High-Temperature Superfluidity

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Abstract

With the recent advancement of the technique of cooling atomic gases, it is now possible to make fermionic atom gases into superfluid state. Although the critical temperature of the superfluidity is only around 50nK, the superfluid state of fermionic atom gas is sometimes called "High-Temperature Superfluidity" in a sense that the critical temperature of the superfluid normalized by the Fermi temperature is high, even much higher than any known high-temperature superconductors. In this paper, I will discuss the basic theory of superfluidity in fermionic atom gases and present several experimental realizations of it. I will also mention the relation between ultracold fermionic gas and high temperature superconductors.

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1 Introduction

Since the realization of Bose-Einstein condensation (BEC) of atomic gases in 1995, the study of ultracold atomic gases, both experimentally and theoretically, has made a remarkable success. One of the important feature of this area is that it enabled physicists to experimentally realize a macroscopic quantum system with great degrees of freedom.

In this paper, among various interesting branches of research in ultracold atomic gases, I will focus on the superfluidity of ultracold fermi gases.

The superfluid transition temperature of ultracold fermi gas, i.e. Li_6 is about 50nK. However, the transition temperature normalized by the Fermi tenperature of the system reaches as high as $T_C/T_F \approx 0.2$. This is a remarkably high value considering the fact that this ratio is at most the order of 10^{-3} for conventional superconductors and superfluid ^{3}He. Even for high temperature superconductors, this value is of order 10^{-2} [1]. (See Table 1 taken from [2].) Thus, considering in terms of normalized temperature, the

Table 1: T_C/T_F in various Fermi superfluids [2]

	T_C/T_F
Conventional superconductors	$10^{-5} - 10^{-4}$
Superfluid ^{3}He	10^{-3}
High-temperature superconductors	10^{-2}
Fermi gases with resonant interactions	~ 0.2

ultracold fermi gases exhibit superfluid transition at much higher temperature than any other known fermion system. From these facts, the superfluidity of ultracold fermi gas is sometimes called "High-Temperature Superfluidity".

The organization of this paper is as following. I will first explain the basic theory of 'High-Temperature Superfluidity'. Then, I will present various experiments concerning the realization of this phenomenon. In the final chapter, I will briefly discuss the connection between 'High-Temperature Superfluidity' and 'High-Temperature Superconductivity'.

Review papers which I found particularly helpful on writing this paper are Giorgini *et al.* [2] and Bloch *et al.* [3].

2 Theory

2.1 Feshbach resonance — BCS-BEC crossover

Ideal fermi gas does not exhibit BEC nor superfluidity because of the Pauli principle. (Fermions cannot 'Bose' condensate by itself!) In conventional superconductors, electrons form Cooper pairs through electron-phonon interaction and condense into superconducting state. To obtain BEC or superfluidity, we need some interaction, or pairing mechanism, that makes two (or more) fermions into 'effective bosons'. In ultracold fermionic gases, this interaction is rather artificially introduced by the experimental setup. The technique is called "Feshbach resonance". A remarkable feature of this technique is that it makes it possible to vary the magnitude and even the sign of the effective interaction between fermions.

What we can directly change by the use of Feshbach resonance is the scattering length between particles. When two particles are in relative scattering state with scattering length a, the effective potential for low energy scattering between these two particles can be expressed by a pseudopotential [4]

$$V_{eff}(\mathbf{r}) = \frac{2\pi\hbar^2 a}{m_r}\delta^{(3)}(\mathbf{r}), \tag{1}$$

where m_r is the reduced mass. This expression is valid when the density of the system is low enough. Thus, being able to change the value of a means that we can vary the effective potential between two particles.

