Power law behavior in designed and natural complex systems: self-organized criticality versus highly optimized tolerance

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Abstract

Power law scalings are abundant in natural and man-made complex systems, but the presence of a power law in itself is insufficient to specify the mechanism that generates it. It would be particularly valuable to understand the origin of the power law distribution of electrical blackout size versus frequency in national high-voltage power grids, in order to better moderate blackouts. The concept of self-organized criticality (SOC), inspired by the critical dynamics of phase transitions, underlies many cascade models for power grid failures; however, the distinct "highly optimized tolerance" (HOT) mechanism can also replicate empirical statistics. This essay surveys recent SOC and HOT models to compare and contrast the two mechanisms in the context of the power grid system.

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1 Motivation: power grid blackouts

Depending on its timing, extent, and duration, an electrical blackout can span the spectrum from minor nuisance or costly delay to severe emergency. The cause of a particular blackout is often mundane: a tree branch breaks a transmission line, and the local power grid cannot maintain or reroute power to some region until repairs are made. Large weather-related blackouts occur in the US on a regular, almost seasonal basis; one recent example is the so-called Halloween snowstorm of 2011 in New England, where snow took down leafy branches and left some 3 million people without power for days. Yet some of the largest recorded blackouts have far less obvious origins: a cascade of failures emenating from a small region in northern Ohio led to the August 2003 blackout that took out 62 GW over several states for 50 million people.[14] The details of the long, complicated cascade of events from that localized failure to a blackout of the entire US Northeast are documented in a Federal report over 200 pages in length.[15]

The high-voltage component of the US power grid alone is an enormous, complicated network of transmission lines connecting diverse nodes including several types of generators and transformers, each with their own nontrivial internal dynamics, interactions, and highly nonlinear behavior (human operators, circuit breakers, outright failure). A physicist's first approximation to such a many-body problem might involve a picture of a randomized network of identical nodes with linear interactions; as the compound pendulum demonstrates, even this simplification leads to chaotic behavior in response to small perturbations. An independent-node theory of such a model might predict that the frequency of blackouts falls off binomially with blackout size; in reality, observed blackouts in power grids worldwide demonstrate power law scaling with size (Fig. 1). The empirical distribution of blackouts has a "fat tail," and blackouts on a scale comparable to the system size, like the August 2003 blackout, can and do occur with non-negligible frequency, even in response to small perturbations. For a complex system like the US power grid, the question of why a particular cascade of failures started gives little insight into the nature of cascades: why they obey power law statistics, and hence how their frequency or severity might be reduced.

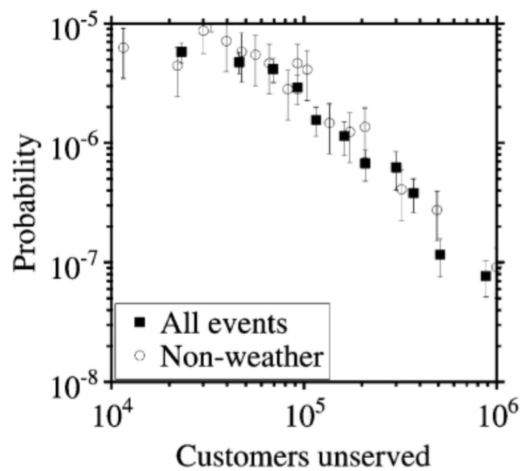

Fig. 1. Log–log plot of PDF of the number of customers unserved comparing the total data set with the data excluding the weather related events.

Figure 1: Extracted with caption from [7]; indicates blackout power law scaling remains after controlling for weather-based accidents.

1.1 Power law behavior by nature and design

Apparent power law scaling is pervasive in nature: it is found in coastlines, earthquakes, rainfall, forest fires, neural networks, and many other areas. In man-made structures, we find power laws in Internet file sizes, data compression techniques, city growth rates, and power grid blackout sizes.

