Stability of Optical Vortices in Turbulent Media

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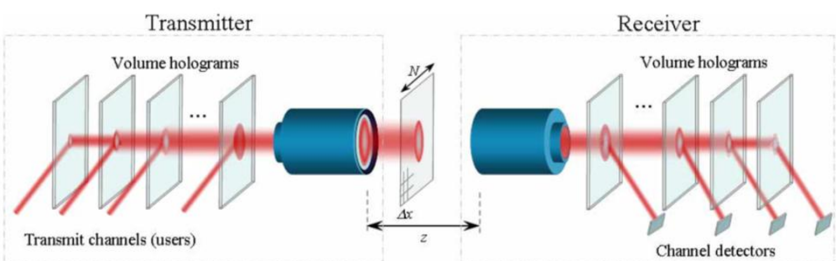
Optical communications rely on light as a carrier of information. One possibility of transmitting information with light is to use topological defects in the phase, an optical vortex, with the associated topological charge as a multi-bit. Due to the defect nature of optical vortices, they cannot be unwound through continuous deformations – potentially a robust way of sending information. However, one must verify the stability of these vortices for the medium that will transmit the information. In this paper I will discuss the stability of optical vortices against atmospheric turbulence by reviewing relevant simulations.

Introduction

With the growth of quantum information methods, such as quantum cryptography, the preservation of the information through methods of transportation of the states is becoming critical. As photons are used as the primary carriers in most implementations, ways to keep these photons alive and the information intact are highly sought after. One potentially robust method could be to use the topical defect nature of the angular momentum states of light. As with topological defects in Bose-Einstein condensates or superfluids, these topological defects cannot be unwound through continuous deformations. Unlike these two examples, light must constantly be propagating, and the transmission through medium couples the photon to the environment, and the stability of these optical vortices could be destroyed. For instance, during transmission through free-space links the photon is subject to turbulence effects. These effects could in principle unwind the defects and cause a loss of information. In this paper, I will begin by giving specific examples of the need to understand the stability of optical vortices. Then I will give an introduction into the mathematical framework, by briefly showing how these vortices arise naturally out of the wave equation using the paraxial approximation (this approximation is valid for situations such as lasers, which is what I am interested in). I will mention how to produce and control these optical vortices, which comes directly from the topological defect nature. Then finally, I will discuss simulations of atmospheric turbulence, and discuss the implications that arise from various changes in the topological defects.

Motivation

One application of optical vortices is in increasing the amount of data, or having a more stable form a data, carried by each photon for quantum cryptography. Quantum cryptography is a way of sending secure one-time pads to two separate parties without leaking of any information. The idea would be if two parties have completely random, but correlated, sets of information, then any message sent from one to the other, encoded with these random bits of information, will be completely random to any eavesdropper. As entangled particles do just that – have correlated but completely random information – they are a prime candidate for this application. Modern systems are limited in practicality due to their secure bit rate speed. Optical vortices potentially help this in two separate ways, by either allowing for the combination of multiple channels or by adding additional bits to each photon. The secure bit rate depends heavily on the efficiency of the system. For most applications of quantum cryptography, any error in the system corresponds to an eavesdropper receiving four times that amount of error (in general, two conjugate bases are used, and only half of the time can an eavesdropper be detected, therefore one error corresponds to an eavesdropper potentially listening in on four bits of information) [1]. Thus if optical vortices are more stable, then say the polarization state, they can be used to lower the amount of errors, consequently reducing the post-data processing that will be required to get a final secure key with minimally lost information. The second option for increasing the final secure bit rate is to use hyper-entanglement [2], entanglement in every degree of freedom – in particular, the polarization, angular momentum (the charge associated with the topological defect), and the timing. By doing so, the unsecured bit rate can be dramatically increased by using the angular momentum states in different ways. One way, is to have an actual multi-bit encoded in the topological charge. This is possible since the charge can, in principle, take on any value – negative or positive – and so one could encode an arbitrarily large amount of bits in one photon. However, since a charge of $m > 1$ is unstable, and will split into m charges of magnitude 1 [3], this will have some upper limit to the amount of charge that can be used. The alternative is to use the optical vortex as a means of combining multiple independent beams. This can be done by using a technique known as mode sorting – holograms are created that have a deflection angle dependent on the angular momentum state of the photon [4]. Therefore multiple beams can be combined into a single beam and then resorted at the final detection system. Again, this method will depend highly on the stability of these vortices (see Fig. 1a). Of particular interest, as we move forward with quantum cryptography, will be turbulence for secure satellite communications. In this case, the knowledge of the stability of optical vortices will determine the design. If the optical vortices are the most stable carriers of information, it would be best to use it as a bit, whereas if your other information carriers have enough associated bits, then it would be best to use optical vortices as a means for combining multiple channels (see Fig. 1b). In the remainder of this paper, I will focus on the optical vortices in the atmosphere.



