BCS Pairing Dynamics

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Abstract. Experimental control over inter-atomic interactions by adjusting external parameters is discussed. Qualitatively different time evolution of the order parameter are predicted theoretically in ultracold atomic fermions with time-dependent pairing interaction. Following an abrupt change of the pairing strength, the order parameter undergoes undamped oscillation, damping oscillation and exponential decay in different dynamical regimes respectively.

1 Introduction

Ultracold atomic gases have become an important medium to realize novel phenomena in condensed matter physics and test many-body theories in new regimes. Of particular interest are pairing phenomena in fermionic gases, which have direct analogies to superconductivity.

The dynamics of the superconducting BCS state in metals has been a subject of long-time active interest. However, until recently, several remarkable experiments, made possible due to the precise experimental control over interactions between atoms in trapped cold gases, have demonstrated Cooper pairing in cold atomic Fermi vapor.[1] Characteristics of a paired state — condensation of Cooper pairs and the pairing gap have been observed. In addition, trapped gases provided a unique tool to explore aspects of fermion pairing normally inaccessible in superconductors. This new discovery has renewed interest in quantum collective phenomena. One of the most exciting prospects is a study of far from equilibrium coherent dynamics of fermionic condensates, which can be initiated by quickly changing the pairing strength with external magnetic field.

Theoretically, response of fermionic condensates to fast perturbations is a long-standing problem. The main difficulty is to describe the time evolution in the *nonadiabatic* regime when a nonequilibrium state of the condensate is created on a time scale shorter than the quasi-particle energy relaxation time. In this case the evolution of the system cannot be described in terms of a quasiparticle spectrum or a single time-dependent order parameter $\Delta(t)$. One has to account for the dynamics of individual Cooper pairs, making it a complicated many-body problem.

In the work of Barankov *et al*[2][3], the authors investigated this problem by using a combination of numerical and analytical methods to solve the Bloch equation of pseudospins. In the work of Yuzbashyan *et al*[4][5], this problem is treated and generalized based on the integrability of the BCS Hamiltonian in this case. In this term essay, I primarily followed the work of the previous group.

Experimentally, the nonadiabatic regime can be accessed in ultracold Fermi gases, where the strength of pairing between fermions can be rapidly changed. Nonadiabatic measurements can also be performed in quantum circuits utilizing nanoscale superconductors where the dynamics can be initiated by fast voltage pulses. This opens up the possibility of exploring the interesting behavior of the pairing dynamics following an abrupt change of the coupling strength.

2 Experimental control over atomic interaction

Dilute fermionic alkali gases cooled below degeneracy temperature are expected to host the paired BCS state. Recent experiments[1] on cold fermion pairing realized the control of interaction strength by using magnetically tuned Feshbach resonances which provides access to the strong coupling BCS regime.

The strength of atomic interactions is characterized by the scattering length a. Scattering lengths for alkali atoms are of the order of $100a_0$, where a_0 is the Bohr radius. In a scattering process, the internal states of the particles in the initial or final states are described by a set of quantum numbers, such as those for the spin, the atomic species, and their state of excitation. One possible choice of these quantum numbers is referred as a *channel*. Coupling between channels gives rise to the so-called *Feshbach resonances*, in which a low-energy bound state in one channel strongly modifies scattering in another channel.[8] Feshbach resonances make possible to tune the magnitude of the effective atom-atom interaction, characterized by the s-wave scattering length a, as well as whether they are, in the mean-field approximation, effectively repulsive($a > 0$) or attractive($a < 0$), they have become a powerful tool for investigating cold atoms.

Feshbach resonances appear when the total energy in an open channel matches the energy of a bound state in a closed channel. From perturbation theory one would expect there to be a contribution to the scattering length having the form of a sum of terms of the type

$$a \sim \frac{C}{E - E_{res}} \tag{1}$$

where E is the energy of the particles in the open channel and E_{res} is the energy of a state in the closed channels. Consequently there will be large effects if the energy of the two particles in the entrance channel is close to the energy of a bound state in a closed channel. From second-order perturbation theory for energy shifts, coupling between channels causes a repulsive interaction if the energy of the scattering particles is greater than that of the bound state, and an attractive one if it is less. The closer the energy of the bound state is to the energy of the incoming particles in the open channels, the larger the effect on the scattering. Since the energies of states depend on external parameters, these resonances make it possible to tune the effective interaction between atoms.