A special feature of such systems is that they are scale invariant; given a power law relationship $f(x) = ax^k$, rescaling $x \to cx$ has only a proportional effect on $f(x)$: $f(cx) \to ac^k x^k \propto f(x)$. The system "looks" (via the function $f(x)$) identically distributed regardless of the length scale x examined, so long as x is within the domain in which $f(x)$ behaves as a power law (e.g., for a finite system of size L, $x \leq L$). A trivial integer power law relation might be $V \propto L^3$; fractal patterns host striking examples of power laws with non-integer exponents.

In physics, power law correlations generally accompany thermodynamic phase transitions, be it a solid to liquid or or paramagnetic to ferromagnetic. These transitions are described by sets of critical exponents; different transitions with the same critical exponents, such as the two transitions just noted, fundamentally obey the same dynamics and are said to belong to the same universality class.

From our experience with power law scaling at critical points in thermodynamic systems, it is tempting to view power law behavior in certain natural and manmade events as signs of criticality, signs that complex systems are balanced on the edge of a phase transition between simple and chaotic dynamics, or relative stability and system-wide cascades. Given the complexity, noise, and myriad initial conditions inherent in each of these systems - e.g., continental plate shapes for earthquakes, or forest types for fires - it seems as if the analogous "critical point" also comprises an attracting set, or dynamic equilibrium, such that the system exhibits universal dynamics with a consistent critical exponent under a wide variety of conditions. This concept, known as "self-organized criticality" (SOC) or "edge of chaos" (EOC) dynamics, is a central theme of much recent research in complex systems and nonlinear dynamics. Studies and simulations of the power grid from the SOC perspective have been made by Dobson, Carreras, Lynch, and Newman,[6, 8, 7, 5, 4, 9, 11] among others.[20, 18, 23, 16, 12, 17] The sandpile toy model of SOC, along with a more sophisticated power grid cascade simulation, is presented in Section 2 of this review.

SOC is not, however, the only mechanism that might explain power law scaling in nature and manmade structures. The ubiquity of power laws may be partly due to the many ways by which they may be constructed, via a variety of simple, non-critical dynamics. Taking an example from [2], suppose we are given an exponentially distributed random variable X: $\mathbb{P}\{X = x\} = \lambda e^{\lambda x}$. Now consider an exponentially distributed function of X: $Y(X) = y_0 e^{\alpha X}$. It follows that

$$\mathbb{P}\{Y = y\} = \mathbb{P}\{X = \alpha^{-1}\ln(y/y_0)\} = \lambda (e^{\ln(y/y_0)})^{\lambda/\alpha} = \underline{\lambda \left(\frac{y}{y_0}\right)^{\lambda/\alpha}}.$$

Any exponential process whose argument X is in turn exponential, or an exponential interaction between exponential variables, is liable to yield a power law. There also exist several other means by which statistics can conspire to produce a power law in the absence of criticality.[Scholarpedia] For a heterogeneous, strongly interacting power grid with many interacting linear subcomponents, SOC may not be the best explanation.

The leading alternate mechanism put forth to explain power law scaling in certain complex systems is known as highly optimized tolerance (HOT). According to Carlson and Doyle, who coined the term,

> "Unlike SOC or EOC, where the external forces serve only to initiate events, and the mechanism which gives rise to complexity is essentially self-contained, our mechanism takes into account the fact that designs are developed and biological systems evolve in a manner which rewards

successful strategies subject to a specific form of external stimulus. In our case uncertainty plays the pivotal role in generating a broad distribution of outcomes. We somewhat whimsicially refer to our mechanism as *highly optimized tolerance* (HOT), a terminology intended to describe systems which are designed for high performance in an uncertain environment."[3]

At first glance, the distinction between HOT and SOC may be unclear. In SOC, the dynamics generally contain two competing forces: one gradually maximizing an order parameter, and another that switches on once the order parameter passes the EOC, tending to drive the order parameter back. The order parameter can be seen as something that is extremized under random perturbations, monotonically increasing up to the cliff-edge of chaos. In HOT, as discussed in Section 3, external design based on specific knowledge of risk seems to replace dynamical feedback, but the concept of optimizing some parameter under uncertainty remains. The goal of this literature review is to present and evaluate SOC and HOT with respect to understanding power grid complexity, somewhat more broadly than is done in [3], but in more detail than [14].

