

# Cosmic Strings

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## **Abstract**

Cosmic strings are similar to other one-dimensional topological defects familiar from condensed matter systems such as vortex lines in type II superconductors. The goal of this paper is to provide a brief introduction to what cosmic strings are and how they come about in the early stages of the universe highlighting similarities to defects from condensed matter systems when possible. After will be a discussion of their possible observational consequences and their importance to cosmology; especially gravitational lensing and the effect on their cosmic microwave background (CMB). I will conclude by attempting to give a brief overview of the current state of cosmic string research.

# 1 Introduction

The term “cosmic strings” refers to 1D topological defects formed in symmetry breaking phase transitions in the early universe. Such defects are familiar from systems studied in condensed matter such as disclination lines in nematic liquid crystals, vortex lines in liquid  $^4\text{He}$ , and flux tubes in type II superconductors. Interestingly, analogous defects occur in field theories normally relegated to high energy physics and with similar dynamics.

The idea that strings could be stable and have observational consequences was first proposed by Tom Kibble in the 1970’s [1]. 2D domain walls and 0D monopoles are also possible but are ruled out by experience. Due to gravitational effects domain walls would ruin the isotropy of the cosmic microwave background (CMB) [1][2]. Stable, ultra-heavy monopoles are generic predictions of grand unified theories (GUTs). Once formed, they would soon dominate the energy density of the Universe. This is known as the “monopole problem” because we don’t see any. Cosmic inflation was developed, in part, to solve the monopole problem by reducing the monopole density to undetectable levels. Inflation should also then reduce the density of cosmic strings. However, it is possible to reconcile these theories by adjusting the coupling between the inflation field and the field responsible for the cosmic strings so that string formation occurs during the late stages of inflation. For more details see the references given in [3, sec. 5.2].

The study of cosmic strings is a very rich field and encompasses aspects of high energy physics, cosmology, numerical simulation, and the experimental efforts of observational astronomers. I hope to tell a complete, if simplified, version of how strings come about, how they evolve, and how they may affect our Universe today.

Please note that in this paper I will be using natural units in which  $\hbar = c = k_B = 1$ .

## 2 String solutions in field theories

Much of the discussion in this section follows closely the terrific review article by Hindmarsh and Kibble [3]. In this section I will attempt to provide simple examples of string solutions in field theories and show how they are similar to more familiar condensed matter analogs.

## 2.1 Global strings

The simplest example of a field theory with string solutions is that of a complex scalar field  $\phi(x)$  with a Lagrangian density

$$\mathcal{L} = \partial^\mu \phi^* \partial_\mu \phi - V(\phi) \quad V(\phi) = \frac{1}{2} \lambda (|\phi|^2 - \frac{1}{2} \eta^2)^2$$

This Lagrangian possesses a global U(1) symmetry  $\phi \rightarrow \phi e^{i\alpha}$  with  $\alpha$  a constant. It also exhibits a double well potential with a ground state  $\phi = (\eta/\sqrt{2})e^{i\alpha_0}$  where  $\alpha_0$  is a constant. This ground state is not unvariant under the global U(1) symmetry. The equation of motion following from this Lagrangian is

$$\partial_\mu \partial^\mu \phi - \frac{1}{2} \lambda \eta^2 \phi + \lambda |\phi|^2 \phi = 0.$$

The mass of the scalar particle in the symmetry-breaking vacuum is given by  $m_s^2 = \lambda \eta^2$ . In natural units mass has dimensions of L<sup>-1</sup> and  $m_s^{-1}$  is analogous to the correlation length in superconductors as it determines the scale of fluctuations in  $\phi$ .