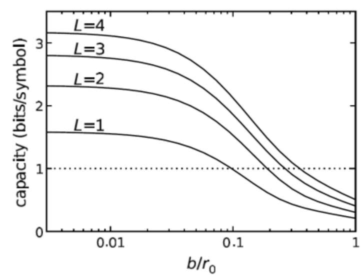


Figure 1: a) A diagram showing the multiplexing of multiple systems using angular momentum states and holograms to combine or pick off the specific angular momentum component. Depending on the stability of the other source of bits, either the angular momentum can be used to combine multiple channels or as additional bits. Figure from Ref. [5].
b) Plot of the channel capacity for the different angular momentum states. The dashed line at 1 is for a polarization based system. b/r_0 is the beam width relative to the coherence scale of the aberrations. Narrower beams result in less scattering of the angular momentum state, allowing for higher channel capacity. Figure from Ref. [6].

Mathematical Framework

A brief mathematical framework will be given here, described more in detail in Ref. [3], to show how phase singularities arise naturally out of the wave equation with the paraxial wave approximation, used for beams that have a small divergence. This approximation is accurate for most applications of laser beams. It should be noted that some nonlinear effects will invalidate this approximation, for instance self focusing of a laser in nonlinear crystals. However, in cases such as those considered in this paper, this approximation is valid. We start from the standard wave-equation for the electric field,

$$\nabla E^2 = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}$$

This can be simplified by looking at the scalar electric field in spherical coordinates, and considering the propagation in the $\hat{z}$-direction. Then with the paraxial approximation, namely that the angle between the direction of propagation of the beam (the k-vector) and the direction of propagation (the z-direction) is small, or that $\hat{k} \cdot \hat{z} \ll 1$. We then obtain the paraxial Helmholtz equation,

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 E}{\partial^2\varphi} - 2ik\frac{\partial E}{\partial z} = 0$$

which has the time dependence implicitly in the z coordinate. One general solution to this equation is of the form [3],

$$E(\rho,\varphi,z) \sim \left(\frac{\rho}{w(z)}\right)^{|m|} \exp\left(-\frac{\rho^2}{w(z)^2}\right) \mathrm{L}_{\mathrm{p}}^{|\mathrm{m}|}\left(\frac{\rho^2}{\mathrm{R(z)}}\right) \exp[i\,\Phi(\rho,\varphi,z)]$$

where $\mathrm{L}_{\mathrm{p}}^{|\mathrm{m}|}\left(\frac{\rho^2}{\mathrm{R(z)}}\right)$ are the Laguerre Polynomials with $p \geq 0$ describing the radial modes and m describing the angular modes; w(z) is the waist size of the beam (basically the width of the beam); and R(z) describes the radius of curvature of the wavefront. Of particular interest from this solution is $\Phi(\rho,\varphi,z)$, which is given by

$$\Phi(\rho,\varphi,z) = -(2p + |m| + 1)\arctan\left(\frac{2z}{k\rho^2}\right) + \frac{k\rho}{2R(z)} + m\varphi + kz$$

Therefore, when $m \neq 0$, the phase will be undefined at the origin. This can only make sense then if the amplitude also equals zero. This is plotted in Fig. 2a, and Fig. 2b. Here, we can see the phase singularity, which can only make sense if the amplitude at the topological defect is zero. Thus, we can define an optical charge inside of a region as [3],

$$t = \frac{1}{2\pi}\oint \nabla\Phi \cdot \mathrm{d}\mathbf{l}$$

This topological charge can be discriminated efficiently by methods discussed by Berkhot *et al.* [7].

