Glashow-Weinberg-Salam Model: An Example of Electroweak Symmetry Breaking

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In this essay, The generalization of the gauge theory for electromagnetic interaction of superconductivity to that for the weak interaction is discussed in detail , which is named as Glashow-Weinberg-Salam Model. It turns out that the concept of symmetry spontaneous breaking plays an important role for the theory of eletroweak interaction. Consequencly, the broken symmetry $SU(2) \times U(1)_Y \rightarrow U(1)_{EM}$ gives the prediction of the massive particle W and Z bosons, which have already been found at the correct mass in 1983.

I. INTRODUCTION

During the late 1950s, studies of the helicity dependence of weak interaction cross section and decay rates has shown that the weak interaction involves the coupling between vector current built of quark and lepton fields. It was thus natural to assume that the weak interaction is due to exchange of very heavy vector bosons. In contrast with electromagnetic interaction, this is due to exchange the photons. Such vector bosons, now named as W and Z particles were discovered at CERN in 1983. However the most difficulty for building the complete theory of weak interaction comes from the massive boson fields themselves. Any mass term appearing in the Lagrangian will spoil the gauge-invariance property because gauge symmetry prohibit the generation of a mass for the vector field. The nonzero W and Z masses turns out to require incorporating into the theory of symmetry spontaneous breaking. The resulting theory is known as GWS model. It was first formulated by Weinberg in 1967 and by Salam in 1968 independently.

The essay is formulated as follows. First in order to built the theory of weak interaction, some properties of weak interaction are discussed. We will argue that why the symmetry spontaneous breaking is essential to approach the theory of weak interaction.

Then we will use the $U(1)$ symmetry as a simple example to illustrate some basic ideas about symmetry spontaneous breaking. In this theory, one massless boson-Goldston boson is generated.

Thirdly, I will discuss how to generate a mass for a gauge boson—Higgs mechanism. The basic idea is to promote $U(1)$ from global to a local symmetry. It turns out that after symmetry spontaneous breaking, two scalar fields and one massless photon with two helicities become one massive scalar field and one massive photon with three helicities.

Next, 'Georgi-Glashow model' will be introduced. Historically, this might be the first attempt to built a theory of the weak interaction. It is the theory with broken $SU(2)$ local symmetry. I will briefly discuss this theory and explain why this is the wrong theory for weak interaction.

Finally, I will discuss the remarkable theory of weak interaction: Glashow-Weinberg-Salam model in detail. This is the theory with broken symmetry $SU(2) \times U(1)_Y \rightarrow U(1)_{EM}$, which predicts the massive vector bosons W and Z. This theory also unifies the weak and electromagnetic interaction between elementary particles. The correction of GWS model was experimentally established at CERN in 1983 by the discovery of the W and Z gauge bosons in proton-antiproton collisions at the converted Super Proton Synchrotron. Abdus Salam, Sheldon Glashow and Steven Weinberg were awarded the Nobel Prize in Physics in 1979 "for their contributions to the theory of the unified weak and electromagnetic interaction between elementary particles, including, inter alia, the prediction of the weak neutral current".

II. WEAK INTERACTIONS

As we all known, there have four kinds of interactions in our nature: strong interaction, weak interaction, electromagnetic interaction and gravity. Although strong, electromagnetic and weak interactions are all gauge theories, they are remarkably different.

Electromagnetic interaction is the one we are much familiar with. The gauge theory of electromagnetic field is the key point to deal with the property of superconductivity. The electromagnetic field is long ranged field and the classical equation of motion for electromagnetic field is well-known Maxwell equations, hence it predicts the existence of massless photon.

The theory of strong interaction is called QCD. Strong interaction is short ranged, the typical interaction range is approximately near $\Lambda_{QCD}^{-1} \approx 10^{-13} cm$. Now, we know this short ranged property of strong interaction is due to confinement of massless gluons, which is known as asymptotic free. We don't discuss the theory of strong interaction here.

What about the properties of weak interaction? Experimental evidences also show that weak interaction is short ranged: typical range is around $M_W^{-1} \approx 10^{-15} cm$. Hence we can ask ourselves a question: Is the mechanism of weak interaction the same as the strong interaction? The answer is NO!! Although, both strong interaction and short interaction are short ranged, the corresponding mechanism is quite different. The short ranged property of strong interaction is due to confinement of some gauge field (gluon in this case), but it turns out that short ranged weak interaction is caused by another mechanism: symmetry spontaneous breaking. Consequeclely, it predicts two massive particles :W and Z bosons.