Feshbach resonance happens when the energy level of a scattering state (open channel) between two particles becomes close to the energy level of a bound state of a different channel (closed channel). For example, an open channel and a closed channel may be relative energy levels by different spin states. If the magnetic moments of the particle pairs in these two channels are different, by applying magnetic field, we can control the relative energy levels between two channels. If there is a coupling between these two channels in the process of collision, the scattering length becomes large if the energy levels of two channels are close. This phenomenon is called Feshbach resonance. Depending on the relative sign of energy levels of two channels, the sign of the scattering length can be positive or negative. As a function of the external magnetic field B, the scattering length can be written

$$a = a_{\mathrm{bg}}\left(1 - \frac{\Delta B}{B - B_0}\right), \tag{2}$$

where a_{bf} is the scattering length far from resonance, and ΔB is the width of the resonance. At $B = B_0$, resonance happens and the scattering length diverges.

By means of Feshbach resonance, we can effectively tune the interaction between two fermions from the negative infinity to the positive infinity.

Let's first consider the case $a > 0$, i.e. effective potential between two fermions is repulsive. In general, positive scattering length implies an existence of a bound state with the binding energy [5]

$$E_b = \frac{\hbar^2}{2m_r a^2}. \tag{3}$$

Thus, in the limit $a \to +0$[1], two fermions form one molecule-like bound state with the size of order a. In this limit, fermionic gas will become a gas of molecules with each molecule consisting of two fermions of the original gas. Since a molecule composed of two fermion behaves like a boson, the low-temperature behavior of this molecule gas would be just like a gas of bosons and it exhibits BEC in the low enough temperature. This limit $a \to +0$ is thus called 'BEC limit' and the region $a > 0$ is called 'BEC region' or 'BEC side'.

Next, consider the case $a < 0$, i.e. effective potential is attractive. The ground state of ideal fermi gas is unstable against even little attractive perturbation between particles and the system tends to form Cooper pairs. Especially, in the limit $a \to -0$[2], the system is described by the celebrated BCS wavefunction. In this limit, the low-temperature behavior of the system is pretty much alike the behavior of electrons in conventional superconductors. They form Cooper pairs and become a condensate yielding superfluidity, in the case of ultracold fermions, or superconductivity, in the case of superconductors. This limit $a \to -0$ is thus called 'BCS limit' and the region $a < 0$ is called 'BCS region' or 'BCS side'.

The two limits $a \to \pm 0$ are well-known to physicists through the study of BEC or BCS theory. However, the area where $|a|$ is big is not an easy area to explore. In this region, the interaction between particles is large so we cannot regard the interaction strength as a perturbative parameter. It is a convention to describe the system in terms of a dimensionless parameter $v = 1/k_F a$, where k_F is the Fermi wavevector of the system. In terms of this parameter, $v \to -\infty$ limit corresponds to 'BCS limit' and $v \to +\infty$ corresponds to 'BEC limit'. The theoretical and experimental challenge of the last decade was to explore the region where $|v|$ is not so big. The study of this area is called 'BCS-BEC crossover'. Expecially, the point $v = 0$ is called the 'unitarity point'. At this point, from the dimensional analysis, the only relevant dimensionless parameter is $\theta = T/T_F$, where T is the temperature of the system and T_F is the Fermi temperature, and the system exhibits universal properties. See Figure 1 for a schematic diagram of this BCS-BEC crossover. The phase diagram shown in Figure 1 will be discussed more in detail in the next subsection.

[1]This limit should be understood as $a \to a_{\text{bg}} + 0$. However, comparing the value of a near $B = B_0$, a_{bg} is significantly small, so I adopted a notation $a \to +0$.

[2]Again, this limit should be understood as $a \to a_{\text{bg}} - 0$.