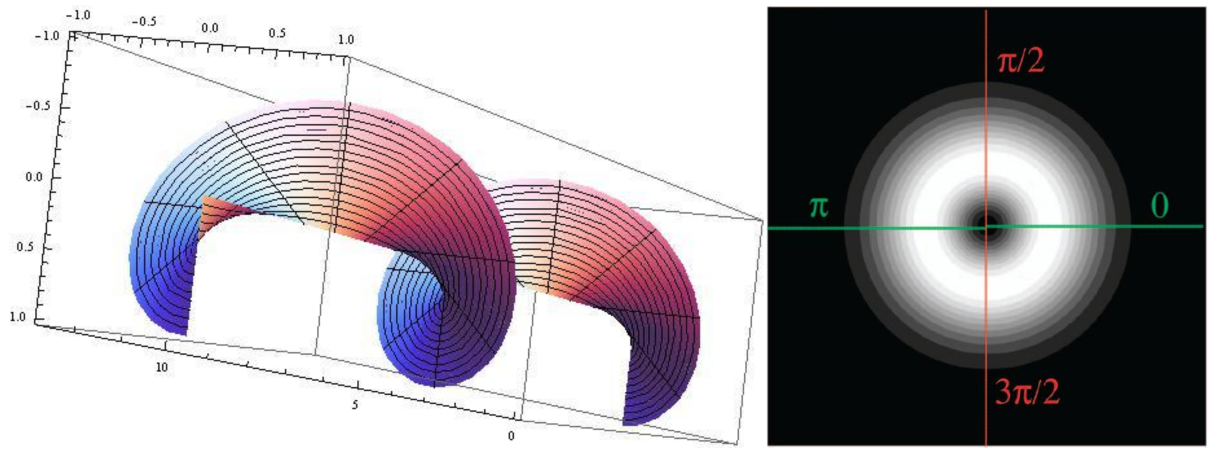


Figure 2: a) An image of the wavefront of a photon with a topological charge of 1. A larger charge would cause the rotation to be faster, whereas a negative charge would cause the rotation to be in the reverse direction.
b) An image of the intensity pattern of a photon with a charge one optical vortex. The dark hole in the center is the topological defect in the phase, as can be visualized using Fig. 1a. The lines in the figure represent lines of constant phase, which results in a singularity in the phase at the origin. These defects can be detected using holograms and reference beams, or more recent methods as discussed in Ref. [7]. Figure from Ref. [8].

As a final remark, it is interesting to note that the design of the hologram mode detection described earlier has the design shown in figure 3. Diffraction off of this hologram changes the angular momentum state of the photon. By coupling into a single-mode fiber (an optical fiber that only allows a Gaussian beam to enter), the exact state of the original beam can be determined [4].

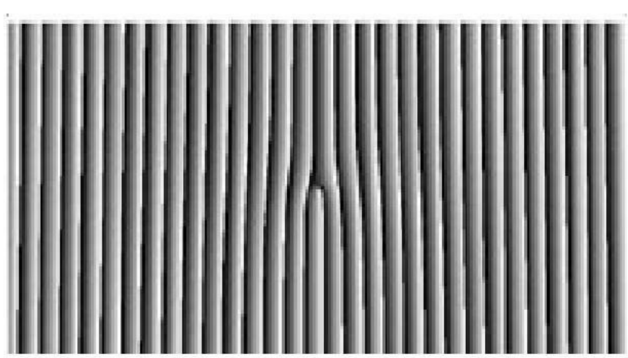


Figure 3: Image of a computer generated hologram that shows. Diffraction off of this hologram changes the angular momentum state of a photon. By doing so, the angular momentum state can be detected. Figure from Ref. [4].