As we have seen in class that the gauge symmetry explains why the photon (same for gluon) are massless. Then the weak bosons (W and Z) should also be massless if we require our theory for weak interaction has the gauge symmetry. Then the question is what endows them with mass? This is also the basic question of the origin of the mass. The answer is spontaneous symmetry breaking.

In order to understand this, it is instructive to first study spontaneous breaking of a global symmetry. This is the most simple case ,which can give us some basic ideas about symmetry spontaneous breaking. Then we will move on to spontaneous breaking of a local (gauge) symmetry.

III. BROKEN GLOBAL U(1) SYMMETRY

There has a well-known theorem related to the global symmetry breaking: Goldstone theorem.

Goldstone theorem: Every broken generator of a global symmetry group has a corresponding massless Goldstone boson (spin zero).

I don't want to prove this theorem here, I will use a simple physical example to illustrate the Goldstone theorem. Let's consider a thin rod with circular cross section, and apply a force F on the end points of the rod. If the force F is small, nothing happens, if F exceeds a critical value $F_{critical}$, however, the rod bends in a plane which it chooses at random as shown in Fig 1. The symmetric (unbent) configuration becomes unstable when $F > F_{critical}$, and the new ground state is unsymmetric. Also, there are infinitely many possible new degenerate ground states, which are related by a rotational symmetry. The rod can only, of course, choose one of them, but the others are all reached by a rotation without causing any energy. This example tells us

1) A parameter (in this case the force F) has a critical value.
2) Beyond this critical value, the symmetric configuration becomes unstable.
3) the new ground state is degenerate.

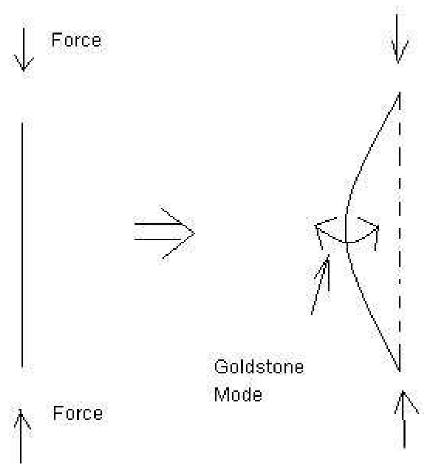


Figure 1: Compressed rod

All these properties is just the restatement of Goldstone theorem. In this case, the rotational symmetry about z axis (U(1)) spontaneously broken.

Now we can consider one simple filed theory example: broken U(1) symmetry. The lagrangian for this example is

$$L = \partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi - \lambda(\phi^* \phi)^2 \tag{1}$$

where ϕ is the complex scalar field and m is the mass term for this field. It's obvious that this lagrangian is invariant under global U(1) symmetry: $\phi \to e^{iQ\theta}\phi$. Q here is the conserved charge for field ϕ.

The conjugate momentums for the field ϕ and ϕ^* are

$$\Pi = \frac{\partial L}{\partial \dot{\phi}} = \dot{\phi}^*; \quad \Pi^* = \frac{\partial L}{\partial \dot{\phi}^*} = \dot{\phi} \tag{2}$$

By using the conjugate momentums, we can write down the corresponding Hamiltonian.

$$\begin{aligned} H &= \Pi\dot{\phi} + \Pi^*\dot{\phi}^* - L \\ &= \dot{\phi}^*\dot{\phi} + \vec{\nabla}\phi^* \cdot \vec{\nabla}\phi + m^2\phi^*\phi + \lambda(\phi^*\phi)^2 \geq 0 \end{aligned} \tag{3}$$

Now consider the case $\mu^2 = -m^2 < 0$. Then we can rewrite the Hamiltonian as

$$H = \dot{\phi}^*\dot{\phi} + \vec{\nabla}\phi^* \cdot \vec{\nabla}\phi - \mu^2\phi^*\phi + \lambda(\phi^*\phi)^2 \tag{4}$$

We can read off the potential for this Hamiltonian: $V(\phi) = -\mu^2\phi^*\phi + \lambda(\phi^*\phi)^2$. The shape of this potential looks like wine bottle or Mexican hat and is drawn in Fig. 2.