For example, consider an external magnetic field by including Zeeman terms in Hamiltonian.[7] If the Feshbach resonance occurs for a particular value of the magnetic field B_0, the scattering length is given by

$$a = a_{nr}\left(1 + \frac{\Delta B}{B - B_0}\right) \tag{2}$$

where a_{nr} is the non-resonant scattering length whose energy dependence may be neglected, and the width parameter ΔB is proportional to the square of the matrix element of the coupling part of the Hamiltonian. Eq.(2) shows that because of the dependence on $1/(B-B_0)$, large changes in the scattering length can be produced by small changes in the magnetic field. It is especially significant that the sign of the interaction can be changed by a small change in the field.

One of the most important applications of this technique is to experimentally probe what is known as the BCS-BEC crossover, as the strength of the effective attractive interaction between particles is increased continuously from condensation of delocalized Cooper pairs to condensation of tightly bound bosonic molecules. In the work cited in reference, however, the authors are mostly interested in the BCS pairing dynamics initiated by a sudden change to the coupling strength on the BCS side of the Feshbach resonance. BCS-BEC crossover will not be considered in this term essay.

3 To pose the problem...

Write down the Hamiltonian of BCS model,

$$H_{BCS} = \sum_{j,\sigma} \varepsilon_j c^{\dagger}_{j,\sigma} c_{j,\sigma} - g \sum_{j,k} c^{\dagger}_{j\uparrow} c^{\dagger}_{-j\downarrow} c_{-k\downarrow} c_{k\uparrow} \tag{3}$$

Initially, the gas is in the BCS ground state at zero temperature with a coupling constant $g = g_i > 0$. At $t = 0^+$ the coupling constant is suddenly changed to another value $g = g_f > 0$: $g_i \to g_f$. Ground state of the system at the old and new values of the coupling are characterized by corresponding BCS gaps, Δ_i and Δ_f, respectively.

Immediately after the coupling constant is abruptly changed, the initial state is no longer the BCS ground state for the system, what we are interested in is the time evolution of order parameter $\Delta(t)$ of the fermionic condensate in response to such a sudden change of interaction strength, on a time scale $\tau_\Delta = 1/\Delta_i$. This time-dependent behavior depends critically on the strength of the perturbation, namely, the relative magnitude between the old and new coupling constants, or BCS gaps Δ_f/Δ_i. Before discussing this time-dependent behavior of order parameter, we shall first introduce the method to be used.

3.1 Pseudospin Representation

The most convenient way to derive the BCS mean field dynamics is based on Anderson's pseudospin representation. Make the following identification between fermionic states(empty or occupied) and pseudospin states(up or down)

$$|1_{j\uparrow}1_{-j\downarrow}\rangle = |\uparrow\rangle, \quad |0_{j\uparrow}0_{-j\downarrow}\rangle = |\downarrow\rangle \tag{4}$$

and define the pseudospin operators

$$s_j^z = \left(n_{j\uparrow} + n_{-j\downarrow} - 1\right)/2, \quad s_j^- = c_{-j\downarrow}c_{j\uparrow}, \quad s_j^+ = c_{j\uparrow}^\dagger c_{-j\downarrow}^\dagger \tag{5}$$

It can be proved easily that these operator indeed satisfy the commutation relations of the Lie algebra of $SU(2)$. Up to an additional constant, the BCS Hamiltonian can then be written in terms of these pseudospin operators

$$H_{BCS} = \sum_j 2\varepsilon_j s_j^z - g\sum_{j,k} s_j^+ s_k^- \tag{6}$$

The BCS order parameter is the expectation value of the ladder operator: $\Delta(t) = g\langle\sum_k s_k^-\rangle = \Delta_x - i\Delta_y$. In mean field approximation, each spin evolves in the self-consistent field

$$H_{BCS} = \sum_j 2\varepsilon_j s_j^z - 2\sum_j \left(\Delta_x s_j^x + \Delta_y s_j^y\right) = -\sum_j \mathbf{h}_j \cdot \mathbf{s}_j \tag{7}$$

where $\mathbf{h}_j = (2\Delta_x, 2\Delta_y, -2\varepsilon_j)$. This is done by writing the each spin variable as the sum of its mean value and fluctuations, expand the products and neglect the fluctuation terms. The time evolution of expectation values of these spin operators is governed by the equation

$$\frac{d}{dt}\langle\hat{O}\rangle = i\left\langle[\hat{H},\hat{O}]\right\rangle \quad \Rightarrow \quad \dot{\mathbf{s}}_j = \mathbf{s}_j \times \mathbf{h}_j \tag{8}$$