Optical Turbulence

Since optical fibers, in general, do not allow for more than just a few modes (or angular momentum states), free space links are required for transmission using topological charge as the information. With the improvement of mode sorting [7] and generation of angular momentum states using hyper-entanglement [2], the effect of free-space links on angular momentum states has begun to be looked at in much more detail. Refractive index fluctuations, such as those from turbulence in the atmosphere, result in random phase aberrations on propagating optical beams [6]. As any free-space link for optical communications will undergo these phase aberrations, their consideration for the quantum channel for such processes as quantum cryptography requires knowledge of the deviation of the measured angular momentum state for the expected value. In the following, I will review two relevant articles, the first which gives a simulation for the expected outcome of the propagation over a large distance, and the second which gives a theoretical model will be used to check the first paper. In particular, I will discuss the papers written by Paterson [6] and Gbur and Tyson [8]. Gbur *et al* make simulations based on the evolution of the optical vortex through the atmosphere using Kolmogorov turbulence, described by $\phi_n(k) = 0.033C_n^2k^{-11/3}$, where ϕ_n is the refractive index power spectrum and C_n is the strength of the turbulence [8]. The effect of this turbulence on a beam can be seen in Fig. 4. Gbur *et al* analyze two cases of importance. The first is where the detection system can only pick off a part of the beam. This is realistic because as a beam propagates, the beam waist continues to get larger from diffraction. Thus, after traversing a distance of a few kilometers, the beam size will have drastically increased (~50cm), so most detection systems won't be able to pick up the whole beam [8]. If the turbulent media scatters the vortex outside of the detection region (the paper assumes a detection radius of 4cm, although this could be potentially larger, e.g., having a large lens focusing the beam onto the detector), then it will not be detected. The second case considered is a detection system that can scale with the size of the beam; this could be realized by having the lens system as mentioned before.