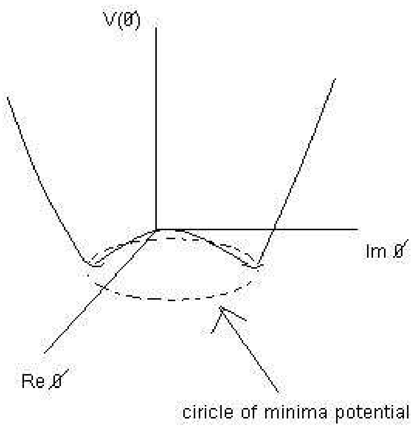


Figure 2: shape of potential

The minimum point of H is located at $|\phi(\vec{x},t)| = \frac{\mu}{\sqrt{2\lambda}}$ for all $\vec{x}$ and t. Notice that there is a continuous circle of minima, physically, this means that the ground state of our system has infinite degeneracy.

Of course, we have right to arbitrarily choose one of them as our ground state, say $Re\phi = \frac{\mu}{\sqrt{2\lambda}}$ and $Im\phi = 0$. Hence we have the 'vacuum-expectation value' of ϕ

$$<\phi>_0 = \frac{\mu}{\sqrt{2\lambda}} \equiv \frac{v}{\sqrt{2}} \tag{5}$$

In order to get the mass of the particle, it's natural to define what is the definition of particle in quantum field theory. The oscillations around the ground state or vacuum correspond to real particles. Hence we need take variation around the ground state given before. We write

$$\phi = \frac{1}{\sqrt{2}}(h+v)e^{i\pi/v} \tag{6}$$

where $h(\vec{x},t)$ and $\pi(\vec{x},t)$ are the fluctuation fields with $<h>_0 = 0$ and $<\pi>_0 = 0$.

Thus the lagrangian becomes

$$\begin{aligned} L &= \partial_\mu \phi^* \partial^\mu \phi - \mu^2 \phi^* \phi + \lambda(\phi^*\phi)^2 \\ &= \frac{1}{2}(\partial_\mu h \partial^\mu h + \partial_\mu \pi \partial^\mu \pi) + \frac{1}{v^2}(h^2 + 2hv)\partial_\mu \pi \partial^\mu \pi - \lambda v^2 h^2 - \frac{1}{4}\lambda(h^4 + 4h^3 v) + \frac{1}{4}\lambda v^4 \end{aligned} \tag{7}$$

We can find that the mass for the h field is $m_h = \sqrt{2\lambda}v$ and mass for the π field is $m_\pi = 0$. Hence we get a massless particle, which corresponds to the Goldstone boson as described by Goldstone theorem.

Now we have explicitly shown that there exist the Goldstone boson when breaking the global U(1) symmetry. Next we can ask what is the broken generator in this U(1) case? A broken generator is the one which does not annihilate the vacuum.

In order to see which generator is broken, we can write down the general U(1) transformation:

$$\phi \to e^{iQ\theta}\phi \tag{8}$$

where Q is the generator in this U(1) theorem.

We will see that the ground state is NOT invariant under U(1) transformation, although the lagrangian is invariant. Under an infinitesimal transformation, the ground state transforms like

$$|0> \to e^{iQ\theta}|0> \approx (1 + iQ\theta)|0> \neq |0> . \tag{9}$$

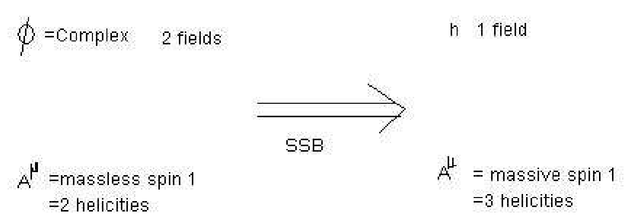
Therefore $Q|0> \neq 0$, this means that Q is the 'broken generator' in this global U(1) symmetry according to the definition.

Summary: In this simplest global U(1) case, theory has only one broken generator Q, and this corresponds to one Goldstone boson: massless π field. There is no remaining unbroken symmetry , so there only exists one Goldstone boson. The symmetry breaking patten in this case is '$U(1) \to nothing$'.

