Dynamics and Self-organization in Aeolian Ripples

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Aeolian ripples arise instantaneously from sandy beds subjected to winds sufficient to cause saltation to surface grains. It is now widely believed that the process of ripple formation is a self-organizing phenomenon involving non-linear dynamics in the variation of flux of secondary grains that are ejected through saltation. In addition, the metric entropy and interaction matrix of the system alter governs the dynamics and emergent behavior in the patterns. In this paper I will review the dynamics, self-organization and entropy pertaining to aeolian ripples. Finally, I will briefly discuss difficulties in studying dune fields.

1 Introduction

The evolution of complex structures in landscapes from aeolian sand transport has fascinated and drawn the attention of many scientists. For geographers, dunes and ripples are of prominent interest since it is a typical landform where aeolian processes dominate. Such aeolian landforms are estimated to cover about one-fourth of the earth surface. Furthermore, dune fields and ripples are also found on Mars and Venus[1]. For physicists the evolution of patterns presents rich information on non-linear dynamics, entropy and self-organization. While in engineering, knowledge of these complex patterns is still important today for exploiting vast areas of land when introducing networks of roads and pipelines.

Wind driven ripples form from the rearrangement of loosely packed sand grains on sand beds into ridge crest and troughs. The crests lie perpendicular to wind direction and with asymmetrical cross sections. The upwind slopes are much less steep than downwind slopes. These ripples are small, with a wavelength of a few centimeters, and with heights roughly one-fifteenth or one-twentieth of their wavelength (fig. 1).

Figure 1: sand ripples atop a dune in the Wadi Rum desert in Jordan. The wavelength is about 10 to 15 cm and the height is a few mm.

The mechanism responsible for the formation of aeolian ripples is thought to be the indirect action of wind on loose sand. When the wind strength is large enough, individual sand grains are lifted by the direct action of the shear stress exerted by the wind on the sand surface. This process is known as *saltation* in which grains hop along the surface in low angle trajectories (fig. 2).

During their flight, the grains reach a velocity that is approximately that of the wind and upon their impact with the surface, they impart their energy and momentum to the sand, ejecting other grains by a process known as *reptation*. These impacts further eject a number of grains on short, quasi-ballistic trajectories that are largely unaffected by wind. For sufficiently large wind velocities, the bombardment by sand grains accelerated by the wind generates a cascade process, and an entire population of saltating grains hopping on the sand surface emerges [2]. During strong winds, the layer of saltating grains can reach a thickness of more than 1 m.

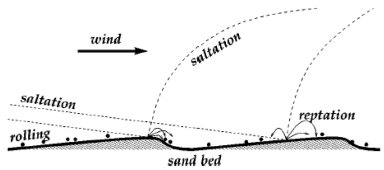

Figure 2: Interaction of saltating, reptating, and rolling sand grains in the model. Saltating particles follow by long trajectories (dashed lines), strike the sand bed.The impacts cause ejection of reptating grains from the bed surface. These grains trajectories (solid lines with arrows) are short. The landing reptating grains may roll for some distance upon the bed surface (small circles) before they stop.

In the late 19th and early 20th century, it was thought that ripples formed on the surface of sand beds similar to water waves, if sand were assumed as a highly viscous fluid capable of bulk deformation. Another perception assumed a pre-existing waviness in the airflow over the bed which acted as an external template for the ripple pattern. This explanation is now dismissed because there is no evidence of sand motion inside the sand bed itself. Other versions of the wave hypotheses considered the effect of wave-like perturbations in the dusty overlying air [3]. Although these possibilities cannot be ruled out, at present there is not much empirical evidence to support them.

2 Aeolian ripple phenomenology

Wind driven ripples and dunes were extensively studied by R.A. Bagnold in his classic reference *The physics of blown sand and desert dunes*. Bagnold explained that ripples formation involved grain-by-grain motion of the bed rather than a continuous deformation. Bagnold hypothesized that the ripple wavelength is equal to the mean length of saltation jump. However, careful experimental results by numerous researchers challenged this claim since the saltating grains trajectory is many times longer than the ripple wavelength.

It is now commonly accepted that the mechanism of ripple formation arises from the nonlinear dynamics in variation of the reputating grain flux. Wind and saltating grains, whose trajectories are many times longer than the ripple wavelength, play a complex and indirect role. The oblique, almost unidirectional bombardment by saltating particles supplies the energy necessary for reptation while wind strength determines the intensity of saltation. However, the probability of direct entrainment of particles into reptation by the wind is small.

