force and part of the condensate leaving the lattice due to Landau-Zener tunneling at successive crossings of the Brillouin zone edge. Holding times in the lattice are (a) 0, (b) 3, (c) 5, (d) 7, and

(e) 10 ms, respectively. In (f), an integrated profile of the absorption image (e) is shown together with a theoretical fit (solid line). Taken from Anderson and Kasevich, Science **282**, 1686 (1998).

When the nonlinear term in the Gross-Pitaevskii equation is not negligible any longer, the behavior of a BEC in an accelerated lattice deviates appreciably from the linear case (Morsch and Arimondo, 2002 in *Dynamics and Thermodynamics of Systems with Long-Range Interactions*, Berlin, pp. 312–331). In particular, performing Landau-Zener tunneling experiments as a function of the nonlinear parameter *C*, Morsch *et al.* (2001) found that the tunneling probability increased with increasing *C*. This can be explained in the effective potential approximation introduced by Choi and Niu (1999) as a decrease in the effective potential depth and hence the band gap at the Brillouin zone edge, leading to increased tunneling (see Fig. 3).

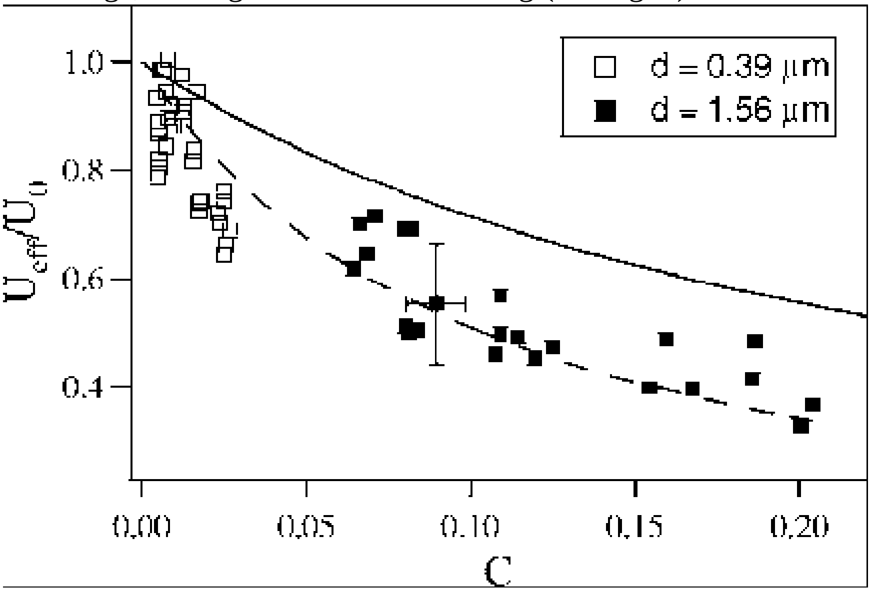

FIG. 3. Variation of the effective potential with the nonlinear parameter *C*. The square symbols are experimental data points and the solid and dashed lines are the theoretical prediction by Choi and Niu (1999) and a best fit with a rescaled nonlinearity parameter, respectively. From Morsch *et al.*, Phys. Rev. Lett. **87**, 140402, (2001).

There are lots of other important experiment work in this field such as Instabilities and breakdown of superfluidity; Dispersion management and solitons; Chemical potential of a BEC in an optical lattice; Josephson physics in optical lattices; Number squeezing and the Mott-insulator transition etc. Because of the limitation of the paper, I can not discuss all of them. Please see the references I gave at the end of this paper for more detail information.

4. Future directions

There are many possible topics of interest, most of which have yet to be discovered, so I will mention a few to display the richness of this research area. Among the future areas of exploration will be a further understanding of the Bose-Hubbard system, including the development of spectroscopic tools to spatially resolve the number distributions in individual lattice sites.

Optical lattices are inherently disorder-free, making for extremely clean systems, but we can expect to see disorder introduced into optical lattices with a degree of control unheard of in condensed matter physics. Unlike condensed systems, the optical lattice system will have knobs to be able to change the amount of disorder at will. We can anticipate many possibilities including the study of localization, the Bose glass phase and perhaps a Bose metal phase.

Because the atom-atom interaction has been the short range van der Waals interaction, well described by a simple contact term, we can expect research in periodic systems where the scattering length as well as change the interaction from repulsive to attractive and long range anisotropic dipole-dipole interactions. So far the lattice work has been confined to single spins, but following on a number of spinor condensate experiments, we can expect two and three component spinor Bose loaded into optical lattices.

5. Conclusion

In this brief review article, I have shown some theoretical and experimental advances in Bose-Einstein condensates in optical lattices. Because of the limit of the paper, I just can give a rough description on some important results. Bose-Einstein condensates in optical lattices have already spawned several different subfields such as nonlinear matter waves, strongly correlated many-particle systems, and quantum computation [10]. In the first two categories, the full control over the system's parameters is exploited in several ways. By changing the geometry of the lattice and combining, e.g., different atomic species, one can realize many-body Hamiltonians that are not easily accessible in condensed matter systems and hence use BECs in lattices as a model system in order to test theoretical predictions. In the latter, optical lattices are used mainly as a tool for preparing and "engineering" quantum states in a controlled way so that they can then be used for the implementation of quantum algorithms.

In the future I intend to do some research on the following areas: by trapping atoms in optical lattices to probe the single photon efficiency which is related with my current research. Because we want to obtain the high detector efficiency of single photons, we should trap the atoms in order to reduce the dark count. I think that optical lattice is one of the candidate media [11].

6. References

[1]. P. Verkerk, it et. al., Phys. Rev. Lett. **68**, 1116 (1992)
[2]. Abdullaev, F. K., et. al., Phys. Rev. A. **64**, 043606 (2001)
[3]. Some review papers such as Morsch et. al., Rev. Mod. Phys., **78**, 179 (2006); Bloch, I., J. Phys. B **38**, S629. (2005); Jaksch, D., and P. Zoller, Ann. Phys. N.Y. **315**, 52 (2005)
[4]. C.Orzel, et. al., Science. **291**, 2386 (2001)
[5]. D. Jaksch, et. al., Phys. Rev. Lett. **81**, 3108 (1998)
[6]. M.Greiner, et. al., nature. **415**, 39 (2002)
[7]. as an example, T. Storfele, et. al., phys. Rev. Lett. **92**, 130403 (2004)
[8]. B.L.Tolra, et. al., Phys. Rev. Lett. **92**, 190401 (2004)
[9].T.Kinoshita, et. al., Science. **305**, 1125 (2004)
[10]. Deutsch, I. H. et. al., Fortschr Phys. **48**, 925 (2000)
[11]. Daniel F. V. James Phys. Rev. Lett. **28**, 183601 (2002)