We now insert a cylindrically symmetric trial solution  $\phi = \frac{\eta}{\sqrt{2}} f(m_s \rho) e^{in\varphi}$  where  $(\rho, \varphi, z)$  are cylindrical coordinates and  $n$  an integer. The equation of motion reduces to a single non-linear ODE

$$f'' + \frac{1}{\xi} f' - \frac{n^2}{\xi^2} f - \frac{1}{2} (f^2 - 1) f = 0$$

where  $\xi = m_s \rho$ . To preserve the continuity of  $\phi$  at the origin we must have that  $f \rightarrow 0$  as  $\xi \rightarrow 0$ . We also need  $f \rightarrow 1$  at infinity so that  $\phi$  approaches its ground state value. If we write  $f = 1 - \delta f$  and plug this into our ODE we see that  $\delta f \sim n^2/\xi^2$  at large  $\xi$ . A numerical solution for  $f(\xi)$  is shown below.

Notice that if  $n = 0$  then  $f = 1$  is a valid solution and we lose this behavior at the origin. These solutions are closely tied to vortices in superfluid <sup>4</sup>He where  $n$  is the topological charge of the vortex and tells us the number of times the phase winds around.  $n = 0$  corresponds to there being no vortex.

If we now examine the energy density

$$\begin{aligned} \mathcal{E} &= |\dot{\phi}|^2 + |\nabla \phi|^2 + V(\phi) \\ &\propto (f')^2 + \frac{n^2}{\xi^2} f^2 + \frac{1}{2} (f^2 - 1) f^2 \end{aligned}$$

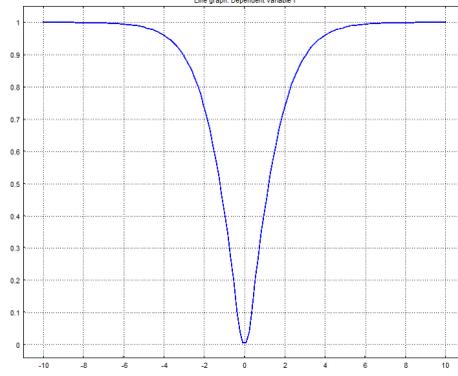


Figure 1:  $f(\xi)$

we see that it is infinite because it has a  $\xi^{-2}$  tail at large  $\xi$  due to the the angular derivative part of the gradient. Inside a cylinder of radius  $R \gg m_s^2$  the energy per unit length is approximately  $\pi n^2 \eta^2 \ln(m_s R)$ .

## 2.2 Local strings

Let us now add a gauge field  $A_\mu$ . Our Lagrangian density is then

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |D_\mu\phi|^2 - V(\phi)$$

where  $D_\mu = \partial_\mu + ieA_\mu$  and  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ .

This is the Abelian Higgs model. This Lagrangian has a local U(1) symmetry and is invariant under the transformation

$$\phi \rightarrow \phi^{ie\Lambda(x)} \quad \text{and} \quad A_\mu \rightarrow A_\mu - \frac{1}{e}\partial_\mu\Lambda(x)$$

The equations of motion for this Lagrangian are

$$D_\mu D^\mu \phi + \lambda(|\phi|^2 - \frac{1}{2}\eta^2)\phi = 0$$

$$\partial_\nu F^{\mu\nu} + ie(\phi^* D^\mu \phi - D^\mu \phi^* \phi) = 0$$

The scalar Higgs particle, once again, has a mass  $m_s^2 = \lambda\eta^2$  in the symmetry breaking vacuum. The vector field also acquires a mass of  $m_v = e\eta$ .

Now let us take the radial gauge  $A_\rho = 0$  and try the cylindrically symmetric solutions

$$\phi = \frac{\eta}{\sqrt{2}} f(m_v \rho) e^{in\varphi} \quad A^i = \frac{n}{e\rho} a(m_v \rho) \hat{\varphi}^i$$

The resulting coupled differential equations do not have an analytic solution but their asymptotic behavior is as follows:

$$f \simeq \begin{cases} f_0 \xi^{|n|} & \text{as } \xi \rightarrow 0 \\ 1 - f_1 \xi^{-1/2} \exp(-\sqrt{\beta} \xi) & \text{as } \xi \rightarrow \infty \end{cases}$$

$$a \simeq \begin{cases} a_0 \xi^2 - \frac{|n| f_0^2}{4(|n|+1)} \xi^{2|n|+2} & \text{as } \xi \rightarrow 0 \\ 1 - a_1 \xi^{1/2} \exp(-\xi) & \text{as } \xi \rightarrow \infty \end{cases}$$