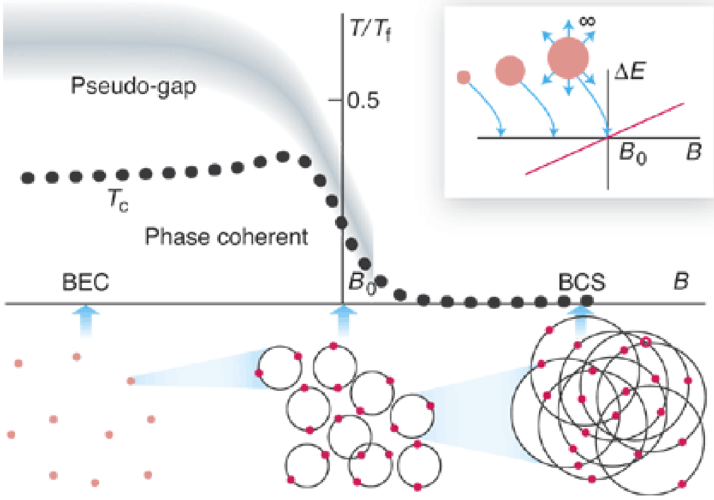

Figure 1: BCS-BEC crossover. Roughly speaking, $B \propto -v$.[6]

2.2 Phase diagram

Let's try to draw a phase diagram of ultracold fermi gas with tuned interaction.

In the weak coupling ($k_F|a| \ll 1$) BCS regime, the condensate transition temperature is known to be [7]

$$T_C = \left(\frac{2}{e}\right)^{7/3} \frac{e^\gamma}{\pi} T_F e^{-1/N(\epsilon_F)|U_0|} = \left(\frac{2}{e}\right)^{7/3} \frac{e^\gamma}{\pi} T_F e^{\pi/2k_F a} \approx 0.277 \cdot T_F e^{\pi/2k_F a}, \tag{4}$$

where $\gamma = 0.577$ is Euler's constant and $N(\epsilon)$ is the density of state at the Fermi energy and $|U_0|$ is the magnitude of the interaction. Therefore, in the limit $a \to -0$, the transition temperature goes to zero: $T_C \to 0$.

The BEC transition tempearture of ideal bose gas is [7]

$$T_C = \frac{2\pi\hbar^2}{k_B m_b}\left(\frac{n_b}{\zeta(3/2)}\right)^{2/3}, \tag{5}$$

where m_b and n_b are the mass and the density of bosons. BEC-limit of fermion gas should also obey this relation. For convenience, consider the transition temperature normalized by the Fermi temperature [8]

$$T_F = \frac{\hbar^2}{2k_B m_f}(3\pi^2 n_f)^{2/3}, \tag{6}$$

where m_b and n_b are the mass and the density of fermions. At the BEC-limit, two fermions form one boson. Therefore, $m_b = 2m_f$, $n_b = n_f/2$. Then, we can see

$$\frac{T_C}{T_F} = \left(\frac{2}{9\pi\zeta(3/2)^2}\right)^{1/3} \approx 0.218. \tag{7}$$

Compared to the exponential decay of the transition temperature in BCS side, on the BEC side, T_C is comparable to T_F. We may call it high temperature superfluid. However, considering the fact that even the BEC of alkali bose gas is actually a BEC of bosons composed of many fermions (electrons, neutrons, and protons), I am not sure if we can call it high temperature superfluidity. As we will see soon, the significance of superfluidity in fermi gas is not at this BEC limit, but at the BCS side.

To predict the transition temperature of the region where the magnitude of $v = 1/k_F a$ is not big is still an active research area. An interesting prediction was made by Stajic *et al.* [9]. According to their paper, above the transition temperature near unitarity point (v = 0) in the BCS side, there exists a large range of temperature where Cooper pairs are formed but they don't condensate. From the analogue with the high T_C superconductor, this regime is called 'pseudogap'. We can define a new temperature T^* as a temperature below which Cooper pairs start to form. In this pseudogap regime $T_C < T^*$, there exists pair breaking energy gap but we cannot see clear superfluid.

There are several attempts to determine T_C/T_F as a function of v. A recent numerical result by Huassmann *et al.* [10] showed the phase diagram in Figure 2[3]. As seen from this figure, close to the unitarity point on BCS side, there is a region where T_C/T_F is of order 10^{-1}. (At the unitarity, they obtained the result $T_C/T_F = 0.16$) It in this region that we may truely call the system 'High Temperature Superfluid'.