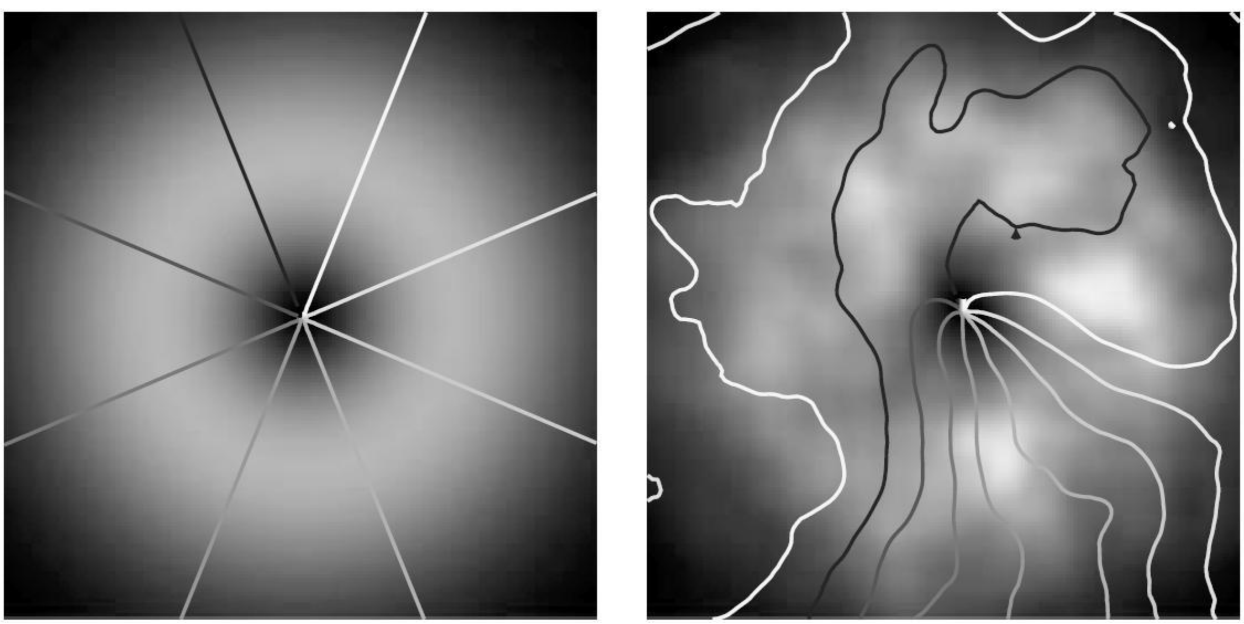


Figure 4: Images showing lines of constant phase on a beam with topological charge of 1. The image on the left shows the initial beam, without propagating through any atmospheric turbulence. The right image shows the effect of the phase aberrations on the beam. Figures from Ref. [6].

Both cases are simulated for weak ($C_n \approx 10^{-14} m^{-2/3}$) to strong ($C_n \approx 10^{-17} m^{-2/3}$) turbulence. The results are summarized in Fig. 5.

