Exotic phases of superfluid Helium 3

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Abstract

In this term paper I describe the experimental data which led to the discovery of superfluidity in Helium 3 and then discuss mostly the Leggett's theory of the exotic phases in ^3He and how his theory justified the experimental data of nuclear magnetic resonance in ^3He. I will discuss that how he used the concept of the spontaneous symmetry breaking and also spin dynamics of the system to find the spin state of the Helium atoms in two different superfluid phases. I will briefly explain the rival theory introduced by Anderson and Brinkman which could do the same job as Legget's did and mention some of the advantages, applications and usefulness of superfluid ^3He.

1 Introduction and Background

In composing this term paper, I decided to be as conservative as possible and follow the term paper instruction almost exactly as given. The reason was that I used a regular paper style for writing my term paper last semester in Phase Transition course and I got a grade for my term paper that I did not expect. Therefore, I change the format for this time to the given format and first of all try to answer the questions given in the instruction. I won't go beyond answering the questions until I get to the last one and if I find some space at that point within the 12-page limit, I start to add other parts. Thus, the structure of this term paper is as following: There are three chapters "Introduction and background", "Methods", and "Results and discussion". In each, I write the given questions and then answer them. Let us start with the first question.

1.1 What hypotheses are being tested in this paper?

In this paper I am going to talk briefly about the experimental data of a group at Cornell at 1972 which led to the discovery of superfluidity in ^3He. The group consisted of D. Osheroff, D. Lee and B. Richardson. The hypotheses that are being tested in this paper is the spin dynamics theory and spontaneous broken spin-orbital symmetry idea introduced by Anthony Leggett at 1972-1973 to justify the nuclear magnetic resonance (NMR) data of the Cornell group who observed anomalies in the aforementioned data [1, 2, 3, 5]

What the Cornell group observed was that in the pressure versus time curves of the mixture of solid and liquid ^3He at low temperatures around 2.6 mK and 2 mK there are two anomalies which are seen in compressional cooling or decompressional warming at almost, in one case, or precisely, in the other case, the same pressures and temperatures (See Figures 1, 2, and 3)[2,1].

They well recognized that the first anomaly at about 2.6 mK as a second-order phase transition and the one at about 2 mK as a first order transition. However, what they were wrong about was the component which bears the transition in the mixture. They thought that the transition occurs in the solid ^3He [4]. Next by applying magnetic field to the ^3He system they did nuclear magnetic resonance experiment [6]. Their finding was in contradiction to their previous interpretation of the origin of the phase transitions. The conclusion was that the transition occurs in the liquid not in the solid. This was the moment when Anthony Leggett did his significant contribution and tried to solve the problem theoretically and his work is the hypotheses that are being tested and discussed in this paper. He started with using the sum rules for susceptibility and then using the spontaneous symmetry breaking approach to

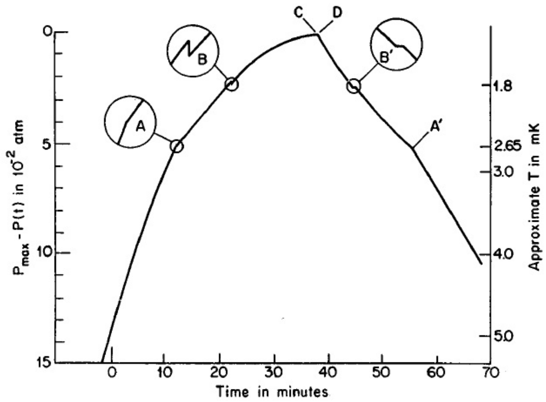

Figure 1: Pressure vs. time curve. In cooling and warming of the solid-liquid mixture two features were observed. A and A' occur at the same pressure and temperature. Their feature was the change of the slop. The pressure at B is greater than that at B' and their feature was a sudden pressure drop and a small plateau respectively. (Courtesy of D. Lee)