IV. HIGGS MECHANISM

We already know that Goldstone boson is massless, but almost all the particles in the nature are massive except for photons. Hence one may ask that what's the goodness for the Goldstone boson? The answer is it will enable us to generate a mass for a gauge boson! This is the Higgs mechanism.

Let's demonstrate this on our simple U(1) model, where we now promote U(1) from a global to a local symmetry. We will see that



So the number of the fields (4) is preserved. The Goldstone boson π is 'eaten' by A^μ to give its mass! This is the origin of the mass W and Z in the GWS model discussed later: the Goldstone bosons are 'eaten' by the gauge field and gain the mass.

Now let's discuss the gauge theory for U(1) case. The lagrangian for local U(1) symmetry is

$$L = (D_\mu \phi)^*(D^\mu \phi) + \mu^2 \phi^* \phi - \lambda(\phi^*\phi)^2 \tag{10}$$

where $D_\mu = \partial_\mu + ieQA_\mu$ as usual definition of covariant derivative.
Under local U(1) transformation,

$$\begin{aligned} \phi &= e^{iQ\theta(x)}\phi \\ D_\mu\phi &= e^{iQ\theta(x)}D_\mu\phi \end{aligned} \tag{11}$$

The minimum point of the potential is at $<\phi>_0 = \frac{v}{\sqrt{2}}$, so we can again write

$$\phi = \frac{1}{\sqrt{2}}(h+v)e^{iQ\pi/v}$$

where $\pi = \pi(x)$ is the Goldstone field.
Since the lagrangian L is gauge invariant, so we are free to make the gauge transformation:

$$\phi \to e^{iQ\theta(x)}\phi, \quad A_\mu \to A_\mu - \frac{1}{e}\partial_\mu\theta \tag{12}$$

Let $\theta(x) = -\frac{\pi(x)}{v}$, then we have

$$\phi = \frac{1}{\sqrt{2}}(h+v)e^{iQ\pi/v} \to \frac{1}{\sqrt{2}}(h+v) \tag{13}$$

We have eliminated Goldstone field π by taking advantage of the gauge invariance. This is what I mean the Goldstone boson is 'eaten' by the gauge field.
What is its consequence? We can look at the kinetic term in lagrangian:

$$\begin{aligned} (D_\mu\phi)^*(D^\mu\phi) &= \frac{1}{2}(\partial_\mu - ieQA_\mu)(h+v)(\partial^\mu + ieQA^\mu)(h+v) \\ &= \frac{1}{2}\partial_\mu h\partial^\mu h + \frac{1}{2}e^2Q^2v^2A_\mu A^\mu + \frac{1}{2}e^2Q^2(h^2+2hv)A_\mu A^\mu \end{aligned} \tag{14}$$

Therefore the gauge field (photon) has acquired a mass $M_A = eQv$. And π field has come back as the helicity zero component of the massive photon:

$$A_\mu \to A_\mu + \frac{1}{ev}\partial_\mu\pi \quad since \quad \theta = -\frac{\pi}{v} \tag{15}$$

This mechanism, by which spontaneous symmetry breaking generates a mass for a gauge boson, was explored and generalized to the non-Abelian case by Higgs, Kibble, Guralnik, Hagen, Brout and Englert, and is now known as the Higgs mechanism. However, this mechanism had an earlier application to the theory of superconductivity. Since the gauge field acquires a nonzero mass, external electromagnetic fields penetrate a superconductor only to the depth M_A^{-1}. This is just the Meissner effect: the exclusion of macroscopic magnetic fields from a superconductor.
The role of Goldstone boson in the Higgs mechanism is intricate and seems mysterious. First, in order for the gauge bosom to acquire a mass, the Goldstone boson is necessary. However, we also see that the Goldstone boson can be formally eliminated from the theory. However, the Goldstone boson is not completely disappeared. As we saw before, the massless vector boson (photon) has only two physical polarization states, but a massive vector boson has three. It is tempting to say that the gauge boson acquires its extra degree of freedom by eating the Goldstone boson.

V. GEORGI-GLASHOW MODEL

Experimentally, we know the weak interaction is short ranged, it implies that the theory of weak interaction must require some massive intermedial particles. Therefore the Higgs mechanism mentioned above might be a correct way to approach the weak interaction problem. The minimal model of the symmetry spontaneous breaking for the weak interaction is $SU(2) \to U(1)$. This is known as the Georgi-Glashow model of the weak interaction. Let me discuss this model briefly as an example of non-Abelian gauge field.