Here,  $\xi = m_v \rho$  and  $\beta = \lambda/e^2 = (m_s/m_v)^2$ . In the case where  $\beta > 4$ ,  $\xi^{-1/2} \exp(-\sqrt{\beta} \xi)$  is replaced by  $\xi^{-1} \exp(-2\xi)$ . Notice that everything is much more localized on the string. In fact, it can be shown that the energy density is  $\mu = \pi \eta^2 \epsilon(\beta)$  where  $\epsilon$  goes as  $\log(\beta)$  for  $\beta > 1$  [6][7] and  $\epsilon(1) = 1$  [5]. The energy density here is finite because the gradients have been replaced by covariant derivatives so that the angular derivative part now goes down faster than  $\xi^{-2}$ .

Now consider a circle  $\mathcal{C}$  of very large radius centered on the string encircling a surface  $S$ . On a circle whose radius tends towards infinity  $D_\mu \phi$  must vanish. We can use this to calculate the magnetic flux which passes through the string.

$$\int_S \mathbf{B} \cdot \hat{\mathbf{n}} d^2 x = \int_{\mathcal{C}} A^i dx^i = \frac{2\pi n}{e}$$

The magnetic flux is quantized much like that of vortices in superconductors.

There is another analogy that can be made. Vortices in the Abelian Higgs model are stable for any  $n$  if  $\beta < 1$ . For  $\beta > 1$  a vortex with  $|n| > 1$  is unstable to splitting into  $n$  vortices with charge  $n = 1$ . This can be interpreted in much the same way as distinguishing between type I and II superconductors. There are two forces at work here. The scalar field would like to push the vortices on top of each other since it wants to minimize the area where  $\phi = 0$ . However, the gauge field wants to push the vortices apart since the magnetic field lines want to spread out. The long-range force, i.e., the field with the lower mass, is the one that dominates. So, the  $\beta$  parameter is somewhat analogous to the Landau-Ginzburg parameter  $\kappa$ .

### 3 Phase transitions in the early universe

Now that we have seen what strings solutions look like, let's look at how they might form in the early universe. Let us begin with the simple complex scalar field Lagrangian from section 2.1. At a finite temperature the expectation value of  $\phi$  must be obtained by minimizing the free energy or, equivalently, the effective temperature dependent potential. If we assume the coupling constant  $\lambda$  is small then the leading temperature dependent terms at high  $T$  can be found by calculating the one-loop diagrams [4]. Our potential then becomes

$$V(\phi) = \frac{1}{2}\lambda(|\phi|^2 - \frac{1}{2}\eta^2) + \frac{1}{6}\lambda T^2|\phi|^2.$$

If we now define  $T_c \equiv \sqrt{3}\eta$  the potential can be rewritten as

$$V(\phi) = \frac{1}{2}\lambda[|\phi|^2 - \eta^2(1 - T^2/T_c^2)]^2 + \frac{1}{2}\lambda\eta^4$$

which now looks like the homogenous part of the Ginzburg-Landau free energy. Above  $T_c$  the potential has a singular minimum at  $\phi = 0$ . Below  $T_c$ ,  $\phi$  will acquire a vacuum expectation value such that  $|\phi|^2 = \eta^2(1 - T^2/T_c^2)$ . Also notice that in section 2 we saw that a string's energy density  $\mu \sim \eta^2$ . This means that  $\mu \sim T_c^2$ .

This simple model illustrates a second order transition. In this type of transition, when the temperature falls below critical, the field will assume a vacuum expectation value everywhere at about the same time. Due to causality reasons, the value of the phase at widely different positions must be uncorrelated. This differs somewhat from the behavior in superconductors where one normally thinks of the correlation length extending to infinity at the critical temperature. Strings will then form where the phase winds a multiple of  $2\pi$  around a loop.