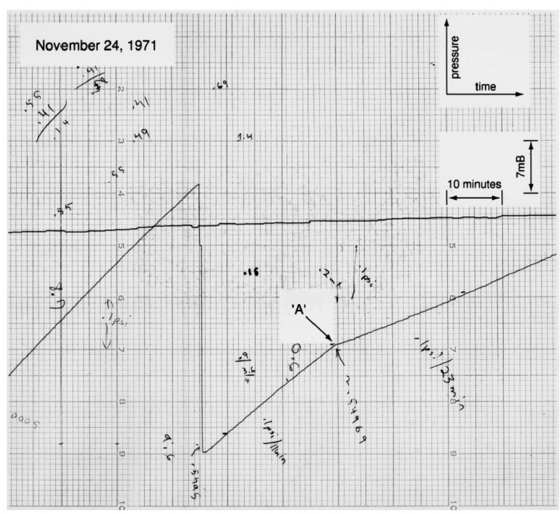

Figure 2: The original P-t plot taken in Nov 24, 1971 by Osheroff. The phase transtion A is observed in this plot. (Courtesy of D. Osheroff)

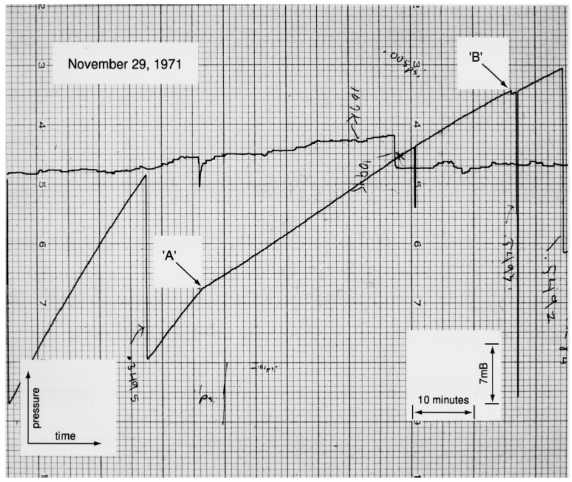

Figure 3: The original P-t plot taken in Nov 29, 1971 by the Cornell group. The phase transtion B is also observed. (Courtesy of D. Osheroff)

find the correct form of the resonance frequency in phase A and phase B. He later modified his work by thinking about the actual microscopic physics of the system and bringing up the spin dynamics of the Heliium atoms to find the correct states of the pairs in the two superfluid phases and also to adjust the justification of the frequency shifts and transitions (on which much below) [3,5].

1.2 What information induced the authors to perform the experiments or theory?

The experiment I mentioned above measured the radio frequency power absorption as a function of the frequency ω of the radio frequency magnetic field. In the normal phase of a liquid like ^3He the profile of absorption versus frequency have a sharp resonance at the Larmor frequency $w_{res} = \gamma H_{ext}$ where γ is the gyromagnetic ratio of the free ^3He atoms, and H_{ext} is the dc component of the external magnetic field. That was normal and observed above the transitions. However, between transition at 2.6 mK and 2 mK , which now called phase A, the resonance frequency is shifted from the Larmor frequency by an amount which is temperature dependent. Amazingly, below the second transition, which is called phase B, things apparently goes back to normal and the resonance frequency is again the Larmor frequency. While in the normal phase of liquid ^3He (N phase) and B phase the resonance frequency is proportional to the external radio frequency field, in A phase it obeys a Pythagorean law

$$\omega_{res}^2 = \gamma^2 H_{ext}^2 + \omega_0^2(T) \tag{1}$$

where

$$\omega_0^2(T) = A(1 - T/T_A). \tag{2}$$

T_A is the transition temperature to A phase from N phase and

$$A/(2\pi)^2 = 5 \times 10^{10}\ Hz^2 \tag{3}$$

This finding and, based on that, calculating the dc susceptibility which drops by about 50% at $A-B$ transition point indicated that the transitions cannot occur in the solid and must be a feature of the liquid ^3He [1,2,3,5] (see Figures 4, 5).

