

# Magnetic Phases of the Cuprates

Leslie Ross

The University of Illinois, Urbana-Champaign

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## **Abstract**

The cuprate superconductors have a far more complicated magnetic phase diagram than the traditional BCS superconductors. This essay will describe the experimental evidence of and theoretical basis for the novel phases associated with these high- $T_c$  superconductors, including vortex lattices and the pseudogap, and discuss the extent to which these phases (and their associated phase transitions) are universal, and to what extent they are determined by both intrinsic material properties and material doping.

# 1 Introduction

Since the discovery of the cuprate superconductors in 1986, these materials have been of great interest to both condensed matter theorists and experimentalists. In addition to the manifold practical possible applications, the cuprates exist at a sort of material inflection point: the tuning of any of a substantial number of parameters can result in a completely different state, with measurably different parameters. This underlying theoretical richness indicates that the cuprates are an excellent test case for many methods and hypotheses in condensed matter physics.

In this paper, we will discuss the known phases of the cuprates near the superconducting point, along with their properties, possible theoretical explanations, and experimental exploration and verification of their behaviors. We will look at the effects of magnetic fields, temperature, disorder, and doping level on the cuprates, and discuss the various vortex phases, the traditional "true" superconducting phase, and the pseudogap phase, along with their implications for the underlying physics. We will begin by considering the optimally doped cuprates and the subphases of superconducting behavior, and then move on to the changes in behavior when the cuprates are under- or over-doped.

## 2 Optimally Doped Cuprates

### 2.1 Factors influencing Type II Behavior

In the case of optimally-doped cuprates, we are mostly interested in the roles of temperature and magnetization with regard to magnetic phase, though disorder also plays a role. Whereas a normal superconductor displays only normal and superconducting phases, the cuprates show one or more intermediate states as well. It is believed that these intermediate states result from the much smaller superconducting lengthscale,  $\xi$ , found in the cuprates. Among other things,  $\xi$  is roughly inversely proportional to the magnetic field (adjusted for temperature) associated with the transition between the normal and intermediate phase; if this field is not substantially larger than the field associated with the transition between superconducting and intermediate phases (inversely related to the magnetic penetration lengthscale  $\lambda$ ), there is in some sense no "room" in the phase diagram for the intermediate phase

to occur [1].

In the case that an intermediate phase does exist, the superconducting length scale is generally on the order of or smaller than the magnetic penetration depth (a ratio of  $\lambda/\xi \geq 1/\sqrt{2}$  is the typically cited cutoff)[1]. In this case, it becomes more energy efficient, in some sense, to allow the existence of small vortices without superconductivity to "channel" the magnetic field through the material, rather than to allow the more typical penetration. It should be noted that the cuprates, which contain layered sheets of varying composition, reflect the anisotropy of their build in their magnetic behavior; it is generally easier for magnetic fields to penetrate the cuprate in the direction perpendicular to the sheets than along the plane parallel to them, so smaller magnetic fields are required to induce a vortex state when perpendicular to the state than when parallel to it[1].

## 2.2 Vortex Lattice

The vortex phase displays several different subphases. The most easily theoretically studied of these is the vortex lattice, where in a perfectly ordered crystal, the vortices self-arrange into a perfect lattice. This actually leads to some counterintuitive behaviour: though the vortex phases are essentially superconducting phases, seeing as they do, in fact, "expel" magnetic field, the perfect vortex lattice does not display the most publicly-recognized trait of superconductors, in that it is not a zero-resistivity conductor. If the lattice is perfect and there are no defects which prevent its motion, a current can cause the entire lattice to move as a single body; since the lattice vortices "contain" the magnetic field, such motion requires energy and leads to dissipation. However, this is not actually a commonly-seen scenario: crystals perfect enough to attain such a state are not easily available[1].