2 Sand pile model

In 1988 Bak, Tang, and Wiesenfeld introduced the prototypical model of self-organized criticality, known as the BTW sand pile in 1 to 3 dimensions.[1] The basic sandpile in two dimensions consists of an integer grid, where each point is randomly assigned an initial $h \in [0,4]$ grains of sand. At each time step in the simulation, all points (x,y) for which $h(x,y) = 4$ collapse, depositing sand on its nearest neighbors: $h(x,y) \to h(x,y) - 4$, $h(x\pm 1,y) \to h(x\pm 1,y) + 1$, and $h(x,y\pm 1) \to h(x,y\pm 1) + 1$.

This rule is iterated until all sites have height $h < 4$. At the beginning of the next time step, each point on the grid receives an additional grain of sand with probability $P = P(x,y)$. The grid has open boundary conditions so that sand may exit the system. Alternately, a single grain may be added at each time step, with a location determined uniformly or according to some PDF. Note that this is a simplified version of the original BTW model, in that the absolute height at (x,y), rather than local slope, is the determining factor in avalanches. the resulting dynamic equilibrium after a number of time steps exhibits self-similarity in space and time, notably in the fractal shapes of the regions experiencing an avalanche in a given time step.

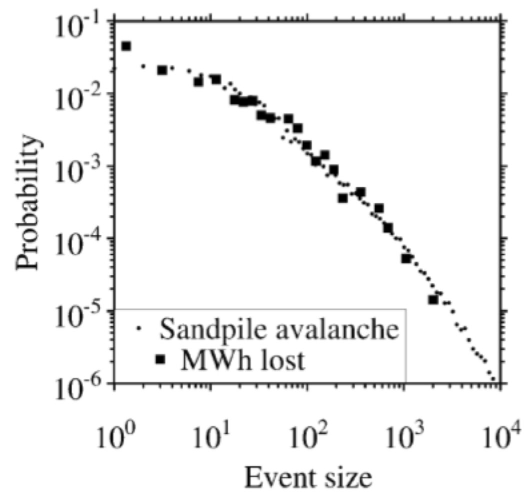

Figure 2: "Rescaled PDF of energy unserved during blackouts superimposed on the PDF of the avalanche size in the running sandpile"; from [7].

In [7], the authors compare the statistics of a particular sand pile model of the above form with $P = P_0$ and size $L = 800$ (the authors do not make the dimension clear, even in the referenced paper [19]; it is most likely a two-dimensional model). They choose P_0 to match the average frequency of outages in the US power grid, and the avalanche size statistics are gathered once the simulation reaches dynamic equilibrium. They then rescale the simulated sandpile and empirical power grid probability density functions according to

$$P(X) = \lambda F\left(\frac{X}{\lambda}\right),$$

where X is the dimensionful event size, λ is a rescaling parameter, and $F(X/\lambda)$ is the dimensionless universal function. In the power law regions of the PDFs, λ would only shift the PDFs vertically on a log-log plot; the rescaled results are shown in Fig. 2. The correspondence is good, and the authors claim that other statistical properties and measures of blackout sizes also match well with the sand pile statistics; that said, not all SOC toy models fit real-world data so precisely,[13] and the authors are sufficiently vague that this reader is inclined to try to determine how replicable and nontrivial is this correspondence.

The analogy between sand piles and power grids made in [7] might be paraphrased thusly. The grid exists as a network of nodes and transmission lines that deliver and support certain loadings (points on a grid with N grains of sand). Over time, the demand on the grid increases gradually (sand is added to the grid); at some point a component is overloaded (the sand height at a point reaches a critical value), and distributes its load to its nearest neighbors. In a real power grid, the overload or blackout cascade takes places in less than a day, whereas relaxation (the reduction of the ground zero sandpile to zero height and its ability to accept loadings from other avalanches) is not instantaneous, and can take several days. In this sense, and given the ability of the sand pile simulation to replicate power grid statistics, a power grid might be in a self-organized critical state.

