The Emergence of Polarization in Flocks of Starlings

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Society is very familiar with the concept of a flock or herd moving in one direction. This paper seeks to introduce the reader to the fundamental models and observations that are present in the community. These models describe the organization of the flock in the normalized average velocity or polarization of the members of the flock. The polarization of the flock can be treated as the order parameter that describes the organization of the flock. Models are presented that show the dependence of the polarization on bird density and the random noise added to the interaction between birds.

Introduction

Many children have experienced the phenomena of herding and flocking from watching The Lion King. The evil hyenas scare a herd of wildebeests turn into a flowing river of hooves and fur that envelopes Simba and ends up taking Mufasa's life. Although this picture of herding is imprinted in the minds of many children, the general phenomena of herding, applies to a wide range of biological and physical systems. These systems range in scale and dimensionality from single cell bacteria moving on an auger plate to schools of fish swimming around in seemingly random directions over [1]. Other formulations of the same phenomenon include synchronized clapping in crowds and waves of people standing up that travel around assembly hall and memorial stadium during athletic events [1]. These systems all have the common attribute that a few simple short range interaction rules between organisms create self organized long range order from the breaking of a continuous symmetry [1]. One of the best-studied examples of the phenomenon is the flocking behavior of Starling birds over their nesting site during dusk. [1] This paper will present the motivation for researching this problem, as well as the main theoretical and observational findings to date.

Motivation

The wide spread interest in flocking behavior has stemmed from the generality of the phenomenon and the impact a better understanding could have on people's lives. Thousands of years ago cave dwellers wanted to have a better understanding of when and how herds of animals move so that they could more reliably provide food for their clans. In the modern day work has changed to searching for the fundamental rules that influence the dynamics of flocks. These underlying rules will be applied to a variety of systems such as groups of autonomous robots, slime mold growth, swarms of insects and bacteria growing [1]. A better understanding of the underlying principles that govern the long-range dynamics of flocks is an interesting problem with deep physical implications [1].

The deep physical implications come from the emergence of net flock velocity despite rotationql symmetry of the polarization. This polarization increases with stronger interaction between the neighboring birds in the flock and decreased random noise passed along with each interaction [2]. Condensed matter physicists have been working on many body interactions for many years searching for fundamental explanations of magnetism, super fluidity and superconductivity. In these condensed matter systems long-range coherent behavior gives the macroscopic phenomena such as paramagnetism, the Meisner effect and zero viscosity in super fluid He. The methods that were developed to describe these quantum systems can be applied to the flocking problem despite its inherent dynamical nature [3]. Using the machinery from condensed matter physics, researchers have been able to quantitatively predict the behavior of flocks.

Theory of flock formation

The underlying mechanism

The underlying principle of the current models on flocking is the formation of collective behavior due to short-range interactions between birds in the flock rather than following decisions made by a leader in the flock [1]. After each time step, all the birds [4] adjust their velocity to the average velocity of all the birds within an interaction radius r_I. When an obstacle is added to the model in the form of a predator or an object that the birds must fly around, the information about where the obstacle is gets passed from bird to bird through the short-range interactions. The more recent models also include some form of errors and inaccuracies in the interaction between birds to simulate inaccuracies that are inherent in biological systems [1]. The movements can be correlated over a coherence length λ, which corresponds to the distance in the flock over which the information can be passed before it is lost [5]. This loss of information is due to the noise introduced with each bird's alignment with the average velocity of birds in its interaction radius. The models below investigate the behavior of flocks in different regimes of parameter space.

An example of another physical system that obeys similar rules is a paramagnet. Each bird with speed and orientation corresponds to a particle with spin and direction. A system of spins at zero temperature that start in a random orientation will eventually form domains and then have an overall magnetization. The introduction of thermal noise to the magnetic system is analogous to the errors that are introduced in the alignment of the birds with each of their neighbors. The difference between the two systems is that the birds flocking are inherently dynamical and therefore are not an equilibrium system. In the limit where no birds have velocity, the system becomes the magnetization problem [1].

Describing the flock

To build and characterize the behavior of a flock of birds several quantities have been introduced to quantify the interactions and movement of the individual birds that collectively form the flock. The position and velocity of the i_{th} bird in the flock are the vector quantities $\vec{r}_i$ and $\vec{v}_i$ respectively. With these quantities for each bird, a quantity describing the degree of collective flight of the flock known as the polarization is defined [5].

$$\phi = \frac{1}{N}\left\| \sum_{i=1}^{N} \frac{v_i}{|v_i|} \right\| \tag{1}$$

Where N is the number of birds in the flock. For a flock of birds flying in exactly the same direction $\phi = 0$. When the direction of the bird's movement is in random directions that all cancel out, $\phi = 1$. One thing of note about the polarization is that there is no preferred direction for the flock to fly in.