After taking expectation values, the spin variables in equation (8) are replaced by classical spins. The above equation is usually called *Bloch equation*. The ground state of the Hamiltonian (7) corresponds to the configuration that each spin is parallel to its own local magnetic field. Without loss of generality, we can appropriately choose the x-axis so that the order parameter is real: $\Delta_i = \Delta_x$, thus $s_j^y(t=0) = 0$.

3.2 Initial value problem

Now we have a well-defined problem as follows

$$\frac{d}{dt}\mathbf{s}_j = \mathbf{s}_j \times \mathbf{h}_j, \quad \text{with} \quad \mathbf{h}_j = (2\Delta_x, 2\Delta_y, -2\varepsilon_j), \quad \text{for} \quad t > 0 \tag{9}$$

where the effective magnetic field depends on the pairing amplitude Δ, which is defined self-consistently with new coupling g_f:

$$\Delta = \Delta_x - i\Delta_y = g_f\sum_k s_k^- = g_f\sum_k \left(s_k^x - is_k^y\right) \tag{10}$$

The initial condition is the ground state with old coupling g_i:

$$s_j^x(t=0) = \frac{\Delta_i}{2\sqrt{\varepsilon_j^2 + \Delta_i^2}}, \quad s_j^z(t=0) = -\frac{\varepsilon_j}{2\sqrt{\varepsilon_j^2 + \Delta_i^2}} \tag{11}$$

This is a set of coupled nonlinear ordinary differential equations.

3.3 Method to be used

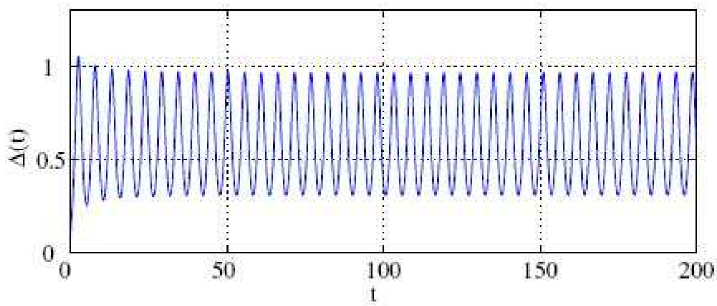
Two primary kinds of methods are used to analyze the problem posed in the previous section.
(1) Numerical method[2]: The Runge-Kutta method of the 4th order was used to integrate the differential equations with initial conditions (9)-(11). The advantage of the purely numerical method is completeness, by which a uniform treatment to different cases is allowed. On the other hand, qualitative physics picture is obscured by the numerics.
(2) Analytical method[4][5][6][2][3]: By using linear analysis or asymptotic analysis in some extreme cases, approximate solution can be obtained respectively. Qualitative but rigorous information can be obtained to determine the dynamics of the system.

4 Classification of dynamical transitions

Three qualitatively different dynamical regimes are observed.

4.1 $\Delta_i < e^{-\pi/2}\Delta_f$: undamped oscillation

If the initial state at $t = 0^-$ has a relatively small BCS gap Δ_i compared to the new gap Δ_f such that $\Delta_i/\Delta_f < e^{-\pi/2} \approx 0.21$, the time evolution of the order parameter $\Delta(t)$ is an undamped oscillation between a lower limit Δ_- and an upper limit Δ_+.



The above plot of the time evolution of the order parameter is obtained by using Runge-Kutta method of the 4th order.[2] The order parameter is in the unit of Δ_f, and the time is in the unit of $1/\Delta_f$. The order parameter in the initial state is $\Delta_i = 0.05\Delta_f < 0.21\Delta_f$, and then it oscillates between $\Delta_+ \approx 0.97\Delta_f$ and $\Delta_- \approx 0.31\Delta_f$.

