Exotic Goldstone Particles: Pseudo-Goldstone Boson and Goldstone Fermion

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Abstract

This essay describes two exotic Goldstone particles. One is the pseudo-Goldstone boson which is related to spontaneous breaking of an approximate symmetry. The other is the Goldstone fermion which is a natural result of spontaneously broken global supersymmetry. Their realization and implication in high energy physics are examined.

1 Introduction

In modern physics, the idea of spontaneous symmetry breaking plays a crucial role in understanding various phenomena such as ferromagnetism, superconductivity, low-energy interactions of pions, and electroweak unification of the Standard Model. Nowadays, broken symmetry and order parameters emerged as unifying theoretical concepts are so universal that they have become the framework for constructing new theoretical models in nearly all branches of physics. For example, in particle physics there exist a number of new physics models based on supersymmetry. In order to explain the absence of superparticle in current high energy physics experiment, most of these models assume the supersymmetry is broken spontaneously by some underlying subtle mechanism. Application of spontaneous broken symmetry is also a common case in condensed matter physics [1]. Some recent research on high T_c superconductor [2] proposed an approximate SO(5) symmetry at least over part of the theory's parameter space and the detection of goldstone bosons resulting from spontaneous symmetry breaking would be a 'smoking gun' for the existence of this SO(5) symmetry.

From the Goldstone's Theorem [3], we know that there are two explicit common features among Goldstone's particles:
(1) they are massless;
(2) they obey Bose-Einstein statistics i.e. they are boson particle.
However, there could be exception to these rules if we loosen the precondition of the Goldstone's Theorem. Frist, if the symmetry to be broken is only an approximate symmetry rather than an exact one, the Goldstone particle can gain a small mass due to the existence of the explicit symmetry breaking term. Second, If a fermion-type symmetry such as supersymmetry is spontaneous broken, the Goldstone particle would be fermion called goldstino [4].

In the following sections, we will explain in detial the two exotic goldstone particles mentioned above. Section 2 is devoted to the pseudo-goldstone boson. This particle is generated from spontaneous breaking an approximate symmetry. We consider the application of this formalism to the low energy pion theory. In section 3, we introduce the concept of supersymmetry and its breaking mechanism. Then we will discuss the implication of local supersymmetry. Finally, the role played by supersymmetry in grand unified theory (GUT) is reviewed.

2 Pseudo-Goldstone Boson

The standard theory of the strong interactions is quantum chromodynamics (QCD). QCD is described by the lagrangian density

$$\mathcal{L}_{QCD} = -\frac{1}{4}G^{a}_{\mu\nu}G^{\mu\nu}_{a} - \sum_{n}\bar{q}_n(\gamma^{\mu}D_{\mu} + m_{q_n})q_n$$

where $G^{a}_{\mu\nu},\ a = 1, 2, ..., 8$ is the field strength tensor for the gluon fields. The quarks are represented by Dirac spinors q_n, where $n = 1, ..., 6$. In order of the increasing mass, these are: u, d, s, c, b and t. The quark masses, in GeV, are $m_u = 0.0015 - 0.005, m_d = 0.003 - 0.009, m_s = 0.06 - 0.17, m_c = 1.1 - 1.4, m_b = 4.1 - 4.4$ *and* $m_t = 173.8 \pm 5.2$. The strong interactions are believed to bind the quarks and gluons into bound states, which correspond to the observed strongly interacting particles (or hadrons). Figure 1 gives the masses and some of the quantum numbers for all of the hadrons who masses are less than $1GeV$.