2.1 The OPA model and blackout mitigation

The sand pile toy model in its simplicity is not presumed to represent actual power grid dynamics. One prominent SOC-based cascade model for power grid failure propagation is known as the OPA model. The acronym represents the three institutions involved in developing it: Oak Ridge, PSERC, and University of Alaska. As described in [10], OPA works from a standard fixed IEEE test network of transmission lines, loads, and generators; after solving a base case under some simplifying approximations for the power flow in the system (DC flow, linearized, no losses, uniform voltages), random line outages are inflicted on the network. With each outage, load is redistributed via linear programming methods (optimization, essentially) that seek to minimize “load-shedding,” that is reducing the total load applied by the generators on the network. If a line is overloaded, it has some probability of outaging. The total load shed represents the power lost or size of the blackout. OPA might be viewed as the next step up from a sandpile model, with rudimentary circuit behavior, a fixed non-uniform network, and distinct lines and nodes.

Qualitatively, at very low loading, very few initial line outages will cascade into a larger blackout in OPA and similar models; these models will then demonstrate blackout sizes roughly proportional to the number of initial outages, and so have an exponential tail for large blackouts. Beyond a critical loading, the network is overloaded and a single outage is liable to cascade through a large fraction or the entirety of the network (Fig. 3, left). More generally, these types of models are iterated in time such that after each

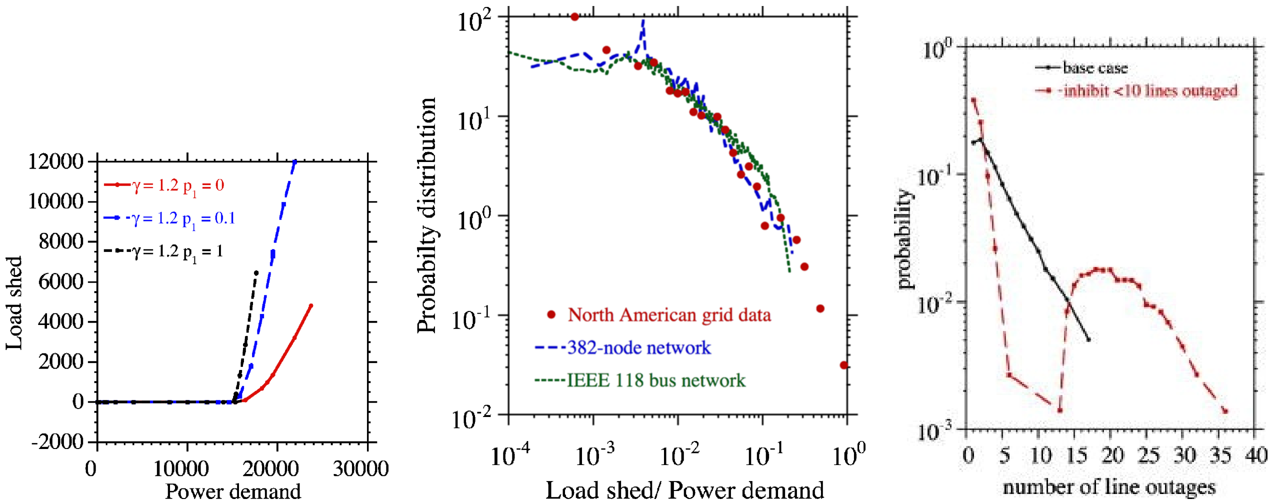

Figure 3: From left to right: demonstration of critical loading, comparison of blackout size to North America grid data, and the simulated effect of small-blackout mitigation efforts. All simulations are with the OPA model; on the right, the simulation is on the IEEE 118-bus test network. Figures from [10].

blackout cascade the outaged components are rebuilt with higher load capability, even as the overall load increases with consumer demand (e.g., the CASCADE model in [10]). With these slow dynamics overlaid on the fast cascading failure dynamics, this class of models settle in equilibrium near the critical point, and the OPA model in particular shows power law scaling consistent with North American power grid data (Fig. 3, center).