Using high-speed photography of saltation occurring on sand filled strip in a wind tunnel, Willett (1986) observed that sand collision typically resulted in one ricocheted grain and a large number of low energy grains (fig. 3). The ricocheted grains impart most of the impact energy, with about two thirds of the impact velocity, and are ejected at an angle larger than the impact angle. The other low energy grains had a mean speed of only 3% of the impact speed. The largest number of grains ejected occurred when the impact angle was near 12°[3].

Numerical simulation of single impacts shed further light on the grain impact process. The kinetic energy of each impact redistributes as 1) rebound of the original grain from the bed 2) ejection of secondary grains from the bed 3) dissipation by inelastic deformation of the bed. The initial grain population is entirely aerodynamic with a delta function distribution of velocities. After tens of hop times, the full range of ejection velocities is populated. At this point, the total number of saltating grains begins to grow rapidly resulting in a chain reaction. Wind causes grains to land with more energy than they initially left the bed and will consequently retain a larger fraction of their energy upon impact.

After many generations, a stable population is achieved. The steady state is characterized by a specific total mass flux, an equal number of impacting and ejected grains, and a stationary wind velocity profile. The steady state saltation population contains no aerodynamically entrained grains. As the population of the splashed and rebound grains increases, the shear stress at the bed is reduced and correspondingly reduces the rate of aerodynamic entrainment (fig. 4).

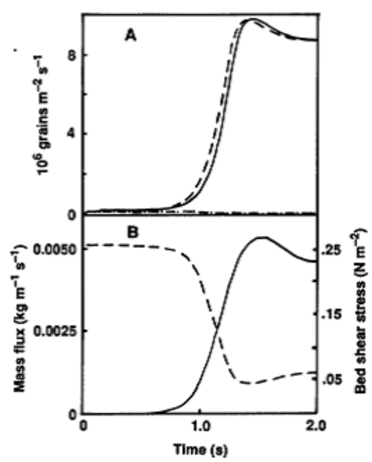

Figure 4: Simulation of a saltation system a) number of grains (dashed) and ejected (solid). b) total mass flux (solid) and shear stress at the bed (dashed).

3 Mathematical model

Models of ripple dynamics started in 1940s with Bagnold. The original idea of Bagnold was based on the existence of a large number of grains that jump from one ripple crest to another, in what is called the ballistic hypothesis. The ballistic hypothesis was later abandoned because experimental observations showed that grains jump over distances that are much larger than the ripple wavelength [4]. Moreover, this model cannot predict the merging of ripples and the resulting coarsening of ripple patterns that is observed in natural settings [5].

Starting from 1980s, a renewed interest in aeolian phenomena caused a surge of many analytical models and has led to a series of papers based on different types of phenomenological and mathematical assumptions [6-12]. Some of these works are based on discrete modeling approaches, such as cellular automata, and include empirical rules of sand grain behavior [6,8, 9]. One particular advance in the theoretical understanding of ripple formation was achieved by Anderson [13,14].

In Andersons model the only role of saltating grains is to bring energy into the system, having extracted it from the wind. The saltating grains energize the reptating population, and ripple formation is entirely due to the spatial variability of the flux of reptating grains. Andersons original model deals with a two-dimensional layer of sand, i.e. one-dimensional ripples, and formulates the problem in terms of an integro-differential equation describing the one-dimensional sand surface. The dynamics of the reptating grains is handled by a probabilistic approach in which the grains take steps of random length, which avoids tracking the detailed dynamics of the impact and reptation flight. In the model, ripples arise as a linear instability of a uniform layer of sand. Unfortunately, the model yields unrealistic results at the nonlinear stage of ripple growth, which begins very early.

A simpler model proposed by Prigozhin (1999) provides a better description of the physical process than the simplified stochastic model. The model produces the expected asymmetrical ripples and is able to simulate the merging of ripples. To allow for metastability, the model needs to include the surface density of the rolling particles in addition to the surface flux by the local surface slope.

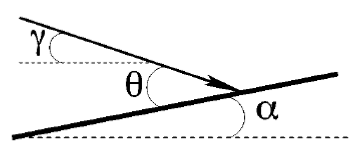

Figure 5: The saltating grain collides with the bed at an angle γ to the horizontal, while the surface is inclined at an angle α. The angle of incidence $\theta = \alpha + \gamma$.