The polarization is similar to the net magnetization in a paramagnet [1]. Since the polarization describes the overall order of the flock, it can be applied as an order parameter describing the motion of the flock. One thing of note about the polarization is that there is no direction that is preferred for the final motion of the flock [5]. Despite this rotational symmetry, the flock spontaneously breaks that symmetry and has an overall direction travel[5]. This spontaneously broken symmetry can be compared between models and observations of real flocks.

Once the development of polarization in the flock has been characterized, the correlation between movements in the flock provides an interesting direction to continue analysis. From the average velocity of the flock, a deviation from that velocity is defined with the average being over all birds in the flock. Each bird also has a deviation from $v_{flockave}$,

$$u_i = v_i - v_{flockave} \tag{2}$$

The deviation is a method for transforming the velocity of each bird into a center of mass reference frame. With this terminology we can examine the long-range coherence of the birds in the flock due to short-range interactions [8]. The correlation between birds with a separation r can be written

$$C(r) = \frac{\frac{1}{c_0}\sum_{i,j} u_i \cdot u_j \delta(r - r_{i,j})}{\sum_{i,j} \delta(r - r_{i,j})}. \tag{3}$$

The random noise in the neighbor interaction makes the information passed between birds decay over space in the flock. A decay length can characterize this decay λ [8]. If the decay length is short compared to the flock size, then the center of mass velocities will not be correlated across the flock. This applies to the ability for the flock to respond as a whole to an obstruction or a predatory attack [8]. The literature uses these parameters to statistically quantify the dynamics of flocks of bird and bird like objects.

Modeling Flocks of Birds

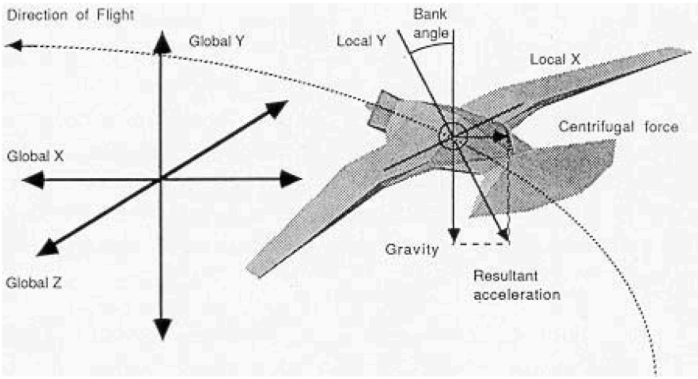
Reynolds Model of Boids

The first group to simulate flocking behavior in birds as interactions between birds was C W Reynolds' group. Their main motivation was to make a computer animation of birds flying. Since the model simulated bird-oid like objects that could be birds or fish or insects in reality, they use boid and bird as synonyms in the paper [7].

The Reynolds model started with three assumptions to set up the problem. All birds were attracted to the other birds by a natural flocking instinct. This was originally implemented as a Hookeian linear attraction between all birds, but they found that a Newtonian gravity like attraction led to a more realistic model [7]. Each bird matched its velocity, both magnitude and direction, with the birds in an area close to itself. The last assumption added to the model dictated that each bird avoids collisions with other flock mates with a short-range repulsive potential. These three beginning assumptions set Reynolds' model apart from previous central potential models of flocking, and put together the fundamental interactions needed to exhibit flocking behavior [7].

Reynolds found that the behavior of the flock could be separated into two different stages that depended on initial conditions. If the starting density of the flock produced a nearest neighbor distance that was less than the repulsion between birds then the initial reaction was for the flock to expand rapidly before the birds aligned directions and formed a flock. If the birds' initial distribution was less dense than the critical density, then the birds would form smaller flocks that then joined together to form bigger flocks. In this way "acceptable approximation of flock like motion"[7] was observed.