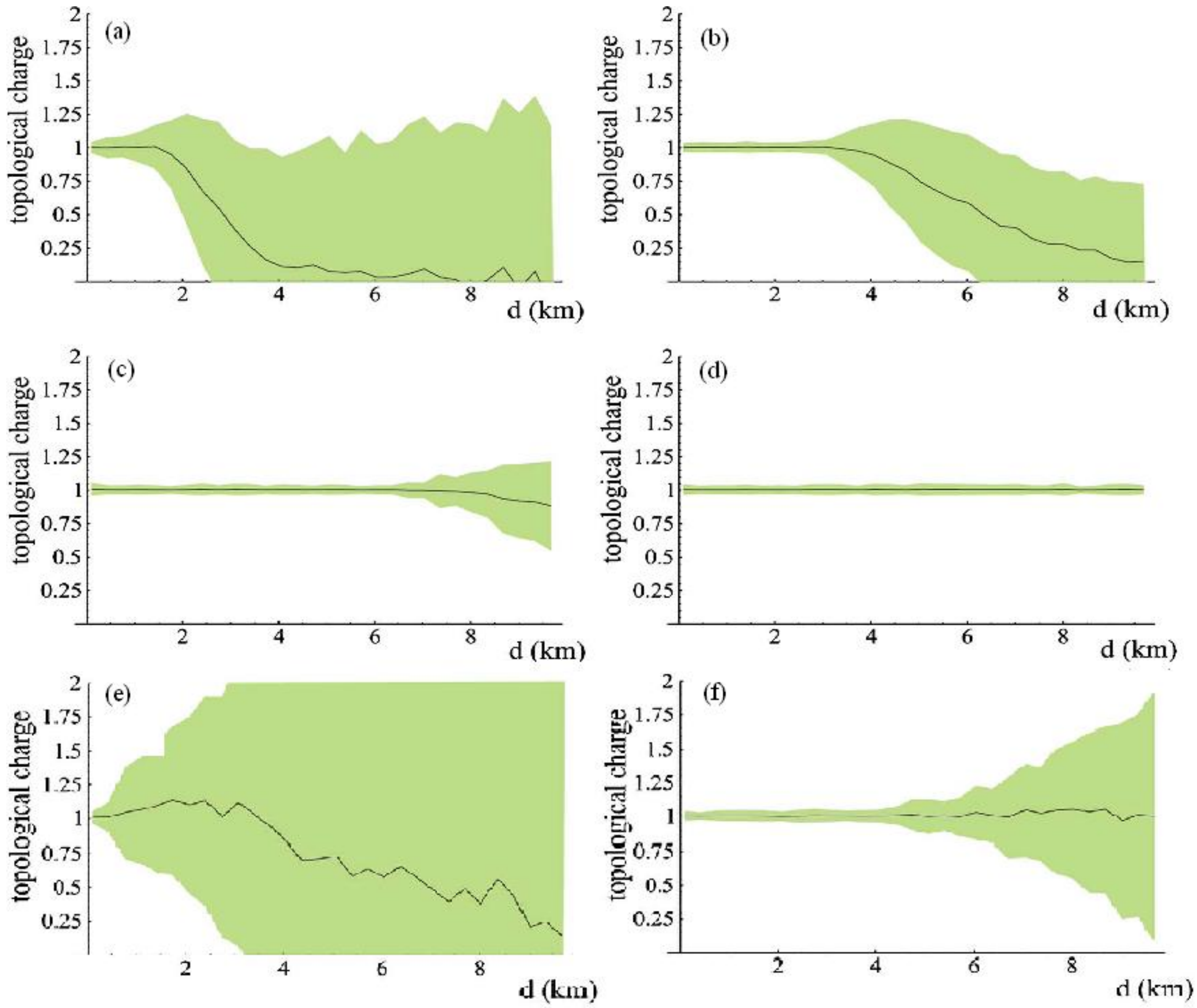


Figure 5: A collection of the relevant plots from Ref. [8]. In each case, the initial beam is the (1,1) Laguerre-Gauss mode. The line represents the average measured value of the topological charge, the green shaded region is the standard deviation. Note that λ is taken to be 1.55 μm and the initial beam waist size is $w_0 = 2cm$. The figures have turbulence strength values of **(a)** $C_n = 10^{-14} m^{-2/3}$, **(b)** $C_n = 10^{-15} m^{-2/3}$, **(c)** $C_n = 10^{-16} m^{-2/3}$, **(d)** $C_n = 10^{-17} m^{-2/3}$, **(e)** $C_n = 10^{-14} m^{-2/3}$, **(f)** $C_n = 10^{-15} m^{-2/3}$. Figures **(a)** through **(d)** are simulations where the detector radius is fixed at 4 cm. Figures **(e)** and **(f)** have a detection aperture that scales with the growth of the beam due to diffraction.

From Figure 5, we can see there is an associated distance before the breakdown of the information. For weaker turbulence, the beam is capable of propagating over a distance of ~7 km before significant deviation from the mean value of the measured topological charge. The most interesting result is a comparison between figures **(a)** and **(e)**, where under extremely strong turbulence, a detector that scales with the radius has a mean value that is closer to the actual topological charge, but the standard deviations become extremely large at much shorter distances than with a fixed detector size. This cannot be explained through simple charge wandering, but instead is the result of two major factors. One is pair production: a positive 1 charge vortex and a -1 charge vortex are produced, conserving angular momentum, and one of these two charges can drift outside the detection region causing an error. A more important error is due to the complexity in the phase profile, which will give a charge counting algorithm additional errors due to the increased complexity of discerning a vortex from a region of rapid variation in the phase [8]. Some detection devices, such as a Shack-Hartmann wavefront sensor, will have these errors. In the paper by Paterson, Ref. [6], turbulence is analyzed in a more theoretical manor. The results of his analysis is plotted in Fig. 6. His parameter of importance is the probability of detecting the correct eigenvalue of the topological charge. This is useful as a check on the simulations by Gbur *et al*.