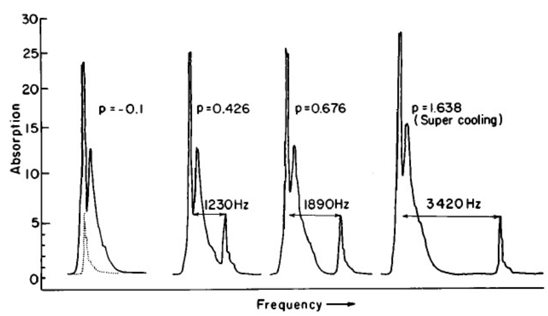

Figure 4: The sequences of nuclear magnetic resonance as the liquid cools down below the A transition and the phase shifts becomes distinguished and increases. (Courtesy of D. Lee)

1.3 What new methods or insights brought to bear on the problem?

In the first part, Leggett used the conventional method of using sum rules to compute the susceptibility. That was not a new method. However, he figured out that in the A phase a spontaneous symmetry breaking happens between the spin and orbital coupling of the system. This was a new fact but still not a new method. But what one can say is a new method used by him in this work goes back to the last part of the work where he came up with a better and complete explanation of the spin dynamics of the helium atoms. In that

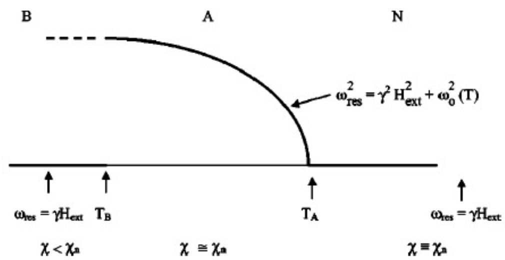

Figure 5: The behaviour of w_{res} and static susceptibility χ below 3 mK in liquid Helium 3. (Courtesy of A. Leggett)

part he generalized a method he him self invented in 1966 [7]. The idea is that for a two-band superconductor one can introduce an internal Josephson effect between the bands. Then one can define the number difference and phase difference between the macroscopic states of the Cooper pairs of the two bands as two conjugate operators and write the Hamiltonian of the system in terms of those canonical variables and solve the problem . This was the new method that he invented for two-band superconductors and later generalized it for the liquid ^3He to find its character. The two conjugate operators in the latter are the total spin vector of the system $\mathbf{S}$ (not just of the Cooper pairs!) and angle of rotation of the spin coordinates around the axis of the filed $\boldsymbol{\theta}$.

1.4 Why did you chose to write about this topic?

I needed to pick a topic for my term paper. I went to the instruction email and fortunately among the suggested topics found "Superfluid phenomena in Helium 3" under the category of "Exotic Superfluidity". That was very exciting to me because I wanted to find some time and sit down and read the previous work of my adviser, Professor Anthony Leggett, which led him to his Nobel prize and this was the best opportunity for me. I could read his work and learn it and also write my term paper for the course that my adviser asked me to take.

Saying that, I should confess that there was a deeper root for my interest in reading his work. I wanted to see how a Nobel prize winner work looks like. How deep should it be ? and what kind of problems should it solve ? Should it be a practical problem which has some benefits to the general public directly or at least in near future ? or can that be a pure abstract work which is not interesting to anyone but physicists. By reading the papers I think I got the answers to my questions.

Finally, I look at the world as human plus the rest. That makes me particularly interested in humans. So when someone makes a significant contribution to science I get interested to know him. Not only to know his works but also to know his character, his moralities, his others ideas, how he lived and how he views the world. To me, professor Leggett is one of those humans and I wanted to see how he viewed the world 40 years ago and how he views it now. To some extend when I read the part of his papers where he mentioned that in University of Sussex after passing his pos doc in UIUC he decided to abandon the conventional physics and devote his life to the foundational studies, I felt that I found the path and the glitches that also happened to me and made me to be interested in foundations of Quantum mechanics and to work only on that.

1.5 Why is this interesting or important?

The first importance of this work was that it led to discovery of superfluidity in ^3He. The Cornell group discovered the amazing phenomena in ^3He but it took theorists to justify that as a superfluid phenomenon. Leggett says that I find this question embarrassing that what is superfluid ^3He good for ? when I am asked by journalists and others particularly when standing next to Paul Lauterbur, a UIUC chemist whose research made MRI possible and obviously had a direct benefit for mankind [5]. The superfluid ^3He does not have any direct and practical usage. But fortunately, it does have some indirect usages and importances.