This kind of undamped periodic oscillation is called *multi-soliton* solution, and can be understood analytically in the following way[3]. Write the differential equation (9) for each spin in the following form

$$\dot{s}_j^+ = 2i\varepsilon_j s^+ + 2i\Delta^* s_j^z, \quad \dot{s}_j^- = -2i\varepsilon_j s^- - 2i\Delta s_j^z, \quad \dot{s}_j^z = i(\Delta s_j^+ - \Delta^* s_j^-) \tag{12}$$

Assume the time evolution of the order parameter of the form $\Delta(t) = e^{-i\omega t}\Omega(t)$, with $\Omega(t)$ real, also $s^\pm \to s^\pm e^{\pm i\omega t}$, eliminate the common phase factor $e^{\pm i\omega t}$, the above equation is transformed into the following form(written in cartesian components)

$$\dot{s}_j^x = -\xi_j s_j^y, \quad \dot{s}_j^- = \xi_j s_j^x + 2\Omega s_j^z, \quad \dot{s}_j^z = -2\Omega s_j^y \tag{13}$$

where $\xi_j = 2\varepsilon_j - \omega$. The above set of equations can be solved by the ansatz

$$s_j^x = C_j \xi_j \Omega, \quad s_j^y = -C_j \dot{\Omega}, \quad s_j^z = C_j \Omega^2 - D_j \tag{14}$$

Then, the first and the third of Eq.(13) is satisfied identically. The second of Eq.(13) is consistent with the normalization condition

$$(s_j^x)^2 + (s_j^y)^2 + (s_j^z)^2 = 1/4 \quad \Rightarrow \quad 2s_j^x \dot{s}_j^x + 2s_j^y \dot{s}_j^y + 2s_j^z \dot{s}_j^z = 0 \tag{15}$$

which can be verified by direct substitution. Thus the normalization condition takes following form

$$C_j^2 \xi_j^2 \Omega^2 + C_j^2 \dot{\Omega}^2 + (C_j \Omega^2 - D_j)^2 = 1/4 \tag{16}$$

The above equation can be cast into the following form for all the spins

$$\dot{\Omega}^2 + (\Omega^2 - \Delta_-^2)(\Omega^2 - \Delta_+^2) = 0, \quad \Delta_- < \Delta_+ \tag{17}$$

if the constants D_j and C_j are appropriately chosen as $(D_j^2 - 1/4)/C_j^2 = \Delta_+^2 \Delta_-^2$ and $2D_j/C_j = \xi_j^2 + \Delta_+^2 + \Delta_-^2$. Now, Eq.(17) is a typical differential equation which defines an elliptic integral

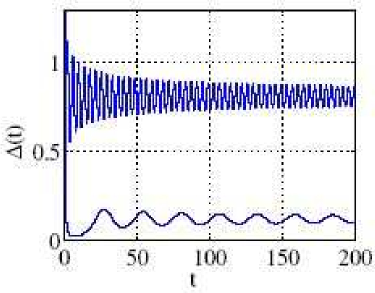
$$t = \int \frac{d\Omega}{\sqrt{(\Delta_+^2 - \Omega^2)(\Omega^2 - \Delta_-^2)}} \tag{18}$$

the solution $\Omega(t)$ is an elliptic function oscillating periodically but non-harmonically between Δ_- and Δ_+. Physically speaking, this is caused by *synchronization* of different Cooper pair states resulting from their interaction with the mode singled out by BCS instability of the initial state.

4.2 $e^{-\pi/2}\Delta_f \leq \Delta_i < e^{\pi/2}\Delta_f$: damping oscillation

Desynchronization occurs when the new BCS gap Δ_f at $t = 0^+$ is small enough such that $\Delta_i > 0.21\Delta_f$. In this regime, two cases are possible, underdamped and overdamped. The former case, in which the time evolution of the order parameter is still an oscillation but with damping amplitude, converging to a constant value Δ_∞, will be considered in the current subsection, which occurs when $0.21\Delta_f < \Delta_i < e^{+\pi/2}\Delta_f = 4.81\Delta_f$.