Particle	Quark Content	Mass (GeV)	Spin	Isospin
$\pi^-(\pi^+)[\pi^0]$	$d\overline{u}(u\overline{d})[u\overline{u}, d\overline{d}]$	0.140 [0.135]	0	1
π^0	$u\overline{u}, d\overline{d}$	0.135	0	1
$K^+(K^0)$	$u\overline{s}(d\overline{s})$	0.494 (0.498)	0	$\frac{1}{2}$
$K^-(\overline{K}^0)$	$s\overline{u}(s\overline{d})$	0.494 (0.498)	0	$\frac{1}{2}$
η	$u\overline{u}, d\overline{d}, s\overline{s}$	0.547	0	0
$\rho^-(\rho^+)[\rho^0]$	$d\overline{u}(u\overline{d})[u\overline{u}, d\overline{d}]$	0.770	1	1
ω	$u\overline{u}, d\overline{d}, s\overline{s}$	0.782	1	0
$K^{*+}(K^{*0})$	$u\overline{s}(d\overline{s})$	0.892 (0.896)	1	$\frac{1}{2}$
$K^{*-}(\overline{K}^{*0})$	$s\overline{u}(s\overline{d})$	0.892 (0.896)	1	$\frac{1}{2}$
η'	$u\overline{u}, d\overline{d}, s\overline{s}$	0.958	0	0
f_0	$u\overline{u}, d\overline{d}, s\overline{s}$	0.980	0	0
a_0	$u\overline{u}, d\overline{d}, s\overline{s}$	0.980	0	1
$p(n)$	$uud(ddu)$	0.938 (0.940)	$\frac{1}{2}$	$\frac{1}{2}$

Figure 1: Masses and Quantum Numbers of the Lightest Hadrons [1]

The most significant feature about this particle spectrum is that the lightest two quarks u and d have masses which are much smalller than all of the masses of the states which make up the spectrum. So the QCD dynamics may be well approximated by taking $m_u, m_d \approx 0$. Under this approximation, the QCD lagrangian acquires the symmetry below

$$\begin{pmatrix} u \\ d \end{pmatrix} \to (U_L\gamma_L + U_R\gamma_R) \begin{pmatrix} u \\ d \end{pmatrix}$$

where U_L and U_R are arbitrary two-by-two unitary matrices having unit determinant. The symmetry group obtained in this way is $G = SU_L(2) \times SU_R(2)$, which is chiral symmetry treating left- and right- handed fermions differently.

If the chiral symmetry G were not spontaneously broken by the QCD ground state $|\Omega\rangle$, then all of the observed hadrons should fall into representation of G. However, this is not seen in the spectrum of observed hadrons. Instead the known particles organized themselves into roughly degenerate representations of theapproximate symmetry of isospin: $SU_I(2)$. It can be understood at the quark level to consist of the diagonal subgroup of G, for which $U_L = U_R$. This suggests that the ground state of QCD must spontaneously break the approximate symmetry group G down to the subgroup H=$SU_I(2)$, for which:

$$\begin{pmatrix} u \\ d \end{pmatrix} \to U \begin{pmatrix} u \\ d \end{pmatrix}$$

From the Goldstones' Theorem, we know that the low-energy spectrum of the theory must include the corresponding Goldstone bosons. If G were an exact symmetry, then the corresponding Goldstone bosons would by exactly masses. But G is only a real symmetry in the limit of vanishing quark masses, so the Goldstone bosons need only vanish with these quark masses. The lightest hadrons $\pi^{\pm}$ and π^0 have the precisely the quantum numbers of the Goldstone bosons for the symmetry-breaking pattern $SU_L(2) \times SU_R(2) \to SU_I(2)$. The small masses of pions are protected by the fact that u and d quarks are much lighter than the natural scale of strong interactions $\Lambda \approx 1\ GeV$. Particles which are light, but not massless, due to the fact that they are the Goldstone bosons of an approximate symmtry of a problem are called $pseudo - Goldstone\ bosons$[7].

Next, we will examine the mass of pions due to the existence of small explicit symmetry breaking terms (quark mass terms). In low-energy effective field theory of pions, we can expand quantities in power of the light-quark masses and this technique is called chiral perturbation theory. Detailed calculation [1, 5] gives the mass relation below

$$m_\pi^2 = (m_u + m_d)\frac{\mathcal{M}^3}{F_\pi^2}$$

where $\mathcal{M}$ is the mass scale of chiral symmetry breaking and F_π is the pion decay constant.