The lagrangian for the Georgi-Glashow model is

$$L = \frac{1}{2}\partial_\mu \phi^\dagger \partial^\mu \phi + \mu^2 \phi^\dagger \phi - \lambda(\phi^\dagger \phi)^2 \tag{16}$$

where $\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}$ and $\phi_{1,2,3} \in Real$.

It is clear that this lagrangian has SO(3) (=SU(2)) symmetry:

$$\phi \rightarrow e^{iT^a\theta^a}\phi \tag{17}$$

where

$$iT_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}; \quad iT_2 = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}; \quad iT_3 = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

is the defining representation of SO(3) (= 'spin 1' representation of SU(2)).

Similar to the case of U(1), the potential has the minimal at $<\phi^\dagger\phi>= \frac{\mu^2}{2\lambda} \equiv v^2$. We can also have right to arbitrarily choose $<\phi>= v$ in the 3 direction:

$$<\phi>= \begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix} \tag{18}$$

Therefore it is obvious that T_1 and T_2 are the broken generators for GG model: $T_1 <\phi> \neq 0$ and $T_2 <\phi> \neq 0$. But T_3 is unbroken: $T_3 <\phi>= 0$. We find that one U(1) symmetry is left unbroken , therefore the symmetry spontaneous breaking for GG model is $SU(2) \rightarrow U(1)$.

According to the Goldstone theorem, we know that there have two massless Goldstone bosons. This can be simply shown if we write the field

$$\phi = e^{[iT_1\frac{\pi_1}{v} + iT_2\frac{\pi_2}{v}]} \begin{pmatrix} 0 \\ 0 \\ h+v \end{pmatrix} \tag{19}$$

where π_1 and π_2 are the massless Goldstone bosons.

As we did in the U(1) case, we can promote the global SU(2) to a local symmetry, then the lagrangian for this case reads

$$L = \frac{1}{2}(D_\mu\phi)^T(D^\mu\phi) - V(\phi^T\phi) \tag{20}$$

where $D_\mu\phi = (\partial_\mu + igT^aA^a_\mu)\phi$, a=1,2,3.

We can also use the gauge transformation to eliminate the Goldstone fields:

$$\phi \rightarrow e^{iT^a\theta^a}\phi = \begin{pmatrix} 0 \\ 0 \\ h+v \end{pmatrix} \tag{21}$$

where $T^a\theta^a = -T_1\frac{\pi_1}{v} - T_2\frac{\pi_2}{v}$, i.e. $\theta^a = (-\frac{\pi_1}{v}, -\frac{\pi_2}{v}, 0)$.

Therefore the kinetic term for the GG model lagrangian becomes

$$\begin{aligned} L &= \frac{1}{2}(D_\mu\phi)^T(D^\mu\phi) \\ &= \frac{1}{2}\partial_\mu h\partial^\mu h + \frac{1}{2}g^2v^2(A^1_\mu A^{1\mu} + A^2_\mu A^{2\mu}) + \frac{1}{2}(h^2+2hv)(A^1_\mu A^{1\mu} + A^2_\mu A^{2\mu}). \end{aligned} \tag{22}$$

Now the gauge field A^1 and A^2 eat the two Goldstone bosons π_1 and π_2, and become massive. The mass for the gauge field A^1_μ and A^2_μ are

$$M_{A^1} = M_{A^2} = gv. \tag{23}$$

The equality of M_{A^1} and M_{A^2} is due to the unbroken U(1) (global) symmetry. The another field A^3_μ remains massless. It is the photon of unbroken (local) U(1) symmetry.

We obtain the massive particle from Georgi-Glashow model, however it turns out that this model is incorrect to describe the weak interaction. In this GG model, we can only have massive W-bosons: $W^\pm_\mu = \frac{1}{\sqrt{2}}(A^1_\mu \mp iA^2_\mu)$ and massless photon: $A_\mu = A^3_\mu$. There has no Z-bosons appearing in GG model. Of course nature could have chosen this model, but it doesn't.

Summary: GG model gives the prediction of W bosons, but nature doesn't choose GG model as the correct model to describe weak interaction because of absence of the Z-bosons.