The plot on the right of the time evolution of the order parameter is obtained by using Runge-Kutta method of the 4th order.[2] The order parameter is in the unit of Δ_f, and the time is in the unit of $1/\Delta_f$. The order parameter in the initial state is $\Delta_i = 0.21\Delta_f$ for the upper curve and $4.5\Delta_f$ for the lower curve. with the asymptotic values $\Delta_\infty \approx 0.81\Delta_f$, $0.12\Delta_f$ respectively.



The simplest way to understand this damping oscillation is to consider a small deviation from the initial state, i.e. $\delta\Delta = \Delta_f - \Delta_i$ is small compared to Δ_f. Here, the linear analysis is used. Write

$$\Delta(t) = \Delta_i + \Delta', \quad s_j = s_j(t=0) + s_j' \tag{19}$$

where the primed variables are assumed to be first-order small quantity. And the initial configuration is the ground state with BCS gap Δ_i which is represented by Eq.(11). Linearize the Bloch equation (9) about the initial equilibrium state, and keep the terms up to the first order, we have the following set of coupled linear equations (note: the original set of equations is coupled and nonlinear)

$$\frac{d}{dt}\begin{pmatrix} s_x' \\ s_y' \\ s_z' \end{pmatrix} = \begin{pmatrix} 0 & -2\varepsilon & -2\Delta_i^y \\ 2\varepsilon & 0 & 2\Delta_i^x \\ 2\Delta_i^y & -2\Delta_i^x & 0 \end{pmatrix} \begin{pmatrix} s_x' \\ s_y' \\ s_z' \end{pmatrix} + \begin{pmatrix} -2\Delta_y' s_{z0} \\ 2\Delta_x' s_{z0} \\ 2\Delta_y' s_{x0} - 2\Delta_x' s_{y0} \end{pmatrix} \tag{20}$$

Here we have suppressed the index j for individual spin. First of all, we can solve the eigenvalue equation for the coefficient matrix to get the oscillating frequency for each spin, denoted by

$$\omega(\varepsilon) = \pm 2\sqrt{\varepsilon^2 + |\Delta_i|^2} \tag{21}$$

Calculation can be simplified by assuming the initial condition according to Eq.(11), i.e. $s_j^y(t=0) = 0$, and $\Delta_i = \Delta_x$. Up to the first order of $\delta\Delta/\Delta_f$, we have

$$\begin{aligned} \Delta(t) &= \Delta_f - 8\delta\Delta \int_0^\infty d\varepsilon \frac{\cos(\omega t)}{\omega[\pi^2 + h^2(\varepsilon)]} && (22) \\ &\to \Delta_f - \frac{2\delta\Delta}{\pi^{3/2}\sqrt{\Delta_f t}} \cos\left(2\Delta_f t + \frac{\pi}{4}\right) \quad \text{as} \quad t \to \infty && (23) \end{aligned}$$

where $h(\varepsilon) = \sinh^{-1}(\varepsilon/\Delta_f)$. The asymptotic expression for the integral at long time is obtained by using stationary phase method. The above power-law decay of the order parameter can be generalized to nonlinear case[5]

$$\frac{\Delta(t)}{\Delta_\infty} = 1 + \text{const} \times \frac{\cos(2\Delta_\infty t + \phi)}{\sqrt{\Delta_\infty t}} \tag{24}$$

where the constant coefficient is time-independent, and $\Delta_\infty = \Delta_f$ if $|\Delta_f - \Delta_i|$ is small.

According to Eq.(9), at long times, each spin $\mathbf{s}_j$ precesses in its own constant field $\mathbf{h}_j = (-2\Delta_\infty, 0, 2\varepsilon_j)$ with its own frequency $\omega(\varepsilon_j)$. For example, for the x-component of spin derived from the linearized equations of motion[4]