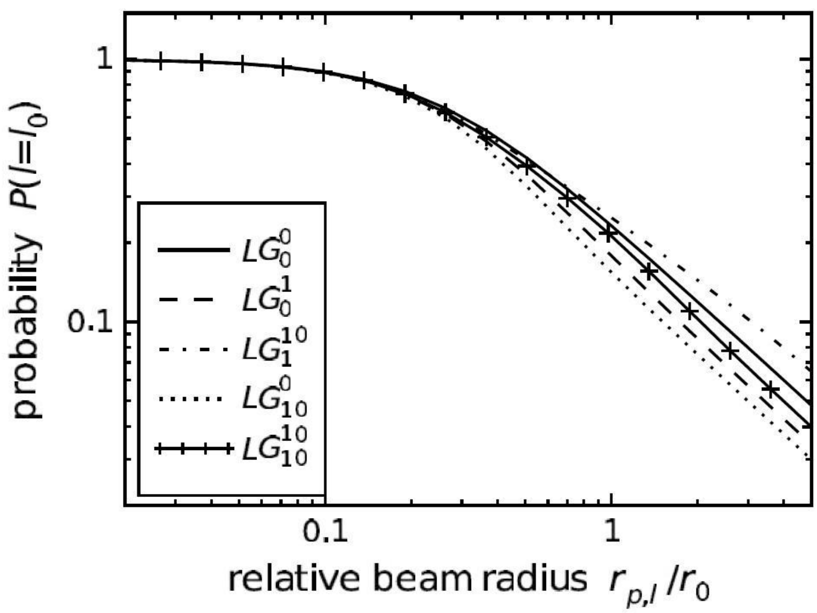


Figure 6: A plot summarizing the relevant analysis by Paterson. Here, the convention is $LG_{|m|}^{p}$, where m refers to the topological charge, and p is the radial mode. The x-axis is the ratio between the rms beam radius and the Friend parameter (the length scale that turbulence becomes important). Figure from Ref. [6].

Paterson considers weak turbulence, which does have similar features to the simulations from Gbur, *et al*, such as the rapid decay after a certain length scale. From these two papers, we can see that even weak turbulence is critical for transmission of angular momentum states. As polarization states are less likely to be destroyed in turbulent media [1], the usefulness of angular momentum states depends on its stability. Luckily most applications of quantum information that require free-space links can utilize satellites as a means of information transfer. In this case, the maximum effective atmosphere will be on an order of 10 km. However, these simulations show that even for weak turbulence, there is a large deviation from the expected value. If one were to design a quantum cryptography system, for instance, it would be unfortunate if it only worked in excellent weather conditions.

Conclusions

In this paper, we have described the potential uses of optical vortices and why the study of turbulence mitigation is a necessary component of continual advancement of quantum information applications using free-space links. We have given a brief mathematical framework, showing how optical vortices arise naturally out of the wave equation in the paraxial wave approximation. Simulations by Gbur *et al* and Paterson have been presented, indicating a direction for experimental studies. As far as my knowledge, I have not encountered any papers having an experimental implementations of the effect of turbulence on angular momentum states. These simulations show that turbulence on optical vortices will have to be considered for free-space links that utilize optical vortices as some form of information carrier. Now that simulations have been completed, giving insight into the expectation of how the beam will behave, the next step is to begin setting up an actual experiment to verify the approximations used in the papers. The experiment, however, will be complicated as two facilities over a distance of at least 4 km will be needed. The beam will then need to be transmitted via free-space, so this will most likely need to be away from any cities. Alternatively, it could be possible to make a highly dense, highly turbulent chamber in a lab with the equivalent parameters of a 5 km path length. However, many other parameters would be unaccounted for, such as the beam expansion. I think the best route would be to set up a system between two facilities, and use a wavefront sensor to see the effect on the vortices. It would also be interesting to see how adaptive optics, such as deformable mirrors or other turbulence mitigation procedures allow for the retention of the optical vortices. By having the two facilities set up, it will be easier to test these techniques to preserve the information.

References

[1] N. Gisin *et al.*, Rev. Mod. Phys. **74**, 145 (2002).

[2] Julio T. Barreiro, Nathan K. Langford, Nicholas A. Peters, and Paul G. Kwiat, Phys. Rev. Lett. **95,** 260501 (2005).

[3] M. S. Soskin, V. N. Gorshkov, M. V. Vasnetsov, J. T. Malos, N. R. Heckenberg, Phys. Rev. A. **56**, 4064 (1997).

[4] Alois Mair, Alipasha Vazirir, Gregor Weihs, Anton Zeilinger, Nature **412**, 313 (2001).

[5] Jaime A. Anguita, Mark A. Neifeld, Bane V. Vasic, Appl. Phys. **47**, 2414 (2008).

[6] C. Paterson, Phys. Rev. Lett. **94**, 153901 (2005).

[7] Gregorius Berkhout, Martin Lavery, Johannes Courtial, Marco Beijersbergen, Miles Padgett, Phys. Rev. Lett. 153601 (2010).

[8] Greg Gbur, Robert K. Tyson, Opt. Soc. Am. A. **25**, 225 (2008)