Based on the phenomenology of saltating grains discussed above, there exists a uniform flux of saltating grains that reach the bed surface $z = h(\mathbf{x}, t)$ at a low angle γ to horizontal. The impacts cause erosion of this surface. The erosion rate f is proportional to the impact intensity, which depends on the surface orientation with respect to the direction of saltation. If the saltating particles strike an inclined surface at an angle θ (fig. 5), then f is proportional to sin θ

and can be written as

$$f = f_0 \frac{\sin\theta}{\sin\gamma} \tag{1}$$

where f_0 is the rate of erosion of a horizontal surface determined by the intensity of saltation.

A reptating grain, ejected by an impact at a point $\mathbf{x}$ and lands on the bed surface at a point $\mathbf{y}$ with probability density p given by the splash function given by $p = p_\alpha(\mathbf{x}, \mathbf{y})$. The splash function is not currently known either experimentally or theoretically although previous studies [6,13] suggest that the system is not very sensitive to splash function behavior and that an approximation is sufficient for simulation. Upon landing, reptating particles do not stop immediately but may roll away. Let $R(\mathbf{x}, t)$ be the effective surface density of rolling particles. When they become stationary, the rolling particles become incorporated into the motionless bed. If the rate of rolling-to-steady-state transition is given by $\Gamma[h, R]$, then the continuity equation for the sand bed can be written as:

$$\frac{\partial h}{\partial t} = \Gamma[h, R] - f \tag{2}$$

$$\frac{\partial R}{\partial t} + \nabla \cdot \mathbf{J} = Q - \Gamma[h, R] \tag{3}$$

Where $\mathbf{J}$ is the horizontal projection of the flux of the rolling particles and the and the source term gives the intensity of the reptating particles.

$$Q(\mathbf{x}, t) = \int f(\mathbf{x}, t) p_\alpha(\mathbf{y}, \mathbf{x}) d\mathbf{y} \tag{4}$$

Assuming that reptating particles lose most of their momentum in collision with the rough bed surface, they impart all their energy to the incline. The particles then roll in the direction of steepest descent and thus the simplest form for the flux $\mathbf{J}$ is given by

$$\mathbf{J} = -\mu_0 R \nabla h \tag{5}$$

With μ_0 is a constant of motion for the particles. Finally, the rate of rolling-to-steady-state transition Γ can be approximated as proportional to the rolling particles R. Since the exchange rate cannot depend on the free-surface slope orientation, Γ can be assumed as a function of $|\nabla h|^2$. The steeper the free surface is, the easier particles roll down and, correspondingly, the lower is the rate of rolling-to-steady-state transition. Combining all these assumptions leads to an expression for Γ.

$$\Gamma = \kappa_0 R \left(1 - \frac{|\nabla h|^2}{tan^2\alpha}\right) \tag{6}$$

where κ_0 characterizes the particle stability upon a horizontal surface.

The model can be further simplified by rescaling the variables by the constants used above, f_0, κ_0 and L the mean length of reptation by letting $h \to h/L, f \to f/f_0$ and $R \to \frac{\kappa_0}{f_0}R$. The new flux and rolling rate become:

$$\mathbf{J} = -\nu R \nabla h, \Gamma = R\left(1 - \frac{|\nabla h|^2}{tan^2\alpha}\right) \tag{7}$$

where $\nu = \mu_0/\kappa_0 L$ is a dimensionless parameter characterizing the competition between mobility and stability of the rolling grains on the surface. In addition, the continuity equation becomes

$$\frac{f_0}{\kappa_0 L}\frac{\partial R}{\partial t} + \nabla \cdot \mathbf{J} = Q - \Gamma[h, R] \tag{8}$$

The time derivative of R can be neglected since $f_0/\kappa_0 L \ll 1$ and we achieve the simplified flux equation

$$\nabla \cdot \mathbf{J} = Q - \Gamma[h, R] \tag{9}$$

By linearizing the above equations and considering up to second order terms, a dispersion relation is obtained for the real part of the ripple wavelength Re(λ). The system becomes unstable when $Re(\lambda) > 0$. The condition relating the incident angle to the horizontal γ to ν is given by

$$cot\gamma > \nu + k_m, k_m = 2.01 \tag{10}$$

This result agrees with the experimental and theoretical prediction where low angle projectiles provide the largest non-uniformity in the ripples (cot γ is large).