Whether or not higher order terms render the phase transition of first-order is a complicated issue. If the phase transition is of first-order it would then proceed by bubble nucleation. The field will roll off the potential hill at different places at different times. One expects the phase of the field in each bubble to be more or less a random variable. When bubbles meet they tend to interpolate between their respective values of  $\phi$  at the boundary. Regions which get trapped in a loop where the phase winds around by a multiple of  $2\pi$  will form strings.

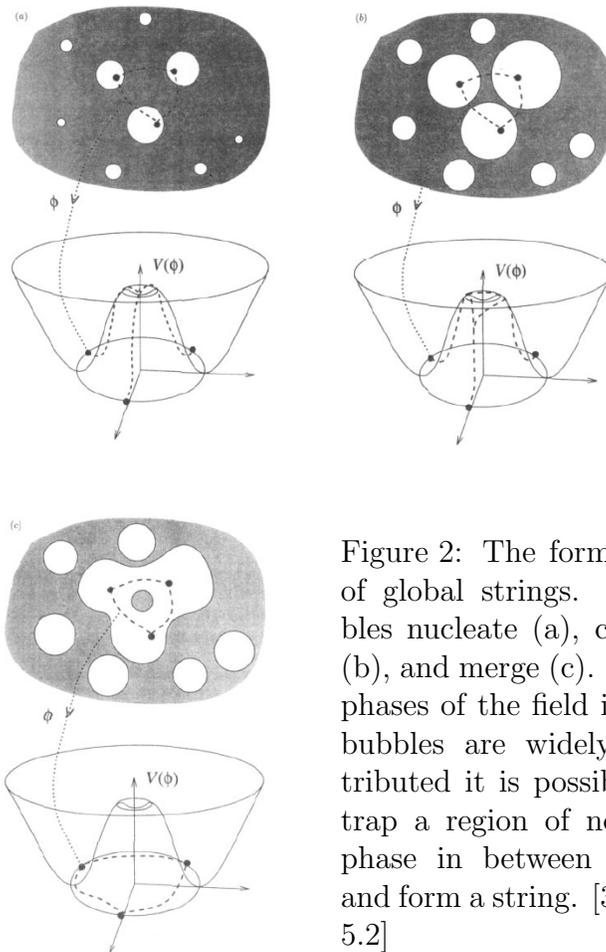


Figure 2: The formation of global strings. Bubbles nucleate (a), collide (b), and merge (c). If the phases of the field in the bubbles are widely distributed it is possible to trap a region of normal phase in between them and form a string. [3, sec. 5.2]

How early in the universe do these transitions take place? One can find a relation between the time and temperature in the early universe when it was dominated by relativistic matter

$$T^2 t = \sqrt{\frac{45}{\pi g_*}} \frac{M_{pl}}{4\pi} = \frac{2.42}{g_*^{1/2}} \text{MeV}^2 \text{s}$$

where  $g_*$  (which we take to be of order  $\sim 100$  during the relevant time period) is the effective number of spin states and  $M_{pl}$  is the Planck mass  $G^{-1/2} = 1.22 \times 10^{28} \text{eV}$  [8]. In the electroweak unified theory there is a phase transition at a temperature of order 100 GeV (the rest mass of the W and Z gauge bosons). This corresponds to a time of  $10^{-5} \text{s}$  after the big bang. In most GUTs, a

phase transition occurs at around  $10^{15}\text{GeV}$  or  $10^{16}\text{GeV}$  corresponding to a time between  $10^{-39}\text{s}$  and  $10^{-37}\text{s}$ . This is, of course, assuming our theories of particle physics work at such high temperatures.

## 4 Observational consequences

Cosmic strings are interesting theoretical objects. The question remains as to whether or not they actually exist. I will briefly sketch a few of the ways in which one might observe a cosmic string along with results from a few more notable searches.