VI. GLASHOW-WEINBERG-SALAM MODEL

Since the failure for building the theory of weak interaction from symmetry breaking of SU(2), it's natural to think about to start from a higher symmetry. Glashow worked out the case for $SU(2)\times U(1)$ gauge theory in 1960, but he didn't get the mass for W and Z bosons. In 1967,68, Weinberg and salam applied the Higgs mechanism to the $SU(2)\times U(1)$ gauge theory. They claimed the unification of weak and electromagnetic interactions.

The general idea is that weak interaction should be mediated by gauge bosons ($W^\pm$), which are to begin with massless. The lagrangian for the theory also contains terms for massless electrons, muons and neutrinos, and is invariant under an internal symmetry group, which is a gauge symmetry. A scalar field (the Higgs field) is then introduced with a non-vanishing vacuum expectation value. The resulting spontaneous symmetry breaking gives masses to e, μ and τ and to the gauge bosons, but not to the photon and neutrino.

The SU(2)group corresponds to weak interaction with coupling constant g , gauge field A^a_μ and $U(1)_Y$ group corresponds to the hypercharge with coupling constant g' , gauge field B_μ.

The lagrangian for GWS model is

$$L = (D_\mu\phi)^\dagger D^\mu\phi + \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2 \tag{24}$$

where $D_\mu\phi = (\partial_\mu + igT^aA^a_\mu + ig'YB_\mu)\phi$ and $\phi = \begin{pmatrix}\phi^\dagger \\ \phi^0\end{pmatrix}$ with $Y=\frac{1}{2}$. The charges of ϕ field are the electric charges, as we will see. This is the Higgs doublet field, comparing with the GG model with Higgs triplet.

As usual we did in U(1) and GG model case, we can find the minimal point of the potential and hence get

$$<\phi^\dagger\phi> = \frac{v^2}{2}. \tag{25}$$

Because the ground state is infinite degenerate,we can arbitrarily choose

$$<\phi> = \begin{pmatrix} 0 \\ v/\sqrt{2}\end{pmatrix}. \tag{26}$$

Next step is to look for the broken generators. The SU(2) group generators for spin 1/2 particle can be expressed as pauli matrices: $T^a = \frac{1}{2}\sigma^a$. Then we have

$$\begin{aligned}
\frac{1}{2}\sigma^1 < \phi > &= \begin{pmatrix} 0 & 1/2 \\ 1/2 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} v/2\sqrt{2} \\ 0 \end{pmatrix} \neq 0 \\
\frac{1}{2}\sigma^2 < \phi > &= \begin{pmatrix} 0 & -i/2 \\ i/2 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} -iv/2\sqrt{2} \\ 0 \end{pmatrix} \neq 0 \\
\frac{1}{2}\sigma^3 < \phi > &= \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ -v/2\sqrt{2} \end{pmatrix} \neq 0 \\
\frac{1}{2}I < \phi > &= \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ v/2\sqrt{2} \end{pmatrix} \neq 0
\end{aligned} \tag{27}$$

Hence all the generators are broken in GWS model. However, one linear combination of these generators remains unbroken:

$$T^3 + Y = (\frac{1}{2}\sigma^2 + \frac{1}{2}I) \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = 0. \tag{28}$$

It is easy to show that this unbroken U(1) is electric charge.

$$Q = T_3 + Y = \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix} + \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{29}$$

That means $\phi = \begin{pmatrix} \phi^\dagger \\ \phi^0 \end{pmatrix}$ has the electric charges as indicated before.
Therefore the symmetry spontaneous breaking for GWS model is $SU(2) \times U(1)_Y \to U(1)_{EM}$.
Now there are three broken generators:

$$\begin{aligned}
&1)T^1 = \frac{1}{2}\sigma^1; \\
&2)T^2 = \frac{1}{2}\sigma^2; \\
&3)T^3 - Y = \frac{1}{2}\sigma^3 - \frac{1}{2}I.
\end{aligned} \tag{30}$$

And hence there have three Goldstone bosons in GWS model.
Next step is to calculate the mass induced by the symmetry spontaneous breaking. We can play the usual game to promote the global symmetry to local symmetry and rewrite the field as

$$\phi = \frac{1}{\sqrt{2}}(h+v)e^{[iT^1\pi^1 + iT^2\pi^2 + i(T^3 - Y)\pi^3]} \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{31}$$