Reynolds was working towards a believable graphical representation of birds flying together so he included aspects in his model that were left out by more quantitative models in later years. To have a graphical animation that is believable to audiences, the individual bird orientation was included in their model, but did not contribute to bird interactions [7]. The form of the interaction between birds was tested and found that a Newtonian gravity like attraction was more believable than a linear Hookeian attraction. Once the density and interaction parameters were tuned to get flocking behavior, the ability to add obstacles was added to the model. Adding obstacles enabled modeling of the dividing features of the flock. In graphical modeling this was important for increased flexibility in application, but no investigation was conducted analyzing the dynamics of flock division [7].



The Reynolds model is based around local interactions of bird-oid objects with their nearest neighbors. The model includes gravity and takes the centrifugal force into account to set the bank angle of each bird to recreate an animation of a small flock of birds [7].

The Viscek Model: adding noise

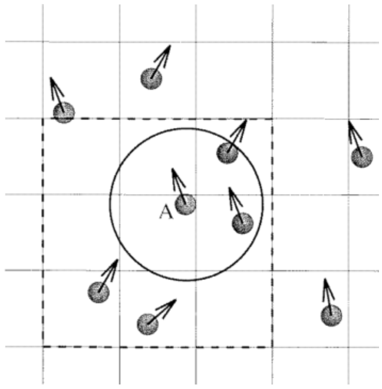
In 1995 Tamas Viscek and collaborators used the Reynolds model as a framework for a new model that looked at the flocking phenomenon from a more scientific viewpoint. Their goal was to build a simple model with tunable parameters, which they could use to characterize the behavior of the flock. The Viscek model used a fixed speed for all birds, but used the nearest neighbor interaction to influence the orientation of each bird's velocity. Since Viscek was more focused on the flocking behavior of the birds and less with a believable animation, the model is run in two dimensions and does not have a nearest neighbor repulsion [5]. With each time step, the position of each bird is updated according to

$$x_i(t + \Delta t) = x_i(t) + v_i(t)\Delta t \tag{4}$$

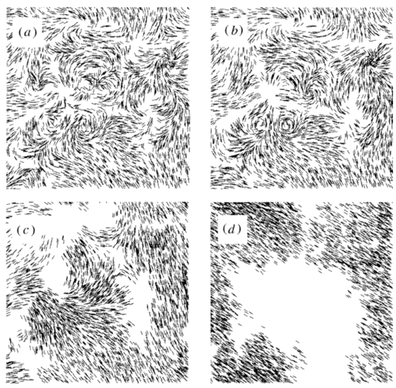
$$v_i(t + \Delta t) = \langle v_i \rangle(t) + \xi \tag{5}$$

where $\langle v_i \rangle$ is averaged over a small region of space in the vicinity of the bird i. ξ is a random number chosen such that $\frac{\eta}{2} > \xi > -\frac{\eta}{2}$. In this way the amount of noise in the system can be tuned in magnitude by changing η [5].

The initial conditions of the birds in the simulation are randomized according to a specified density. The flock size of the simulations is between 10^4 and 10^5 birds. For the interaction range, two different definitions were considered. One considered all birds within a radius R of bird i. The other method creates a lattice of squares with length R, and averaged the velocities of all birds within neighboring squares [5]. By setting the length of each box as 1, and the length of each time step as 1, the velocity is defined as desired to speed up or slow down the rate of development of the flock [3]. Little change was found with $.003 < v_i < .3$. For the results reported, a value of .03 was used [5]. In the limit that $v_i \to 0$ the flock becomes a two dimensional Ising model. If $v_i \rightarrow \infty$ then the flock is completely randomized between each time step [3]. With this model and these parameters, the group set to work characterizing the flock.



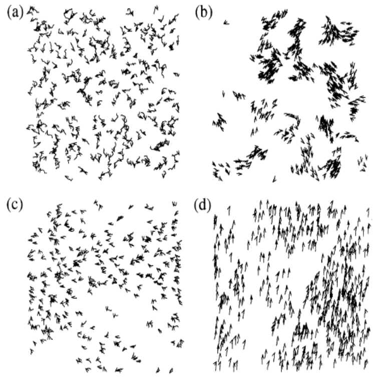
A graphical representation of the short-range interaction between bird A and its neighbors used in the Viscek model. The two short-range interactions are show. The first is where all birds within a radius R are included in the average. The other includes all birds within the nearest neighbor cells of the cell bird A is in. For the purpose of the study the second method was used because it decreased calculation time. [3]