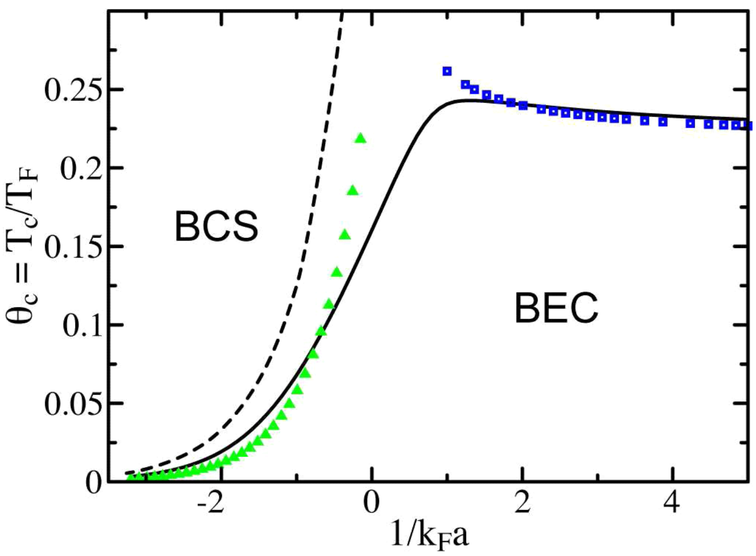

Figure 2: Critical temperature as a function of $v = 1/k_F a$. The solid line represents T_C/T_F as a function of v. The dashed line is a schematical behavior of T^*/T_F which indicates the onset of pseudogap regime. Green triangles in the BCS side represents the behavior of eqn (4). Blue squares in BEC side are asymptotic result at BEC limit.[3]

[3]This figure is actually from [3]. In [3], it is said that this figure is reprinted from [10]. However, they are slightly different and I thought the one in [3] was easier to understand.

3 Experiment

3.1 On BEC side

BEC side of condensate was first observed almost simultaneously by three groups [11], [12], and [13] in 2003. Figure 3 is the time-of-flight image obtained by Colorado group in [11] using ^{40}K.

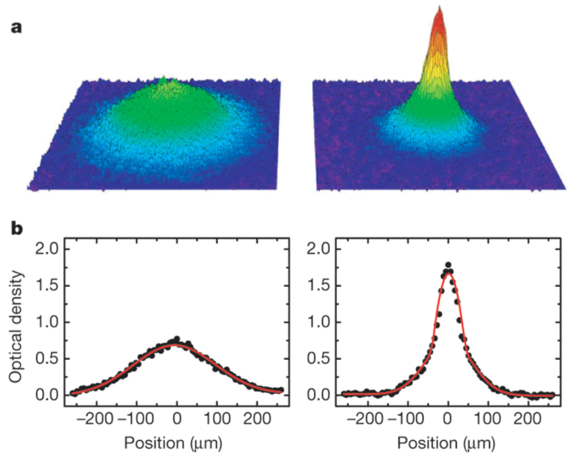

Figure 3: Time-of-flight images of the molecular cloud. The left image was taken above the BEC transition temperature and the right image was taken below the transition temperature.[11]

As pointed out before, BEC of molecules composed of two fermions is not a phenomenon to be called high temperature superfluidity. To obtain superfluidity at BCS side is much more difficult work, and it will be discussed in the next subsection.

3.2 On BCS side

The difficulty of detecting superfluidity at BCS side can be understood from the equation (4). Typically, people have been using time of flight method to measure the momentum distribution of the condensate, namely, they let the condensate expand and measure the position of particles after a certain time. However, since $k_F = (3\pi^2 n)^{1/3}$, as the cloud expands, the transition temperature goes down as $T_C \propto T_F \exp(-\pi/2|a|(3\pi^2 n)^{1/3})$. Therefore, even if originally the superfluid state of ultracold fermi gas is achieved, after the expansion, the condensate is not in the superfluid state anymore.