Finally, by combining (2), (4), (7) and (9), the nonlinear system can be solved numerically assuming periodic boundary conditions and using an implicit finite difference approximations. Initially, the ripples obey the linearized equations but after an initial stage the fastest growing mode dominates.

The simulations show the mechanism of ripple merging in fig. 6.The merger takes place only if the overtaking ripple is much smaller than the bigger overtaken ripple, which move more slowly. As the smaller ripple reaches the larger, the trough between them merge and a two-headed long ripple appears. Then the larger ripple crest starts to move forward as a separate small ripple and two new ripples emerge from this recombination. The emergent ripple is larger than the larger of the two ripples before the merger. These mergers are not simple superposition of waves but a complicated soliton like mode of ripple interaction. So how do these non-linearities translate to self-organization? It is still not clear how to proceed past the non-linear dynamics but the discussion below gives an overview of self-organizing systems.

4 Self-organization, entropy and the Lyapunov exponent

Landscapes are self-organizing due to the fact that individual landforms arise independent of external factors. Phillips (1999) reviewed literature in geomor-

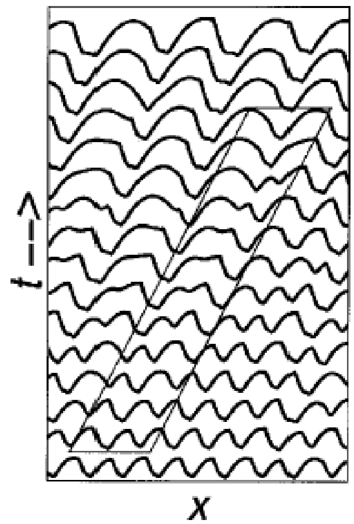

Figure 6: Interaction of simulated ripples with time showing mergers between ripples.

phology and categorized eleven different forms of self-organization [15]. Aeolian ripples and pedogenesis (soil formation) are a form of self-organization where pattern formation arises from chaotic dynamics. Self-organizing geomorphic systems are attracted to stationary end states and are not free to evolve from common starting points into diverse, qualitatively different forms. Thus, geomorphic systems evolve into similar forms over the world.

On the contrary, an opposite effect is seen in self-organizing, far from equilibrium biological systems that are able to evolve into wholly new forms and different states. At broad scales, geomorphic systems indeed develop similar forms. However, at more local scales, landscapes are sensitive to initial conditions and perturbations, whereby common starting points can lead to quite diverse states independent of stochastic forcing and environmental controls.

Self-organizing behavior in geomorphology is governed by the *arrow of history* and the *arrow of time*. The arrow of time applies to irreversible processes which lead towards maximum disorder as observed in chemical weathering that irreversibly transforms rock to saprolite and soil. The arrow of history, conversely, leads towards states of higher order and organization. An example is seen in wind which converts undifferentiated sediment into an increasingly organized soil profile. These examples show that many geomorphic phenomena involve both processes where disorder increased during geochemical weathering followed by self-organization during pedogenesis.

In thermodynamic perspectives, the dynamics of a far- from-equilibrium self-organizing system is governed by an increase in the total entropy of the system and its environment and a decrease in the entropy within the system. However, the system has to order somehow without violating the second law of thermodynamics. If dS is the change in entropy of the system and its surrounding, d_eS is the change due to exchanges between the system and its environment and d_iS is the change associated with entropy production by irreversible processes within the system, then the following statement is always true

$$dS = d_eS + d_iS, d_iS > 0 \tag{11}$$

However, a different type of entropy is known as the metric or K-entropy.

$K_n = -\sum_{i_0 \dots i_n} P_{i_0 \dots i_n} \, lnP_{i_0 \dots i_n}$

In self-organizing systems, entropy would have to decrease to reflect an increase in order and organization. However, since geomorphic systems also include irreversible processes, total entropy must increase with time (second law of thermodynamics). This can only be accomplished within a hierarchy. Letting i denote levels of hierarchy, the following criteria can be put forward for a self-organizing system:

1) Total entropy aggregated over all levels must increase over time.

2) Entropy at all levels i must increase over time.

3) Information (negative entropy) must increase over time.

4) Increase in information (decrease in entropy) at any level i must be offset by increases in entropy at other levels.