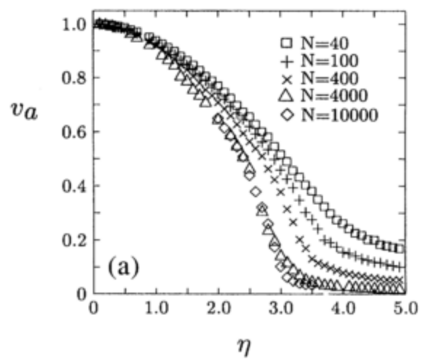
The Figure shows the time evolution of the flock of birds with $N = 4000, L = 40,$ and $v_i = 0.01$. After 50 time steps the flock is in a random state of motion (a). This state relaxes into a state with unstable vortices after 100 time steps (b) from the start. The instability slowly decays into an intermediate semi ordered state after 400 times steps (c) before a long ranged polarized flock emerges at 3000 time steps. [5]

Viscek Results: emergence of movement

The main focus of the Viscek paper is to characterize the influence that noise and density have on the final flocking behavior of the birds as described by the polarization (called normalized average velocity in their paper). After an initial relaxation period, the flock settles into a statistically steady state distribution. This steady state is examined by looking at the resulting formation of the flock after a time period of roughly 10^5 units of time. The Viscek group was the first group to describe the polarization of the flock as an order parameter and find the dependence of ordering on noise and flock density [5].



Examples of the Viscek model with different noise and system size parameters. a) With low density and high noise the flock has not foreseeable order. b) Low noise and low flock density small groups form that travel in random directions. c) With high density and high noise the flock does not have long-range order, but there are correlated movements between birds. d) Long range order develops when high density and low noise parameters are chosen. [3]



The figure shows the polarization (v_a) of the flock as a function of the magnitude of the noise between interacting birds. Across all densities there is a clear transition between an ordered flock at low noise and disorder at high noise. [5]

At first they tested the dependence of the polarization on the magnitude of the noise in the system. This was done by decreasing η from 5 to 0 and observing the effect on the final polarization. They found that for flocks of different densities with the same size, there dependence of the polarization on the magnitude of the noise that was similar to the continuous transitions in condensed matter systems near their critical point [5]. By was a looking at the data with this new insight, the group fit critical exponents to the dependence of the order parameter on noise and flock size according to

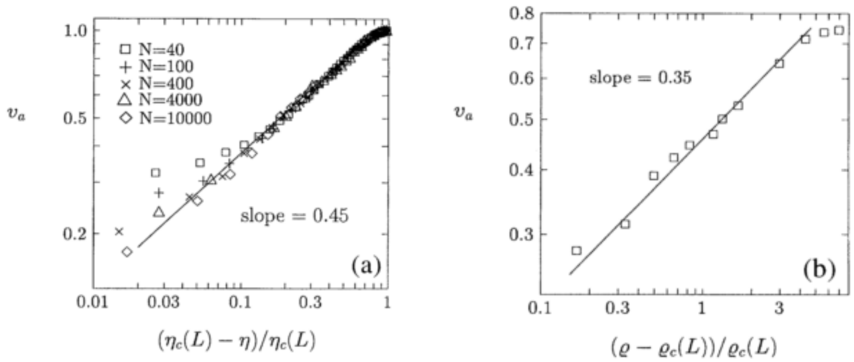
$$v_a \propto [\eta_c(\rho) - \eta]^{\beta} \qquad (6)$$

$$v_a \propto [\rho - \rho_c(\eta)]^{\delta} \qquad (7)$$

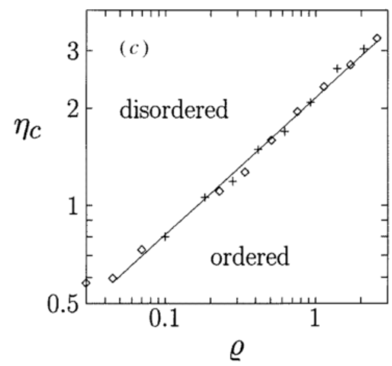
From the fit of the data, $\beta = .45 \pm .07$ and $\delta = .35 \pm .06$. These values describe the effect that the noise and the density have on the emergence of an ordered phase from the noise induced disorder. The phase transition follows a power law to the degree of precision that Viscek could extract from the completed models. This paper provided the first quantitative analysis of a flocking model that analyzed its resulting flocking behavior in terms of a phase transition of the polarization [5].