First, the squared mass of pions only depend linearly on the sum of m_u and m_d. As long as m_u and m_d both are small compared to the characteristic of QCD, so is the mass of pions. Second, the mass of pions does not rely on the isospin-breaking difference $m_u - m_d$ and degenerate for all three pions. The observed mass difference between the charged and neutral pions are mainly due to the isospin-breaking electromagnetic interaction.

Actually, we can combine the next-lightest particles K and η with the pions to form new set of Goldstone bosons for the pattern $SU_L(3) \times SU_R(3) \to SU_V(3)$, which would be the limit case when the masses of the lightest three quark u, d and s vanish. From general effective field theory, we know that although the Goldstone field can only appear as a nonlinear representation of the broken part of the symmetry group G/H, they can serve as a linear representation for the unbroken symmetry group H. Indeed, the lightest eight mesons π, K and η have the exact quantum numbers as an octet representation of the unbroken $SU_V(3)$ symmetry, which is the famous 'eightfold way' proposed by Gell-Mann, see Figure 2.

Since the light meson π, K and η could be regarded as Goldstone boson of the broken chiral symmetry, their low-energy interactions are strongly restricted by the symmetry breaking pattern. So the idea of treating pions as Goldstone bosons is very predictive on low-energy pion-nucleon scattering experiment [6] and indeed it is well confirmed by the experiment results among which the most well-known is the Goldberger-Treiman relation. As clarified by Nambu and Weinberg [5], the Goldberger-Treiman relation is a natural result of spontaneous chiral symmetry breaking and the appearance of the massless or nearly massless pion is a symptom of a broken exact or approximate symmetry.

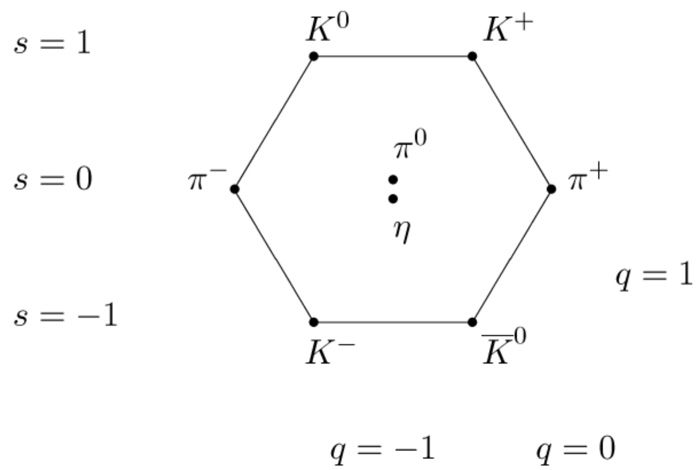

Figure 2: The meson octet: eightfold way

3 Goldstone Fermion–goldstino

The Standard Model of high energy physics provides a remarkable successful description of presently known phenomena. However, the Standard Model is still now a complete story about elementary structure of nature since it doesn't include any quantum description of gravity. The fact that the ratio M_P/M_W (Planck scale $M_P \sim 10^{18} GeV$ and the electroweak scale $M_W \sim 100 GeV$) is so huge is already a powerful clue to the character of physics beyond the Standard Model, because of the "hierarchy problem" or "naturalness problem" . The Higgs potential is disturbingly sensitive to the energy scale of the underlying physics due to the existence of quadratically divergent corrections to the Higgs boson squared mass. One way out of this problem is to introducing supersymmetry to the Standard Model. Supersymmetry can remove the quadratical divergence by requiring that each has a partner particle with different statistical property. However, supersymmetry also requires every particle and its partner have same mass, which is not seen in the observed particle spectrum. So in many versions of supersymmetric standard models, people speculate that supersymmetry is spontaneously broken.

A supersymmetry transformation turns a bosonic state into a fermionic state, and vice versa, so its generator Q must be an anticommuting spinor with the property

$$Q|Boson\rangle = |Fermion\rangle, \quad Q|Fermion\rangle = |Boson\rangle.$$

Since supersymmetry is a fermion-type symmetry unlike those symmetry in the Standard Model, the Goldstone particle with the same quantum numbers as the broken supersymmetry generator must obey Fermi-Dirac statistics, i.e. it must be a fermion rather than a boson, so called goldstino.