K measures the degree of uncertainty and it converse, the degree of order. In random white noise systems, all outcomes are equally likely and entropy is infinite. On the other end of spectrum, a completely deterministic, non-chaotic system where only one outcome is possible, entropy is zero. Thus, finite, positive entropy is associated with a combination of order and random noise and /or deterministic chaos. The metric entropy is related to the dynamics of the system and particularly the Lyapunov exponent λ. λ measures the rate of exponential divergence from the initial condition. A system with negative λ is dissipative and results in an attractive phase space. The metric entropy is related to the Lyapunov characteristic exponents by

$$K_n = \int \sum_{\lambda_i > 0} \lambda_i d\mu \tag{12}$$

If positive entropy is to increase over time in the absence of stochastic forcing, a geomorphic system must have at least one positive Lyapunov exponent since the metric entropy is equal to the sum of all the positive Lyapunov exponents. The Lyapunov exponents are the real parts of the complex eigenvalues of the interaction matrix of the system. Thus the interaction matrix is indicative of whether the system is capable of generating positive entropy over time in the absence of stochastic forcing.

While the positive Lyapunov exponent expresses the metric entropy, a negative exponent reflects the exponential approach of an initial state to an attractor. Thus a positive λ gives the rate of information loss while a negative λ gives the rate of information gain. If a system is to be self-organizing, there must be at least one positive Lyapunov exponent, but $\Sigma\lambda < 0$. In this way, the rate of information gain can exceed the rate of entropy production.

If the system can be represented by an nxn interaction matrix $\mathbf{A}$ which represents the interaction a_{ij} of each component x_i on the other x_j.Then the two criteria for self-organization are as follows:

1) The matrix $\mathbf{A}$ is unstable according to the Routh-Hurwitz criteria.

2) The sum of the diagonal elements of $\mathbf{A}$ is negative ($\Sigma a_{ii} < 0$).

It still remains unclear whether the interaction matrix of non-linear systems such as aeolian ripples and dunes can be measures. If feasible, it will offer new insight on the system and its phase space. Phillips (1995) measured the flow hydraulics at a channel cross section and determined the interaction matrix thus proving that the system is self-organizational. In this case, the interaction matrix was readily measured since scaling laws have been derived for channel/pipe systems.

5 Ripples but not dunes

Although wind driven dunes fall under the same self-organizing behavior as aeolian ripples, it has been much harder to quantify how Aeolian sand transport evolves to dunes. Because of the complicated nature of air flow and of sand erosion, transport, and deposition over a dune, a simple model from the physics of aeolian sand transport to the evolution of a dune field is not currently feasible.

A typical dune field extends for hundreds of kilometers, and takes many thousands of years to form, so that the study of dune-scale geomorphology cannot rely on well-defined experiments. Difficulties arise due to strong nonlinearities in the system, which includes wind flow over a dune field expressed by the Navier-Stokes equation, sand transport induced by wind and sudden deformation by gravity known as avalanching, the latter leading to non-smooth dynamics. Avalanching has been intensively studied in relation to self-organized criticality. Yet, some characteristic features of dunes, such as a strong asymmetric profile, need theory which goes beyond linear analysis.

To describe the evolution of such a system starting, the momentum of the system comprised of air and sand grains has to be properly described. However no such equation is yet available. Even if wind flow and sand transport can be considered separately, it is extremely difficult to solve the Navier-Stokes equation, even for an isolated single dune on a flat surface. In the case of a dune field comprising many dunes with different shapes, it is almost impossible to calculate the wind flow with the same accuracy as in the case of a single dune. Furthermore, these equations have to be solved under constantly changing boundary conditions as the dunes grow, in a three-dimensional space *(x,y,z)*. Tracking dune evolution by calculating wind flow over a dune field each time is therefore practically impossible.

6 Conclusions

Self-organizing phenomena are common in earth surface systems, and self organizing principles have been proposed as general principles applicable to geomorphic systems. Ripples spontaneously emerge from an essentially flat bed, beginning as indefinite bumps on the bed. through a process driven by a dispersion in ripple translation velocity, these small ripples merge, rapidly at first and more slowly later, to arrive at a pattern that translates downwind with little further modification.

Despite the plethora of research on aeolian ripple models and computer simulations using numerous techniques, much less work has been conducted on studying the phase space of the system and its attractors. One reference discussed the phase space of dune fields but only due to scarcity of the analytical models available. Eventually, the merger of the non-linear dynamics and future work on attractors in aeolian ripples will significantly improve our current understanding of self-organization and pattern formation in geomorphology.

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