### 4.1 Gravitational lensing

Space-time around a string is coned shaped as though a wedge of angle  $\delta$  were cut out. The deficit angle is related to the energy density  $\mu$  by

$$\delta = 8\pi G\mu.$$

For strings formed at the electroweak scale this is not observable but for strings formed at GUT energy scales this corresponds to an angle of  $5.18 \times 10^{-5} \sim 10^{-6}$  radians which is.

The conical shaped space time around a cosmic string lends itself to detection by observing gravitational lensing. What one would look for would be a double image of a distant object; one on each side of the string.

In the early part of this decade, an object appearing to be a lensed galaxy called CSL-1 was proposed as a candidate for string lensing [9][10]. The two galaxies observed have nearly the same morphologies, magnitudes, and spectra. However, closer inspection by the Hubble telescope in 2006 revealed them to be two very similar galaxies [11]. The principle axes of the galaxies are misaligned which is not an expected effect of any sort of gravitational lensing.

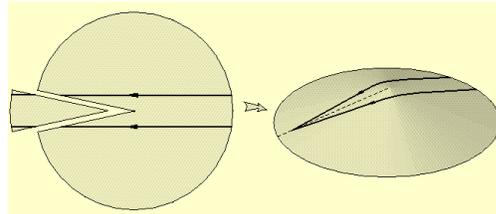


Figure 3: An illustration of the cone-shaped space time and resulting geodesics around a string [12].

Then there is the interesting case of Q0957+561, a system of two quasar images. The images fluctuate in brightness with a delay between them of 417 days. It is a well known lensed system whose delay has been used to measure the Hubble constant. The source of the lensing is a foreground galaxy whose image is clearly visible between the two galaxies. However, in 2004, a group led by Robert Schild reported observing a component which fluctuated

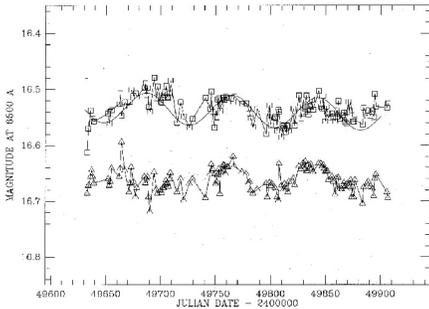


Figure 4: How correlated are these curves? [13]

tions really are; an analysis that has yet to be done.

synchronously, with no delay, during the period between September 2004 and July 2005 [13]. They proposed a scenario where the reason for this synchronous fluctuation is an oscillating cosmic string loop which passed between us and the lensing object. While this explanation may sound far fetched, other explanations involving multiple microlensing stars and undetected, massive binary star systems seem equally unlikely. There is also the question of how statistically significant the correlation between the brightness fluctuations really are; an analysis that has yet to be done.

## 4.2 Effects on the CMB

Measuring the inhomogeneity of the cosmic microwave background was once hoped to provide the most definitive test of the existence of cosmic strings. The effect of a moving cosmic string is to redshift the light in front of it and blueshift the light behind it an amount proportional to  $\delta$  and the velocity of the string perpendicular to the line of sight. This will show up when measuring the spectrum of the CMB. To predict the effect on the CMB one needs to know what the distribution in space and velocity of the strings will be. This inevitably involves a lot of numerical work. The Relativity and Gravitation research group at Cambridge maintains a webpage with plenty of movies of simulated string dynamics and evolution [12].

It became very difficult to produce simulations which would match the large-scale structure and CMB simultaneously. Meanwhile, the theory that the inhomogeneities were caused by the inflation of primordial quantum den-