Then we can take advantage of the gauge invariance and gauge away the three Goldstone bosons π^1, π^2, π^3:

$$\phi \to \frac{1}{\sqrt{2}}(h+v) \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{32}$$

Substituting it into the original lagrangian, we obtain

$$\begin{aligned}
L &= (D_\mu\phi)^\dagger D^\mu \phi \\
&= \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{1}{8}\left[(-gA^3_\mu + g'B_\mu)^2 + g^2(A^1_\mu A^{1\mu} + A^2_\mu A^{2\mu})\right](h+v)^2
\end{aligned} \tag{33}$$

So the fields A^1_μ and A^2_μ have acquired a mass

$$M_{A_1}^2 = M_{A_2}^2 = \frac{1}{4}g^2v^2 \equiv M_W^2 \tag{34}$$

This is the mass for the W bosons, which is generated by the gauge fields A_μ^1 and A_μ^2.

We can also find that the field $-gA_\mu^3 + g'B_\mu$ has also acquired a mass. To find the mass of this field, we need first normalize the field:

$$Z_\mu = \frac{1}{\sqrt{g^2+g'^2}}(gA_\mu^3 - g'B_\mu) \tag{35}$$

where the normalization factor comes from $< Z_\mu | Z_\nu >= \delta_{\mu\nu}$.

Then the lagrangian can be written in terms of Z_μ field:

$$L = \frac{1}{8}\left[(g^2+g'^2)Z_\mu Z^\mu + g^2(A_\mu^1 A^{1\mu} + A_\mu^2 A^{2\mu})\right](h+v)^2.$$

This implies that the field Z_μ acquire a mass

$$M_Z^2 = \frac{1}{4}(g^2+g'^2)v^2 \geq M_W^2 \tag{36}$$

However, for the orthogonal combination field $A_\mu = \frac{1}{\sqrt{g^2+g'^2}}(g'A_\mu^3 + gB_\mu)$, it has no mass term—it corresponds the photon in GWS model!

Therefore we illustrate the details about how the massive particles emerge though the symmetry spontaneous breaking in the GWS model. The gauge filed A_μ^1, A_μ^2 and Z_μ have eaten the Goldstone bosons π_1,π_2 and π_3 to acquire the mass of W and Z bosons. And the another one Goldstones boson still remains massless interpreted as photon.

We can also see that the photon and Z particle are both linear combination of A_μ^3 and B_μ fields. Thus we can write this in matrix form:

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_w & -\sin\theta_w \\ \sin\theta_w & \cos\theta_w \end{pmatrix} \begin{pmatrix} A_\mu^3 \\ B_\mu \end{pmatrix} \tag{37}$$

where $\cos\theta_w = \frac{g}{\sqrt{g^2+g'^2}}$ and $\sin\theta_w = \frac{g'}{\sqrt{g^2+g'^2}}$. θ_w is called weak mixing angle.

Therefore in the tree level, we can relate the mass of W boson and mass of Z boson by the weak mixing angle:

$$M_W = M_Z\cos\theta_w \tag{38}$$

Eqn.(34) and (36) relate the W and Z boson masses to some basic parameters: $g, g', -\mu^2, \lambda$ of the theory. Remarkably, these relations allow the masses of W boson and Z boson to be determined in term of three experimentally well known quantities;

1) Fine structure constant: $\alpha = e^2/4\pi = 1/137.04$.

2) Fermi coupling constant: $G = 1.66 \times 10^{-5} GeV^{-2}$.

3) weak mixing angle: θ_w.

The parameter v in Eqn. (34) and (36) can be expressed in terms of G as $v = (G\sqrt{2})^{-1/2}$. And parameter g and g' can be expressed in terms of electric charge and weak mixing angle as $g\sin\theta_w = g'\cos\theta_w = e$. Therefore the mass of W and Z bosons can be interpret as above three basis physical quantities:

$$M_W = \left(\frac{\alpha\pi}{G\sqrt{2}}\right)^{1/2}\frac{1}{\sin\theta_w}, \quad M_Z = \left(\frac{\alpha\pi}{G\sqrt{2}}\right)^{1/2}\frac{2}{\sin 2\theta_w} \tag{39}$$