Next we will give a rough proof of the existence of the goldstino. From the basic supersymmetry algebra [4], the Hamiltonian operator H is related to the supersymmetry generators through the equation

$$H = \frac{1}{4}(Q_1 Q_1^\dagger + Q_1^\dagger Q_1 + Q_2 Q_2^\dagger + Q_2^\dagger Q_2)$$

where the supersymmetry generator $Q_\alpha,\ \alpha = 1, 2$ is a Weyl spinor under the Lorentz transformation. If supersymmetry is sponteneously broken in the vacuum state, then the vacuum must have positive energy, since

$$\langle H\rangle = \frac{1}{4}(\|Q_1^\dagger|0\rangle\|^2 + \|Q_1|0\rangle\|^2 + \|Q_2^\dagger|0\rangle\|^2 + \|Q_2|0\rangle\|^2) > 0$$

Neglecting spacetime-dependent effects and fermion condensates, we have $\langle H\rangle = \langle V\rangle$, where V is the scalar potential. Under the formalism of superfield V can be written as below

$$V(\phi, \phi^*) = F^{*i} F_i + \frac{1}{2} D^a D^a$$

where F_i is the auxiliary field component of the chiral superfield and D^a is the auxiliary field component of the gauge field of the gauge superfield. To be specific, auxiliary fields means they don't have kinematic term of their own in the Lagrangian, but using field equation, they can be replaced by the products of scalar field components of the superfields, which is

$$F_i = -W^{i*}(\psi), \quad D^a = -g(\phi * T^a \phi)$$

where $W^i(\phi)$ is the functional derivative of the superpotential $W(\phi)$ which an analytic function of scalar fields, i.e.$W^i(\phi) = \frac{\delta W(\phi)}{\delta \phi_i}$, (also write $W^{ij} \equiv \frac{\delta^2 W(\phi)}{\delta \phi_i \delta \phi_j}$), and T^a is the matrix of corresponding gauge transformation. In order to have $\langle V\rangle > 0$ required by the broken supersymmetry , it must be that $\langle F_i\rangle \neq 0$ or $\langle D^a\rangle \neq 0$.

Under the Weyl spinor basis $(\lambda^a,\ \psi_i)$ (where λ^a is gaugino and ψ_i is chiral fermion), after some of the scalar fields in the theory obtain VEVs, the fermion mass matrix has the form [4]:

$$m_F = \begin{pmatrix} 0 & \sqrt{2}g_b(\langle\phi^*\rangle T^b)^i \\ \sqrt{2}g_a(\langle\phi^*\rangle T^a)^j & \langle W^{ji}\rangle \end{pmatrix}$$

Using the gauge invariance of the superpotential and minimum condition of the scalar potential $\langle\partial V/\partial\phi_i\rangle = 0$, we can find the mass matrix m_F annihilates the vector

$$\tilde{G} = \begin{pmatrix} \langle D^a\rangle/\sqrt{2} \\ \langle F_i\rangle \end{pmatrix}$$

So $\tilde{G}$ is a massless Weyl fermion with the same quantum number of the broken supersymmetry generator, which implies it is proportional to the goldstino wavefunction. From the derivation above, we find that if global supersymmetry is spontaneously broken, then there must be a massless goldstino, and its somponents among the various fermions in theory are proportional to the corresponding auxiliary field VEVs.