First of all, the superfluid phases of ^3He are the most complicated physical system that the physicists can claim they have understood. Second of all, the physics of the exotic superfluidity which learnt in the study of superfluidity in ^3He can be used elsewhere and it has actually been used, for example in physics of cuprate superconductors, in particle physics and also cosmology of the early universe, in studies of chaos and turbulence and specially in studying the topological defects in the order parameter generated in quenching through phase transition [5].

There is still a more interesting phenomena to come with superfluid phases of ^3He. Superfluid ^3He like BCS superconductors can be used as an amplifier. That is because of the existence of spontaneous broken spin-orbital symmetry (hereafter SBSOS) there exists a phenomena called superfluid amplification in the superfluid of ^3He. In a normal fluid a tiny effect will be destroyed and quenched by thermal disorder however in superfluid ^3He because all the Cooper pairs should behave identically, the tiny effects may be visible. The advantage of supefliud ^3He over the BCS superconductors and Josephson junctions which also bear amplification property is that in the superfluid ^3He the Cooper pairs have one or more orientational degree of freedom and since all the Cooper pairs have to behave identically and have the same center of mass as well as the same orientions in those internal parameters, we can in principle exploit superfluid ^3He to amplify the properties associated with internal motion.

A potential candidate for usage of the superfluid amplification of ^3He is direct detection of violation of weak interaction. In nature gravitational and electromagnetic interactions can be seen in macroscopic level however the weak interaction can only be seen in microscopic level. Weak interactions violate spatial inversion (P). The question is that can this P violation be observed in macroscopic level. The answer with using superfluid amplification of ^3He can be yes and Osheroff at Standford has being worked on that [5].

2 Methods

2.1 What are the critical methods of the paper?

The first critical method was using the some rules and then finding the spontaneous symmetry breaking in the liquid Helium 3. Leggett started to calculate the resonance frequency using the sum rules for susceptibility

$$\frac{1}{\pi}\int_0^\infty \frac{Im\chi(\omega}{\omega}d\omega = \chi_0 \tag{4}$$

$$\frac{1}{\pi}\int_0^\infty \omega Im\chi(\omega)d\omega = -\langle[S_x,[S_x,H]]\rangle_0 \tag{5}$$

$$\tag{6}$$

where $\chi(\omega)$ denotes the frequency-dependent radio frequency (rf) susceptibility, $S_x = \sum_i \sigma_{xi}$ is the total x component of nuclear spin operator, H is the total Hamiltonian operator of the system in the absence of rf filed, χ_0 denotes dc susceptibility the angular brackets denote the expectation value taken with respect to the unperturbed thermal equilibrium state and ω is frequency.[3, 5]

With those and approximating the peaks of the absorption profiles by delta functions he obtained

$$\omega_{res}^2 = -\chi_0^{-1}\langle[S_x,[S_x,H]]\rangle_0 \tag{7}$$

Now think that you rotate the whole nuclear-spin system by angle θ_x around the $x-$axis. Since the generator of such a rotation is $exp(iS_x\theta_x)$ and quantity $\langle[S_x,H]\rangle$ is zero in thermal equilibrium, you obtain that

$$[S_x,[S_x,H]] = \frac{\partial^2\langle H\rangle}{\partial\theta_x^2} \tag{8}$$

Leggett considered all the significant terms in the Hamiltonian of the system and obtained

$$\omega_{res}^2 = \gamma^2 H_{ext}^2 + \chi_0^{-1} \frac{\partial^2 \langle H_D \rangle}{\partial \theta_x^2} \tag{9}$$

where H_D is the nuclear dipole-dipole interaction Hamiltonian. Then he compared his equations with the experimental data

$$\frac{\partial^2 \langle H_D \rangle}{\partial \theta_x^2} = K(1 - T/T_A) \qquad K \simeq 10^{-3} erg/cm^3 \tag{10}$$

and realized that a single dipole-dipole interaction cannot be responsible for the spectacular frequency shifts. If you call the maximum dipolar energy of a pair g_D, you find that it is only about $10^{-7}K$. It would be a good approximation to say that the second order differential of H_D above is at most of the order of g_D and moreover the advantage of a "right" spin orientation of a pair (parallel to their separation vector) to a wrong one (perpendicular to their separation position vector) is at most g_D. Hence the preference of the right orientation to the wrong orientation should be of order $g_D/k_B T$. The expectation value of $\langle H_D \rangle$ consequently follows to be of order $n g_D \times g_D/k_b T$. However, this is small and cannot account for the above equation [5]. Actually the situation is worse than this because for a degenerate Fermi liquid like Helium 3 the dipole interaction is governed by the Fermi energy not the thermal energy. So the dipolar energy $\langle H_D \rangle$ is of order $n g_D^2/k_b T_F$.