## 2.3 Vortex Glass

The second subphase of the vortices is the vortex glass. In a typical sample, the presence of defects of any sort creates additional energy parameters in the problem. The vortices are now subject to more than just their own interaction forces; they must also deal with the energy landscape induced by the defects, and so the lowest-energy state is no longer a perfect lattice but a

glass that reflects both energy effects of the defects and the state of the system before "cooling" to a steady state. The defects now prevent the vortices from easily changing alignment, but were the system to be heated to above the transition temperature and then re-cooled, it would likely not return to exactly the same state - the precise alignment of the vortices is a fairly initial condition-sensitive problem. In any case, the effective "pinning" of the vortices by the defects means that the vortices can no longer slide and therefore cannot dissipate current energy, and thus the material should have the zero-resistivity effect commonly associated with superconductors. However, it is still possible that thermal fluctuations will provide enough energy for the vortices to follow the movement induced by the current. In this case, the resistivity should be proportional to  $e^{-U/K_B T}$ , where  $U$  is the energy scale associated with the "pinning"; for low temperatures,  $e^{-U/k_B T} \propto T$ , so we expect to see resistivity scaling linearly with temperature[1].

## 2.4 Vortex Liquid

The final vortex phase in the flux or vortex liquid. In this state, the magnetic field induces vortices but thermal fluctuations are large enough to prevent any static ordering. This state is more likely to be existent in systems that display vortex glass rather than lattice behavior, but since any amount of disorder breaks translational symmetry in the system, this likely applies to every experimental sample. This may or may not represent a true phase transition - if it is a second-order transition, it would agree with resistivity measurements found in some samples. In particular, a second order transition from a vortex glass would imply that near the crossover temperature,  $T_i$ , the superconducting lengthscale would scale as some power of  $T - T_i$ , implying that the resistivity would also scale as some (different) power of  $T - T_i$ [1]. Since the dependence of electric field on current should be temperature independent directly above  $T_i$ , we would expect that, to properly cancel factors,  $E \propto J^\sigma$  for some  $\sigma$  in this region, which agrees with experimental results of Koch et al[1, 8].

Whether or not a true phase transition exists, we can measure the "melting" point by looking for a change in the magnetic behavior - in the glass phase, magnetization will essentially be frozen in, but the liquid phase will have a more malleable magnetism. Thus in the glass phase, we will see a non-linear relation between electric field and current, leading to the familiar

hysteresis loop[1].

It is worth noting that, if a vortex lattice to vortex liquid transition does exist, it may occur not only between the normal and vortex lattice phases but also between the vortex lattice and superconducting phases, thus "smoothing" the transition, but this intermediate phase region would likely be too small to be detected. Additionally, the differences between the vortex lattice and vortex glass phases are strong enough that the vortex glass-vortex liquid and vortex lattice-vortex liquid transitions may or may not be related[1].

## 3 Under- and Over-doped Cuprates

### 3.1 Undoped Cuprates

In their native state, the cuprates are not actually superconductors; such behavior arises only when they are doped to allow sufficient holes or electrons as carriers (we consider only hole-doped states here, as they comprise the majority of the literature). For all cuprates there is an optimal level of doping, for which superconductivity persists to some maximum temperature. However, the properties of the cuprates outside that optimum regime are as or more interesting than their behavior in the optimum regime: the reasons why a certain level of doping maximizes superconducting behavior lies in the behavior on the limits.

Even the completely undoped behavior of the cuprates is interesting: in their natural state, they are antiferromagnets at sufficiently low temperatures, harboring a ground state that seems distinctly unfavorable from basic physical principles - spins generally minimize energy by aligning in the same direction. At higher doping levels, they show so-called "pseudogap" behavior both below and above optimal doping, where electrons associated with only certain planar directions see a range between energy bands that contains only a few allowed states. The behavior of the cuprates with increased doping and the characteristics of the pseudogap can be experimentally determined in several ways, and since the pseudogap region seems to be universal and thus possibly key to understanding the cuprates, it has been the subject of a great deal of work.

## 3.2 Pseudogap Universality

One of the first questions investigated with regards to pseudogap behavior was the question of universality: was there some aspect of pseudogap behavior universal to all cuprate superconductors? What were the relevant parameters?