One feature of the OPA model is that, if one simulates engineering measures to reduce the frequency of small blackouts by inhibiting all line outages if the number of overloaded lines is sufficiently small, the frequency of larger blackouts can increase dramatically (Fig. 3, right). The intuitive picture is that of a forest with overzealous firefighters: small fires are prevented, which increases tree and brush density, which makes the forest sensitive to larger fires.[6] Another interesting result from a different cascade model on a standard IEEE test network suggests that the degree to which larger blackouts obey power law statistics, versus an exponential tail, is related to network entropy: simulations on a more randomly connected network exhibit a weaker power law tail.[18]

3 Highly optimized tolerance (HOT)

Though SOC paints an elegant, physicist's picture of power grid dynamics, it is not universally accepted. As part of their motivation for developing the HOT mechanism, Carlson and Doyle argue (as of 1999) that:

> ... while power laws are pervasive in complex interconnected systems, criticality is not the only possible origin of power law distributions. Furthermore, there is little, if any, compelling evidence which supports other aspects of this picture. In engineering and biology, complex

systems are almost always intrinsically *complicated*, and involve a great deal of built in or evolved structure and redundancy in order to make them behave in a reasonably predictable fashion in spite of uncertainties in their environments. Domain experts in areas such as biology and epidemiology, aeronautical and automotive design, forestry and environmental studies, the Internet, traffic, and power systems, tend to reject the concept of universality, and instead favor descriptions in which the detailed structure and external conditions are key factors in determining the performance and reliability of their systems. The complexity in designed systems often leads to apparently simple, predictable, robust behavior. As a result, designed complexity becomes increasingly hidden, so that its role in determining the sensitivities of the system tends to be underestimated by nonexperts, even those scientifically trained.[3]

To support to their qualitative arguments against the relevance of SOC models to engineered and evolved complex systems, Carlson and Doyle introduce a class of optimization problems, termed "probability-loss-resource (PLR)" problems, as simple representations of the HOT mechanism (defined in Section 1).[13] Essentially, the PLR problem is to minimize an expected cost function J subject to resource constraints:

$$J = \left\{ \sum p_i l_i | l_i = f(r_i),\ \sum r_i \leq R \right\},$$

where i indexes a set of blackout or loss events, the probabilities p_i are known from particular expertise and knowledge of system structure or weak points, r_i represents the resources applied to reduce the size of the loss l_i, and $f(r_i)$ is given a simple form:

$$f_\beta(r_i) = \begin{cases} -c\log(r_i), & \beta = 0; \\ \dfrac{c}{\beta}(r_i^{-\beta} - 1), & \beta > 0. \end{cases}$$

Here β is a measure of how the marginal reduction in loss scales with applied resources r_i, as well as the dimensionality of the system; this relationship is the root of the power law behavior Carlson and Doyle extract from the PLR model. That said, the authors make dimensional arguments in support of this functional form and particular choices of β; β is not considered a totally free parameter in fitting. With judicious choice of β, they find the PLR optimization model consistent with a variety of power law phenomena (Fig. 4).

One key feature of HOT, as represented in the simple PLR model, that distinguishes it from SOC is the dependence of power law exponent on dimensionality. In equilibrium critical phenomena, large collective fluctuations are reduced as dimension increases; for example, the