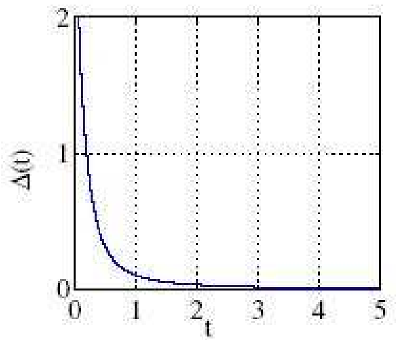
$$s_x(\varepsilon) = \frac{\Delta_f}{2\sqrt{\varepsilon^2+\Delta_f^2}} - \frac{\delta\Delta\cdot\varepsilon}{(\varepsilon^2+\Delta_f^2)\sqrt{\pi^2+h^2(\varepsilon)}} \times \cos[\omega(\varepsilon)t+\phi(\varepsilon)] \tag{25}$$

There is no damping in the precession of individual spin, however, the gap $\Delta(t) = g\sum_j s_j^x(t)$ contains oscillations with many different frequencies from different spins. At large times, they go out of phase and cancel out in the continuum limit. This is the physical reason for the damping oscillation, which is called *collisionless dephasing* by authors in reference[2][3].

4.3 $\Delta_i \geq e^{\pi/2}\Delta_f$: exponential decay of order parameter

The dynamical vanishing of the order parameter occurs in the over-damped regime, when the pairing strength, and hence the BCS gap is suddenly decreased below a certain critical value $\Delta_f \leq e^{-\pi/2}\Delta_i \approx 0.21\Delta_i$ or $\Delta_i \geq e^{\pi/2}\Delta_f \approx 4.81\Delta_f$. In this case, $\Delta(t)$ decays to zero without oscillations.

The plot on the right of the time evolution of the order parameter is obtained by using Runge-Kutta method of the 4th order.[2] The order parameter is in the unit of Δ_f, and the time is in the unit of $1/\Delta_f$. The order parameter in the initial state is $\Delta_i = 4.81\Delta_f$ with the asymptotic value $\Delta_\infty = 0$.



This behavior can be understood in the limit $\Delta_f/\Delta_i \approx 0$, i.e. when the coupling is turned off nearly completely. In this case, according to the evolution equation for s_j^- in Eq.(12) by neglecting the second term on the right hand side

$$\dot{s}_j^- = -2i\varepsilon_j s^- \tag{26}$$

each spin precesses freely and independently, the solution is simply

$$s_j^-(t) = e^{-2i\varepsilon_j t} s_j^-(0) = e^{-2i\varepsilon_j t}\frac{\Delta_i}{2\sqrt{\varepsilon_j^2+\Delta_i^2}} \tag{27}$$

Summing over all spins to get the order parameter

$$\Delta = \sum_j e^{-2i\varepsilon_j t} \frac{\Delta_i}{2\sqrt{\varepsilon_j^2 + \Delta_i^2}} \tag{28}$$

Going to the continuum limit by replacing the summation over j by integral over wave vector

$$\Delta(t) \quad = \quad \int d^3\mathbf{k} \frac{\Delta_i e^{-2i\varepsilon_k t}}{2\sqrt{\varepsilon_k^2 + \Delta_i^2}} \propto \frac{1}{\sqrt{\Delta_i t}} e^{-2\Delta_i t} \tag{29}$$

we get the exponential decay of the order parameter and the system goes into gapless state. In general, the above decay-law can be extended to the following form[4]

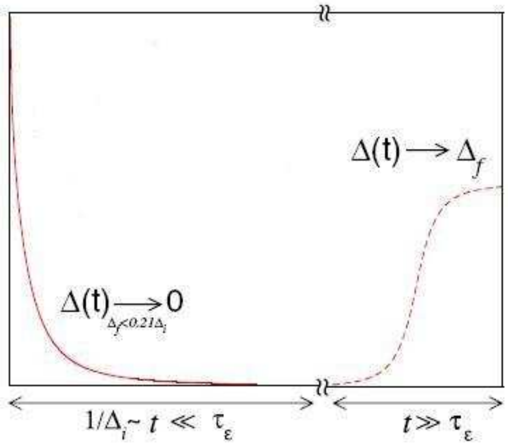
$$\frac{\Delta(t)}{\Delta_i} = A(t) e^{-2\alpha\Delta_i t} + B(t) e^{-2\Delta_i t} \tag{30}$$

where $\alpha = -\cos p$ and $\pi/2 \leq p \leq \pi$ is the solution of $p = \ln(\Delta_i/\Delta_f)\cot(p/2)$. The parameter α has a property $\alpha \to 0$ when $\Delta_f/\Delta_i \to e^{-\pi/2}$ and $\alpha \to 1$ when $\Delta_f/\Delta_i \to 0$. The time-dependent coefficient $A(t)$ and $B(t)$ are power-law decay, $A(t), B(t) \propto 1/t^\nu$ with $1/2 \leq \nu \leq 2$.