Now the spontaneously symmetry breaking comes to the rescue! The idea is that there should be some terms in the energy like the spin-conserving terms such as kinetic energy, van der Walls potential energy, etc. whose effect is to force all the Cooper pairs to have the same relative spin-orbit configuration. If that happens then the dipolar Hamiltanian enhances to $N g_D$. That is because $N g_D \gg k_b T_F$ and hence the degree of polarization is %100. The spontaneous broken spin-orbit symmetry can be understood in analogy with the magnetic system described by Heisenberg Hamiltonian whose spin-spin Zeeman energy is g_Z. (See Table 1)

This was the first method and idea used in solving the problem of exotic phases of liquid Helium 3. The second important one was through studying the microcopic physics of the system in more details. The question is that given the SBSOS is responsible for the anomalies , what kind of physical system can cause this property. The method that Leggett used in this part as mentioned before was his long-forgotten work on the internal Josephson effect in a two-band superconductors [7, 5].

By comparison to that work he could find two conjugate canonical operator, **S** the total spin vector and $\boldsymbol{\theta}$ the operation of rotation through an angle

TABLE I. Analogy between SBSOS and ferromagnetism.

Ferromagnet	Liquid ^{3}He
$\hat{H}=\hat{H}_0+\hat{H}_z$ ↑ Invariant under simultaneous rotation of all spins	$\hat{H}=\hat{H}_0+\hat{H}_D$ ↑ Invariant under ***relative*** rotation of spin + orbital coordinate systems
extl. field ↙ $\hat{H}_z = -\mu_B H \sum_i S_{zi}$ ↑ breaks spin-rot.^{n} symmetry	$\equiv \mu_0\mu_n^2/r_0^3$ ↙ $\hat{H}_D = g_D \sum_{ij} \left(\frac{\underline{\sigma}_i \cdot \underline{\sigma}_j - 3\underline{\sigma}_i \cdot \hat{\underline{r}}_{ij} \underline{\sigma}_j \cdot \hat{\underline{r}}_{ij}}{(r_{ij}^3/r_0^3)} \right)$ breaks ***relative*** spin-orbit rot.^{n} symmetry
Paramagnetic phase ($T>T_c$): spins behave independently, kT competes with $\mu_B H \Rightarrow$ polarization $\sim \mu_B H/kT \ll 1 \Rightarrow$ $\langle H_z \rangle \sim N(\mu_B H)^2/kT$	Normal phase ($T>T_A$): pairs of spins behave independently $\Rightarrow$ polarization $\sim g_D/kT \ll 1 \Rightarrow$ $\langle H_D \rangle \sim N g_D^2/kT$
Ferromagnetic phase ($T<T_c$): $\hat{H}_0$ forces all spins to lie parallel $\Rightarrow k_B T$ competes with $N\mu_B H$ $\Rightarrow \langle S_z \rangle \sim 1 \Rightarrow \langle H_z \rangle \sim N\mu_B H$	Ordered phase ($T<T_A$): $\hat{H}_0$ forces all pairs to behave similarly $\Rightarrow$ kT competes with $N g_D$ $\Rightarrow \langle H_D \rangle \sim N g_D$ $\sim 10^{-3}$ ergs/cm^3!