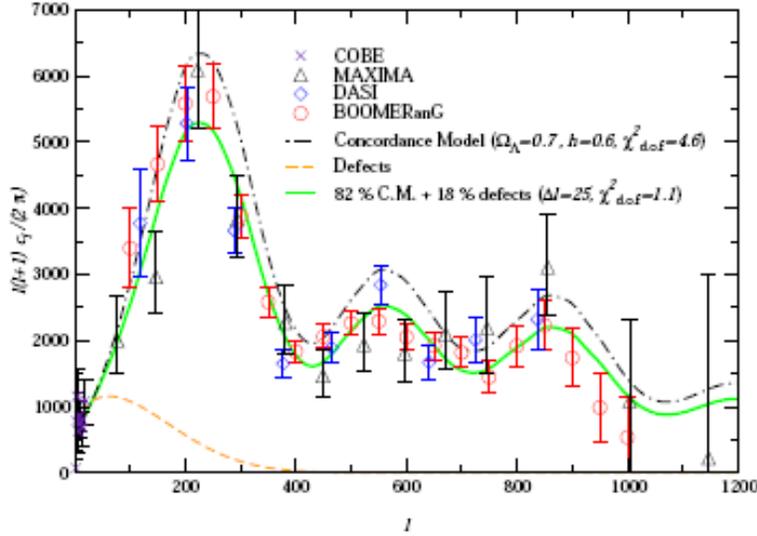


Figure 5: Fits of data to measured CMB power spectrum [14]

sity fluctuations was fitting the data much better [14]. This ultimately led to the decline in interest of cosmic strings in the early 2000's.

### 4.3 Other searches

There are many other proposed methods for searching for cosmic strings. One important way is to test cosmic string theory predictions against observed density perturbations. This leads to similar sorts of analyses that one gets into when measuring the CMB. Another way is the detection of gravitational waves caused by strings, perhaps by LIGO [15]. There are also theories that moving cosmic strings may be the origins of the highest energy cosmic rays (watch some of the movies on the Cambridge website and notice particles 'evaporating' from the cusps of the strings) but this seems unlikely [3, sec. 6.5].

## 5 Current state of cosmic string research

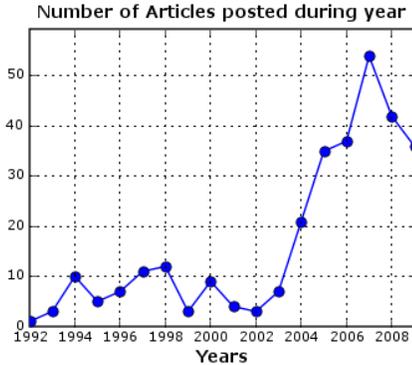


Figure 6: Number of article about cosmic strings and cosmic superstrings on arxiv.org

theory could produce stable cosmic strings of the appropriate energy scale.

Interest in cosmic strings waned in the early part of this decade when it was discovered that they could not play a critical role in structure formation.

However, in 2004, interest in cosmic strings exploded. The new interest in cosmic strings is due to two reasons: one observational, the other theoretical. On the observation side, there was a lot of excitement about the possible discovery of a cosmic string due to CSL-1 and the intriguing case of the anomalous time delay in Q0957+561 A, B. On the theoretical side, there was increased interest due to the realization that string

## 6 Conclusion and future steps

Cosmic strings have been studied in one form or another since the 1970's and as such has become a very rich and detailed field of study. This paper can only be a short introduction at best. Most notable was an omission of any mention of string dynamics and how the numerical simulations are done. This is still an active area of research. The issue of string dynamics and interactions is extremely interesting as a study of how one may go about describing a theory of vortex matter. I refer you section 3 of Hindmarsh and Kibble's review article for more details.

The prospects for the study of cosmic strings beyond being a toy of theorists seems doubtful. The few supposed detections are all questionable at best and completely debunked at worst. The prospect of a confirmed detection of single strings moving near the speed of light seems dim and it seems likely that strings suffered the same fate as monopoles, inflation reducing their density to undetectable levels (if they ever existed at all). Glimmers of hope come from the expectation of more accurate measurements of density perturbations coming from Dark Energy Survey and the Large Synoptic

Survey Telescope. Better measurements of CMB polarization may also hold valuable information. Perhaps with increasingly better measurements of astronomical data we will be able to place better limits on just what kind of contribution defects have on CMB inhomogeneities and large-scale structure.

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