5 Conclusion: applicability of the theory

In BCS pairing dynamics, there are two characteristic time scales, *quasi-particle relaxation time* τ_ε and *order parameter dynamical time* $\tau_\Delta \sim 1/\Delta_i$.

Until now, our discussion is restricted at times much shorter than the quasi-particle relaxation time. The pairing interaction is changed abruptly on a time scale $\tau_0 \ll \tau_\Delta, \tau_\varepsilon$. Definitely, at long time $t \gg \tau_\varepsilon$, the system will reach the BCS ground state with the new coupling constants and the new order parameter Δ_f. For example, in the case of $\Delta_f < 0.21\Delta_i$ where dynamical vanishing of order parameter occurs, shown in the graph below[4]



the order parameter exponentially decays to zero at times $t \sim \tau_\Delta$ and the system goes into a gapless steady state. However, the order parameter will finally recover to its equilibrium value Δ_f at times longer than quasi-particle relaxation time. The theory we outlined in previous sections applies only on the time scale $t \sim \tau_\Delta \ll \tau_\varepsilon$.

Usually in superconducting metals, the variation of external parameter is slow compared to τ_Δ and quasi-particle spectrum evolves adiabatically. In contrast, while relaxation rates in cold fermionic gases are quite slow, the external parameters, such as the detuning from resonance, can change very quickly on the time scale of τ_Δ. This enables the BCS correlations to build up in a coherent fashion while the system is out of thermal equilibrium. In such a situation, theory must account not only for the evolution of the order parameter, but also for the full dynamics for individual Cooper pairs.

To estimate the BCS parameter values for cold fermionic gas, we consider magnetic fields not too close to the resonance where one can neglect the presence of the molecular field and use the weak coupling theory. At particle density $n \approx 1.8 \times 10^{13}\mathrm{cm}^{-3}$, which corresponds to Fermi energy $E_F \approx 0.35\mu K$, and the scattering length $a \approx -50\mathrm{nm}$ for attractive interaction, we have $T_c \approx 0.006E_F$. The coupling constant is related to Fermi energy by $g = \frac{2}{\pi}k_F|a|$ and the order parameter is related to the coupling constant by $\Delta = 0.49E_F e^{-1/g}$. An estimate of the order parameter dynamical time gives $\tau_\Delta \approx \hbar/\Delta_i \approx 2\mathrm{ms}$, while the quasi-particle relaxation time $\tau_\varepsilon \simeq \hbar E_F/\Delta_i^2 \approx 200\mathrm{ms} = 100\tau_\Delta$, consistent with weakly damped oscillations of $\Delta(t)$.

References

[1] Regal C A, Greiner M and Jin D S, 2004 *Phys. Rev. Lett.* **92**, 040403; Zwierlein M W, Stan C A, Schunck C H, Raupach S M F, Kerman A J, and Ketterle W, 2004 *Phys. Rev. Lett* **92**, 120403

[2] Barankov R A and Levitov L S, 2006 *Phys. Rev. Lett* **96** 230403

[3] Barankov R A, Levitov L S and Spivak B Z, 2004 *Phys. Rev. Lett* **93** 160401

[4] Yuzbashyan E A and Dzero M, 2006 *Phys. Rev. Lett* **96** 230404

[5] Yuzbashyan E A, Tsyplyatyev O and Altshuler B 2006 *Phys. Rev. Lett* **96** 097005

[6] Yuzbashyan E A, Tsyplyatyev O and Altshuler B 2006 *Phys. Rev. Lett* **96** 179905

[7] Tiesinga E, Verhaar B J and Stoof H T, 1993 *Phys. Rev. A* **47** 4114

[8] Pethick C J and Smith H *Bose-Einstein Condensation in Dilute Gases* Cambridge