6: (Courtesy of A. Leggett)

$|\theta|$ about the axis $\hat{\theta}$, and justified that the Hamiltonian of the system can be written in terms of those; hence the dynamics of the canonical variables can be obtained. His main argument and method lies on the fact that I just mentioned. That is the Hamiltonian can be expressed in terms of $\mathbf{S}$, $\boldsymbol{\theta}$. The argument goes like the following. It is known from the experiment that the characteristic frequency associated with the dipole forces $\omega_0(T)$ is small compared to the other characteristic frequencies involving in the problem namely the gap frequency $\Delta(T)/\hbar$ and the N-phase quasiparticle relaxation rate τ^{-1}. Thus, all the macroscopic degrees of freedom during the NMR should follow the macroscopic degrees of freedom of $\mathbf{S}$ and $\boldsymbol{\theta}$ adiabatically. An approximation like Born-Oppenheimer's is then can be done in which the effective

Hamiltonian is the minimum value of the free energy for the given values of the two variables [5, 3]. Hence, he found that

$$H = \frac{1}{2}\gamma^2\chi_0^{-1}\mathbf{S}^2 - \gamma\mathbf{S}.\mathbf{H} + H_D(\boldsymbol{\theta}) \tag{11}$$

where **H** is the total external magnetic field. This was the crunch which led him to find time derivatives of the conjugate variables and hence the spin state of the phase A and phase B. [5, 3]

2.2 What are the weaknesses of the methods used?

The weakness of the methods was in exploiting of the sum rules. It turned out that while the SBSOS approach can still be kept the calculations using the sum rules are wrong. That made Leggett to do a more detailed calculation of the full microscopic dynamics that I mentioned as the second method above.

2.3 Are there other or better approaches that could be used?

Yes, there are. Anderson and Brinkman [8] took up the idea similar to the idea of the BCS theory of superconductivity and obtained the Quantum mechanical states of the two superfluid phases A and B. The argument in brief goes like this. In superconductors one electron polarizes the ionic lattice and the second electron feels the induced polarization and hence attracted to the first electron. Similarly in the liquid Helium 3 an atom induces in vicinity a spin polarization parallel to its own spin and the second with the same spin feels an attraction force. There is a crucial difference in the two scenarios. In the first one there was a liaison between the two electrons, i.e. the ionic lattice, while in the second one in liquid Helium 3 there is no liaison. That makes the latter much more sensitive to the onset of pairing and hence the effective attraction modifies by the onset of pairing. Based on this argument Anderson and Brinkman made quantitative calculations and found the states of the superfluid phases A and B. By state I mean the relative spin orientation of the Cooper pairs. It turned out that B phase is in BW state, a superposition of all three Zeeman substates, and phase A is in ABM state, up-up and down-down states of spins of the pairs. This result also obtained by Leggett but Anderson and Brinkman used a different approach.

3 Results and Discussion

3.1 What are the primary conclusions of the paper?

Superfluid ^3He exists below 2.7 mK and it has two phases. In the first phase between 2.7 mK and about 2 mK the superfluid is in ABM phase from the aspect of the microphysical state, and in phase B it is in BW phase. The anomaly of in the phase shift in phase A and lack thereof in phase B can be quantitatively understood by spin dynamics and also SBSOS argument.

3.2 Did the authors prove their hypotheses?

Yes he did. I explained the part of the major arguments above and tried not to go through the technical details and calculations . But for further reading please look at Ref. 3.

3.3 What novel information or directions come from this work?

The novel information is the existence of superfluidty in He-3 and recognizing their spin phases. The novel direction is the method that is used in solving this problem which also used in other branches of condensed matter physics and also other parts of physics as I explained in Subsection 1.5. A possible way to detect a P violation directly in macroscopic world is also a novel direction that comes from this work (Please see Ssubsec. 1.5)

3.4 What control experiments were performed?

The work started with the Cornell group experiment. In solving the problem Leggett, beside justifying the available data, predicted a lack of splitting of the A-phase transverse resonance at low fields and lack of longitudinal resonance at B-phase. These peridications will soon confirmed by Osheroff (1974) and Bozler et. al.(1974) [5, 9, 10]

3.5 What other explanations for the observations are still possible?

As explained in Subsec 2.3 Anderson-Brinkman explanation is also possible. Please see that section for more details.

3.6 What would you do next to advance this field?

The first reaction of Tony Leggett to the result of the experiment was that they are so extraordinary and may indicate breakdown of some fundamental principles of quantum mechanics under very exotic condition [5]. This is the point which can also make me eager to go and solve a problem to check the validity of the conventional Quantum mechanics in the regimes that have not been tested yet. The field that caused Leggett to enter to this problem, foundational physics, and it is not confined to the anomaly of Helium 3 is what I would like to advance which lead us to find the very reality of the nature.

4 References

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