So far, the answers to these questions have been mixed. Initial experimental work on the pseudogap was done largely through NMR and spin relaxation time measurements, with some additional work using inelastic electron scattering and other methods. By 1993, there was enough experimental data available to start drawing conclusions about the generalized behaviour of the cuprates. Sokol and Pines used relaxation time measurements to find evidence for a generic "quantum critical regime" that maintained many properties of the underlying antiferromagnet[2]. Barzykin and Pines similarly used NMR experiments to provide evidence of a transition to a universal regime describing the pseudogap phase where the transition temperature, superconducting lengthscale and transition out of the universal regime are dependent only on each other or the doping level[3].

## 3.3 Further Discussion on Over and Under-doping

As mentioned above, undoped suprates are antiferromagnetic below some critical temperature  $T_A$ . If the system is doped with only a few holes, the holes remain localized, and develop a ferromagnetic interaction[4]. This introduces frustration to the system, resulting in a spin glass and a lowering of the critical temperature for antiferromagnetism.

For all the evidence that the underdoped pseudogap displays universal properties, the status of the pseudogap induced by overdoping is far less settled. The work of Ando et al [7] uses resistivity (specifically, the second derivative of resistivity with respect to temperature) to characterize the pseudogap, and while their results show strong evidence of universality below the critical doping point, they see no such behavior above the critical doping point. Additionally, their "characteristic" pseudogap measurement, a pattern of first decreasing and then increasing resistivity derivatives at temperatures above the superconducting regime, is not present for the expected pseudogap regime above critical doping. This provides some evidence that the pseudogap is the result of distinctly different mechanisms above and below the critical doping

point, a hypothesis with a fair bit of support. Even some of those hypotheses which suggest a shared mechanism include a description of why the character of the pseudogap varies above and below the critical point; Chakravarty et. al.'s discussion of D-density wave order as an additional order parameter influencing the pseudogap, for instance, includes a mention of order parameter competition that leads to different pseudogap character above and below the critical point[6].

There is still active experimental work in this area. In 2006, the first evidence of a distinct magnetic arrangement in the pseudophase was established by Fauque et. al., using polarized elastic neutron diffraction[5]. Their work in a variety of temperature and doping levels indicated this was a generic property of at least the cuprate under investigation, YBCO, and the universality of most pseudogap properties indicates this is likely a universal property of the cuprates as well. Their work provided some of the first experimental support for an additional order parameter governing the pseudogap; further work in similar directions may help firm up this identification and verify the precise nature of the aforementioned order parameter.

## 4 Conclusions

At the time of their discovery in 1986, the cuprates were an entirely unique material, displaying unique behaviors and transitions. Though the discovery of other high temperature superconductors has removed some of their exceptionality, they remain the most studied of the high temperature superconductors and thus the most likely to yield valuable insights about the whole of high temperature superconductivity. In the past 24 years, a great deal has been learned about these materials: there is a fairly solid understanding of the vortex phases of superconductors, and the universality of the underdoped pseudogap has been at least tentatively established. Still, the amount we do know about the cuprates is not nearly so complete as we would like, and many seemingly strong hypotheses have not been as solidly proven as we would like. We still do not have a solid doping-temperature phase diagram that adequately depicts all universal properties and indicates those transitions which are system-dependent, and a single experimentally-confirmed model that explains all aspects along the temperature, doping and magnetic field axes (or an explanation of why such a model is impossible) remains out of reach. In the near future, one major goal could be an

experimentally-verified description of the overdoped pseudogap and explicit list of the system parameters affecting this regime.

There has been a great deal of progress in understanding the physics of these systems which, in what seems like nature's ironic joke, demonstrate both extraordinarily "smooth" (superconducting) and extraordinarily repulsive (antiferromagnetic) behavior. On the whole, the study of the magnetic phases of these materials has been one of the successes of the past quarter-century of condensed matter physics, and remains a very promising arena for the next quarter century as well.

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