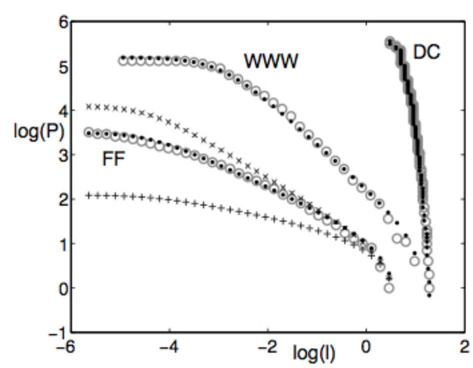

FIG. 1. Log-log (base 10) comparison of DC, WWW, and FF data (gray circles) with the results of the PLR problem (black dots) for $\beta = 0$, 1, and 2, respectively. Results for an SOC FF model with $\alpha = 1/\beta = 0.15$ (+) and an inaccurate PLR FF fit with $\beta = 3/2$ (×) are included for comparison. The cumulative distributions of frequencies $P(l \geq l_i)$ vs l_i describe the areas burned in 4284 fires from 1986–1995 on all of the United States Fish and Wildlife Service lands (FF), 130 000 web file transfers to 591 users on 37 machines at Boston University during 1994–1995 (WWW), and code words from data compression (DC). Both the size units [1000 km^2 (FF), 2 megabytes (WWW), and bytes (DC)] and the logarithmic decimation of the WWW and FF data are chosen purely for convenient visualization.

Figure 4: Figure extracted from [13].

long-wavelength phonon instability predicted for crystals of dimension $d \leq 2$ vanishes for $d > 2$. Under the HOT mechanism, increasing system dimensionality implies a greater probability of large-scale events or fluctuations, whereas SOC, insofar as it represents equilibrium critical phenomena, would predict the opposite.[13]

3.1 Sandpile with design

The three power law systems just described - data compression, the early WWW, and forest fire events - aren't ideal for comparing SOC against HOT; the first two lack an intuitive SOC explanation, and the latter, in the absence of firefighters, relies on the concept of natural selection to justify HOT. Strong intuition for both SOC and HOT exist in the sandpile model, as it relates more directly to the man-made electrical power grid.

To make a HOT sandpile, one introduces the element of design[3]: given a known PDF for where new sand grains will be deposited, what is the optimal initial arrangement of sand that both maximizes the amount of sand in the system, and minimizes the impact of (number of sand grains displaced by) avalanches? The impact of avalanches is extracted from ensemble averaging over the initial designed state, rather than monitoring the dynamic equilibrium in SOC. In HOT, the resulting pattern of sand is intuitive: a region at high risk of receiving new sand grains is enclosed by an avalanche enclosure, a simple closed loop of grid points with sand height $h \leq 2$, so that no single avalanche can propagate beyond it. If the sand input distribution is uniform, as in simple SOC models, then the optimal HOT structure in 2D is a uniform grid of barriers. The result and some of the author's commentary is given in Fig. 5. The authors also note that in HOT, unlike SOC, the configuration is highly structured and not at all scale invariant. The yield (or alternately, avalanche size) is not a function of sand density so much as the known risk distribution and nature of

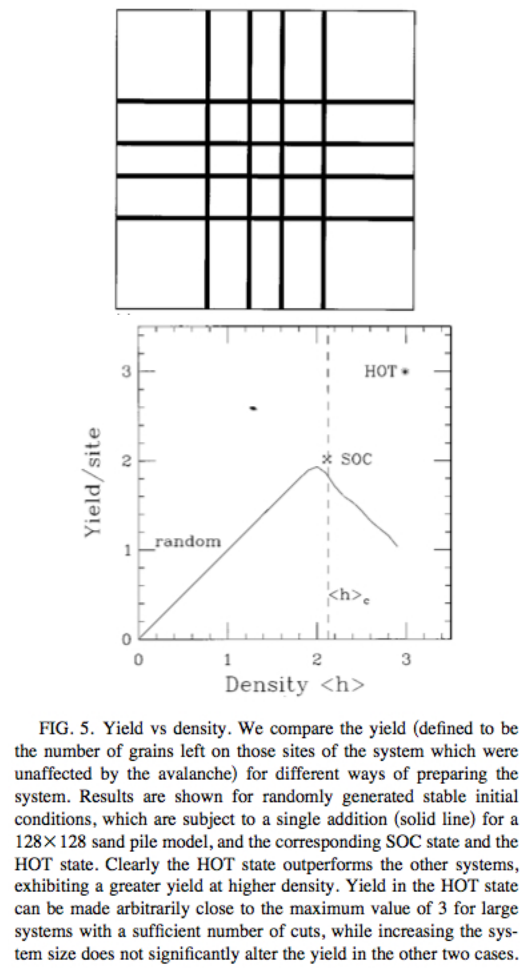

Figure 5: Top: structured HOT pattern on 64x64 lattice for centralized Gaussian distribution of sand hitting probabilities; white pixels represent 3 units of sand, black 0. Bottom: figure with caption extracted from [3].