A technique to circumvent this difficulty was explored and the first observation of condensatin of pairs in BCS side was made by Regal *et al.* [14] in 2004 using ^{40}K. In this experiment, they first made a condensate in BCS side near unitarity and then rapidly sweeped the magnetic field to BEC side. On sweeping rapidly into BEC side, they transformed pairs into molecules, namely, a BCS-like pair with total momentum near zero is converted into a bound molecule with momentum near zero. Thus, BCS pair correlation should be observed through a peak around zero in the molecule momentum distribution. Indeed, that is what they observed (Figure 4).

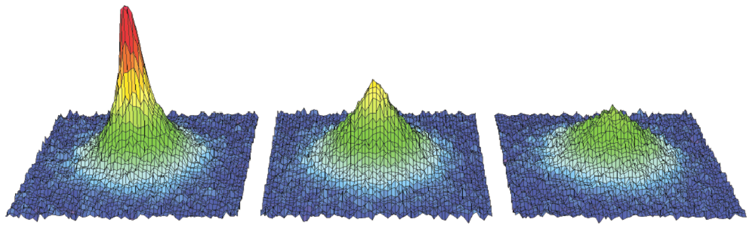

Figure 4: Fermi condensation. The images are taken after the projection of the fermi gas onto a molecular gas. The left image corresponds to the fermi gas closest to unitarity, and the right image corresponds to the farthest from the unitarity. Estimated condensate fractions are $N_0/N = 0.10$, 0.05, and 0.01 from left to right, respectively.[14]

Although this result strongly indicates the existence of Fermi condensate, there are some arguments about its validity [15]. Especially, there is a claim that since condensation does not necessarily imply superfluidity [16], the experiment cannot be regarded as the evidence of the existence of the superfluidity in ultracold fermi gas [17].

The pairing gap on the BCS side was first observed by [18] in 2004 using ^{6}Li. They studied rf-spectroscopy of ultracold fermi gas and obtained the Figure 5. In the figure,

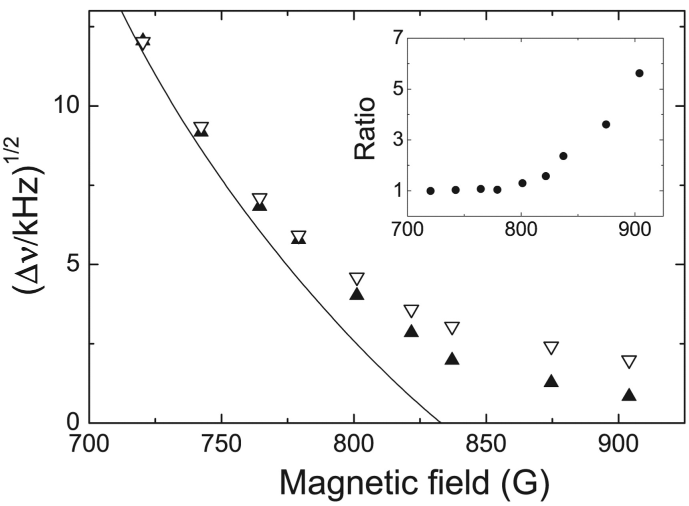

Figure 5: Paring gap in ^{6}Li. The solid line is the molecular binding energy at BEC side. It vanishes at the unitarity point. The closed triangles and the open triangles corresponds to the results of experiments with $T_F = 1.2\mu$K and $T_F = 3.6\mu$K. The inset figure is the ratio of open and closed triangles.[18]

the region where a solid line exists is BEC side. On the right side, there are data in BCS side, and you can observe the energy gap. This observation of energy gap indicates the existence of pairing in BCS side. Note, however, that because of the pseudogap regime, this existence of pairing does not directly mean the superfluidity.