What will happen if we promote the global supersymmetry to a local (gauge) one? Is there any Higgs mechanism related to the spontaneous gauge symmetry breaking? The answer is yes [5]. When dealing with the local supersymmetry transformation, we will inevitably encounter gravity interaction, since supersymmetry generator is related with spacetime transformation in the supersymmetry algebra. Like other gauge transformation, we need to introduce a "metric" superfield to make derivative terms covariant under the local supersymmetry transformation. The metric field naturally appear as the vector component of the "metric" superfield (which has a Lorentz index) and the correponding Weyl spinor component is so called "gravitino". Through detailed derivation, we can find the metric field is a massless tensor with the properties required by general relativity and gravitino is a massless spin$-\frac{3}{2}$ field, with two spin helicity states, which could be thought of as the "gauge" field of the local supersymmetry transformation. So the Localization of supersymmetry simply involves the gravity interacton$-$such formalism is called $supergravity$, which is currently a promising candidate of quantum gravity theory. Once sypersymmetry is spontaneously broken, the gravitino acquires a mass by absorbing the goldstino. The massive spin$-\frac{3}{2}$ gravitino has four helicity states, of which two were originally assigned to the would-be goldstino. This is called the $super-Higgs$ mechanism.

Finally, we want to mention another motivation for supersymmetry from the grand unified theory (GUT). Since supersymmetry introduces new couplings as well

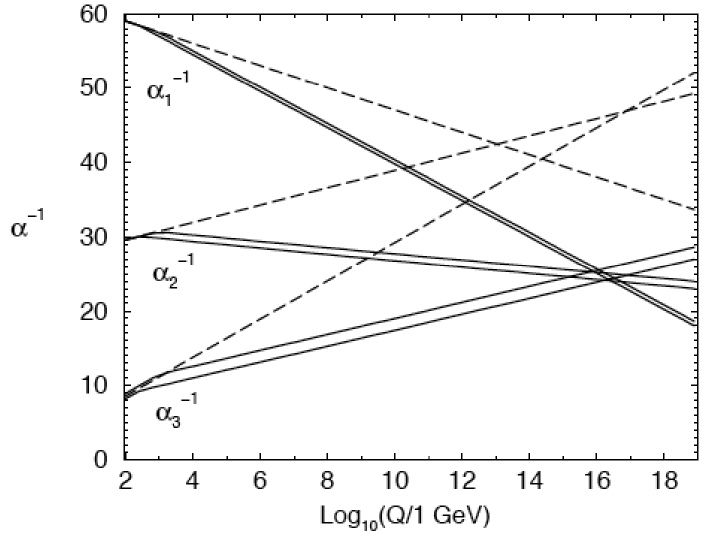

Figure 3: RG evolution of the inverse gauge couplings $\alpha_a^{-1}(Q)$in the Standard Model (dashed lines) and the MSSM(solid lines) [4].

as new particles into the Standard Model, it can change the behaviors of gauge couplings as the energy scale varies. Figure 3 compares the RG evolution of the α_a^{-1}, including two -loop effects, in the Standard Model and the MSSM (minimal supersymmetric standard model). Unlike the Standard Model, the MSSM includes just the right particle content to ensure that the gauge couplings can unify, at a scale $M_U \sim 2 \times 10^{16} GeV$ which is above the lower bound of the GUT scale suggested by the proton decay experiment.

Supersymmetry not only solve some theoretical problems of Standard Model, but also enrich Standard Model with many new phenomenological features. It is well expected that the existence of supersymmetry in TeV energy scale may be confirmed by the coming LHC experiment.

4 Conclusion

Instead of giving a summary of the essay, we will give a comment on the mass of the pseudo-Goldstone boson. In the case of spontaneous breaking an exact symmetry, there is no difference between the vacuum state and the Goldstone mode under the long wavelength limit, so in the low energy effective Lagrangian of the Goldstone mode every term contains a spacetime derivative, which is the origin of the gaplessness and low energy decoupling properties of the Goldsone mode. However, the situation changes when we add a small explicit symmetry breaking term to the Lagrangian. The symmetry breaking term will pick out a special direction of the vacuum in which the perturbative potential is minimized. Otherwise, the vacuum state will not be stable under the perturbation. The process of choosing the direction of vacuum state is called *vacuum alignment*. Due to the existence of the explicit symmetry breaking term, the excitation in the Goldstone mode will cost energy and thus the Goldstone field will gain a small mass which will vanish as the the explicit symmetry breaking term disappears.

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