avalanche breakers, both specific features of a given complex system. There is also no concept of a critical point; in fact, both optimal and slightly suboptimal HOT configurations in a percolation toy model exhibited power law statistics.[3] Intuitively, the HOT sandpile shows power law scaling in absence of criticality because it is a feature of the optimization process: while an avalanche is by assumption less likely to hit an unprotected region, its impact is greater when it does.

3.2 HOT vs. SOC

The HOT sandpile is able to surpass the yield of a SOC sandpile by virtue of one strong assumption: reliable knowledge of the distribution of sand hitting probabilities on the lattice. In a power grid, this would translate to understanding which components are most likely to fail, and which failure or accident scenarios are most likely. This strength is also the greatest weakness of a HOT system: it is hypersensitive to changes in the hitting probability distribution, in other words, to a flawed design or unanticipated accident. Avalanche barriers centered around a central area are counterproductive if significant amounts of sand falls near the lattice edges.

If HOT does give insight into power grid dynamics, the lesson may be that power law scaling in blackouts is a sign of an attempt to optimize certain system features under anticipated uncertainties with limited resources, rather than an emergent property from competing growth and cascade dynamics. A HOT model specific to the power grid, analogous to OPA and similar SOC models, is developed in [21] with apparent success.

The SOC and HOT mechanisms are not mutually exclusive in a particular model system. For a HOT design element imposed on an otherwise SOC sandpile model, allowing time evolution and decay of structure can drive the system back to the dynamic equilibrium near the edge of chaos, albeit after a long transient.[3] Nor is HOT necessarily optimal over SOC, as the latter does not share the former's weakness to unexpected events.

4 Discussion

The distinction between SOC and HOT mechanisms is subtle, yet radical. Both seem to offer the same answer to the non-negligible risk of catastrophic blackouts: they are inevitable or a design feature, respectively. With such outlooks, blackout management, not prevention, may be the most rewarding investment.[22]

Yet to the extent that we do not fully grasp natural or manmade complex systems, the choice can seem somewhat philosophical. SOC is an optimization, or at least stabilization, by feedback within the model, including normally "external" factors like human investment in transmission and generator growth in response to blackouts. The critical dynamic is an emergent phenomenon, and long-term changes to the grid are reflexive and driven by the inexorable economic forces of increasing demand and short-term, decentralized repairs and improvements. HOT represents top-down optimization, a systematic design that pivots on specific details of the power grid.

To reiterate, the pragmatic significance in interpreting a complex system in terms of SOC or HOT reduces to the need for computationally efficient (even feasible) models to isolate the system's essential behavior, with an eye toward better predicting or controlling it.

To this end, the real question isn't which type of toy or intermediate model produces the best or most robust fit to a given power law distribution; as we've seen, both HOT and SOC models can yield very close fits to the same power grid blackout size vs. frequency data sets. Moreover, both HOT and SOC have intuitive appeal, depending on one's perspective on how a power grid is designed and improved. In order to decide between HOT or SOC, our best tools may come from ongoing research into the statistical nature of SOC beyond power law scaling. For example, the original BTW sandpile model also predicts 1/f noise in the temporal spacing of events. However, the power grid blackout data is apparently still too sparse, and covers too short a time span (~15 years), to be definitive on that point, though other temporal correlations have been studied.[7]

Alongside the ubiquity and stubborn obscurity of observed power laws, we may also note with interest the variety of our attitudes toward them. In evolution, the HOT mechanism describes power laws as optimal adaptations, in a sense; in nature, we find beauty in spatial scale invariance and fractals; in financial markets, we feel wary or opportunistic. Yet in power transmission networks, we would like to suppress power law behavior completely.

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