Historically, these equations were used to predict the masses of the $W^\pm$ and Z^0 bosons using the value of θ_w obtained from neutrino scattering experiment: $\nu_\mu + e^- \to \nu_\mu + e^-$. The value of θ_w obtained in this way is $\sin^2\theta_w = 0.235 \pm 0.005$. Substituting this into Eqn.(39), together with the values of α and G, we obtain the masses for W and Z bosons:

$$M_W = 76.9 \pm 0.8GeV, \quad M_Z = 87.9 \pm 0.6GeV. \tag{40}$$

Actually, we have neglected the radiative corrections when we derived the masses of W and Z bosons. The calculation of such radiative corrections requires a discussion of renormalization which goes beyond the scope of this essay, and I shall only quote the result for the renormalized (i.e. physical) masses:

$$M_W = 79.8 \pm 0.8GeV, \quad M_Z = 90.8 \pm 0.6GeV \tag{41}$$

These values are in good agreement with the experimental masses:

$$M_W = 80.22 \pm 0.26GeV, \quad M_Z = 91.17 \pm 0.02GeV \tag{42}$$

VII. EXPERIMENTAL MEASUREMENT

The W bosons were first observed in 1983 in high energy experiments on the CERN $p\bar{p}$ collider. In this machine, protons and antiprotons collide with a total center of mass energy of $540GeV$. Such a collision can lead to a quark (q) and an antiquark ($\bar{q}$) combining to form a W boson which may decay via the weak interaction, for example into electro plus electron-antineutrino. The Feynmann diagram for this process is shown in Figure 3(a). The original experiments obtained a total of 87 such events, and gave the value of the W boson mass: $M_W = 80.22 \pm 0.26GeV$.

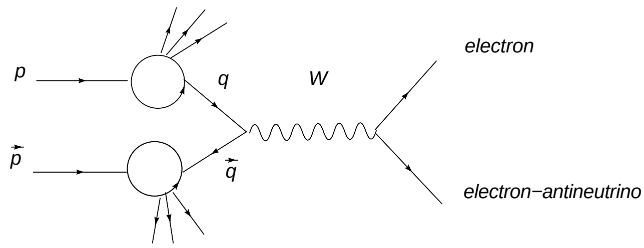


Fig. 3(a) W boson

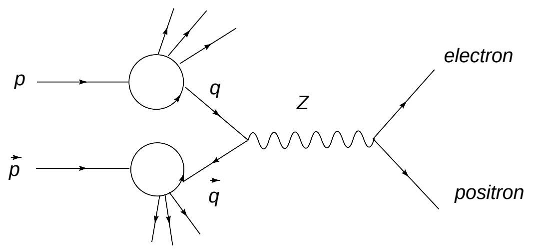


Fig. 3(b) Z boson

The Z boson was first detected in the same $p\bar{p}$ collider experiments in which the W boson was observed. The Feynmann diagram for Z boson production is shown in Fig. 3(b). In this process, a quark (q) and an anti-quark ($\bar{q}$), produced in a $p\bar{p}$ collision, combine to form a Z boson which then decays into a charged lepton pair: e^+e^-. The experimental result gives the mass of Z boson: $M_Z = 91.17 \pm 0.02GeV$.

Both the mass of W boson and Z boson are coincident with the theoretical calculation given above.

VIII. SUMMARY

We discuss the detail of the theory of weak interaction, which is known as Glashow-Weinberg-Salam Model. We start with a massless theory, which is the requirement of gauge invariance. And we carefully illustrate how the massless gauge fields can gain the mass at last–Higgs mechanism. The gauge fields 'eat' the massless Goldstone bosons and become massive. In the GWS model, we find there have three generators break the global symmetries and remaining one doesn't. Hence there have 3 Goldstone bosons in GWS theory and as a result, two gauge fields become massive ($W^{\pm}$ and Z^0) and one remains massless. A short summary of the Glashow-Weinberg-Salam Model is listed below:

$$\begin{aligned}
W_\mu^{\pm} &= \frac{1}{\sqrt{2}}(A_\mu^1 \mp iA_\mu^2) \qquad M_W = \frac{1}{2}gv \\
Z_\mu &= \frac{1}{\sqrt{g^2+g'^2}}(gA_\mu^3 - g'B_\mu) \qquad M_Z = \frac{1}{2}\sqrt{g^2+g'^2}v \\
A_\mu &= \frac{1}{\sqrt{g^2+g'^2}}(g'A_\mu^3 + gB_\mu) \qquad M_A = 0.
\end{aligned}$$

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