3.3 Observation of vortices

A convincing evidence of the superfluidity in ultracold fermi gas was given by the observation of quantized vortices by Zwierlein *et al.* [17] in 2005 using ^{6}Li. Since the existence of quantized vortices is the direct consequence of macroscopic wavefunction for superfluid, this experiment established a new phase of a matter, i.e. high temperature superfluidity. Figure 6 is the image of vortices.

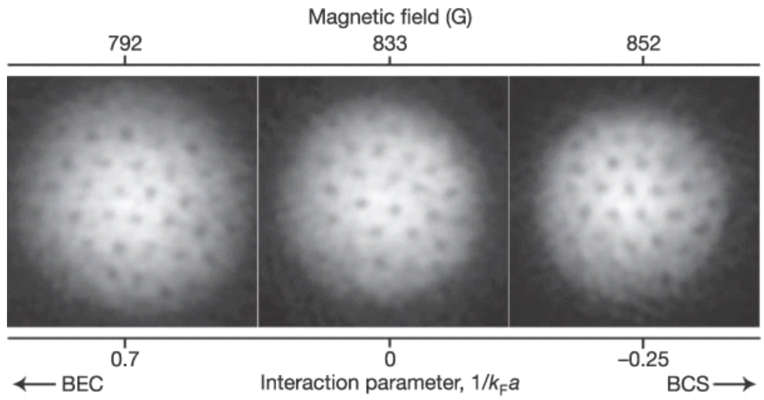

Figure 6: Vortices in both BEC and BCS sides.[17]

They cooled down the ^{6}Li gas around $T/T_F = 0.07$ and obtained the image [4]. Comparing the phase diagram in Figure 2, the observation of superfluidity at this temperature is consistent with theory.

[4]Though they did not give a precise temperature because of the experimental difficulty related with the extraction of actual temperature.

4 Analogy with high-T_C superconductors

From the existence of pseudogap regime in ultracold fermi gas near unitarity, we can expect that there is some analogy between superfluidity in ultracold fermi gas, or BCS-BEC crossover, and high-T_C superconductivity. Indeed there is, and it is one of the most active area of research related to ultracold fermi gas. Chen *et al.* [19] is a good review article on this subject.

An important proposal was given by Hofstetter *et al.* [20]. In that paper, they claimed that ultracold fermionic atoms in optical lattices may exhibit superfluid transition at considerably high temperature $T_C \approx 0.3 T_F k|a|$. Also, properly choosing experimental setup, they claimed that we may be able to realize d-wave pairing of fermions in optical lattices. It gives a direct analogy with electrons in high-T_C superconductors, and its experimental realization may give great insight into the study of high-T_C superconductivity.

Experimentally, Chin *et al.* [21] realized the superfluidity of ultracold fermions in an optical lattice in 2006. In this experiment, they confirmed the existence of long range phase coherence in ultracold fermions in an optical lattice by observing interference peaks when fermions are released from the optical lattice.

5 Conclusion and Future Prospects

As we have seen, the study on superfluid ultracold fermi gas is developing rapidly in the past decade, and high-temperature superfluid was finally observed without any doubt through the realization of vortices in BCS side of the ultracold fermi gas in 2005.

For the future research, I have no doubt that the theoretical and experimental study on BCS-BEC crossover regime, expecially around unitarity point, will continue to be a hot topic in this area. I personally think that the lack of effective theoretically tool in the BCS-BEC crossover is one of the most important theoretical challenges towards condensed matter physicists.

Another important area of future research is ultracold fermions in optical lattices. My impression is that both experimental and theoretical studies of fermions on optical lattices is still in the very beginning stage. As mentioned in the text, there is a direct analogy between superfluidity in optical lattices and high-T_C superconductivity, and it will attract both atomic, molecular, and optical physicists and condensed matter physicists.

In conclusion, the research on this topic is so active now and truely important results are coming out every year. For both theorists and experimentalists, a lot of problems are waiting to be solved. The most important message of this paper for the readers is:

'High-Temperature Superfluidity is actually HOT.'

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