Toner and Tu: Applying Renormalization theory

Toner and Tu published a renormalization group theory based calculation in 1995. In their paper Toner and Tu examine the flocking problem and find that there is a critical number of dimensions at d=4 where the solutions are non trivial. Starting with the equations of motion of the birds in the flock, it is shown that there is a continuous symmetry that is broken in d=2. The symmetry that is broken is the rotational invariance of the polarization. This calculation is more comprehensive than the scope of this project [6].



The Figure shows the power law fit of the polarization (v_a) dependence on interaction noise (η) and density (ρ) according to equations (6) and (7) above. From the fit of the data, $\beta = .45 \pm .07$ and $\delta = .35 \pm .06$. When the noise gets above the critical value, or the density gets below the critical value then the organization of the system breaks down [5].



The graph shows the phase diagram of the flock of birds as functions of critical noise magnitude and density of the birds from the Viscek model [5].

Viscek 2.0: Calculating the critical line in the phase diagram

Viscek et al published a continuation of the work they had started in the 1995 paper by calculating the phase diagram of the polarization of the flock and adding a flocking preference direction to the model [3]. They found that the scaling of the polarization of the flock with respect to noise in the system can be described by a universal function if the noise is normalized around a critical noise value that is density dependant. The relation was found to be

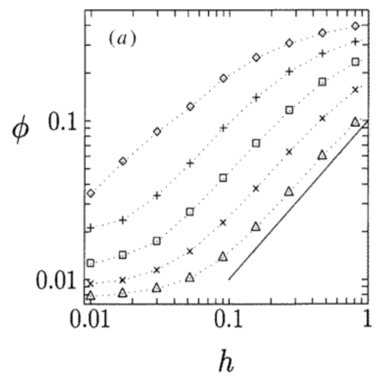
$$\eta_c(\rho) \propto [\rho]^{\kappa} \quad (8)$$

with $\kappa = \frac{1}{2}$ within experimental uncertainties. This exponent corresponds to the mean field approximation of the flock [3].

In their continuation paper Viscek et al also added an overall flocking preference to the model. This flocking preference is akin to a wind that blows a flock in one direction. To account for this, a term is added to the velocity averaging formula with strength h in the direction of the unit vector $\hat{e}$

$$v_i(t + \Delta t) = \langle v_i \rangle + \xi + h \cdot \hat{e} \quad (9)$$

The results of the addition of this term are clearly visible below in the tendency of the flock to develop a polarization despite high levels noise magnitude [3].



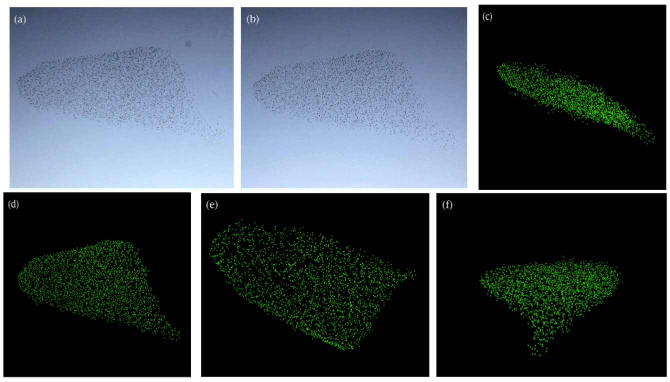
As the noise magnitude (η) increases from 3.9 to 5.2 (from top to bottom) in steps of .3 there is a clear progression of increased polarization ϕ in the system with an increased flocking bias h. Without a flocking bias, all of these levels of noise would yield very unpolarized states [3].

Observations and Analysis of real flocks:

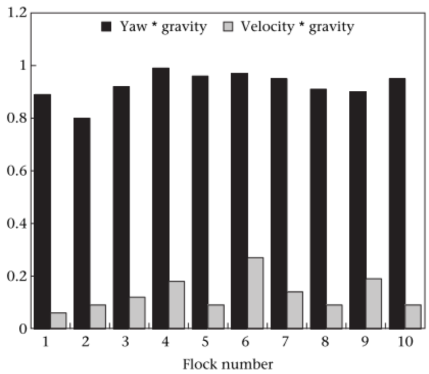
A benchmark study from Rome

A study of the European Starling (sturnus vulgaris) flocks in the city of Rome was published in 2007. The main motivation for the study of the large flocks is to populate the lack of observational data in the scientific community. More data will enable better understandings of the dynamics of flocks as well as the development of flocking in the evolution of animals. Once the flocks had been digitized, global properties of the flock and correlations of motion in the flocks were investigated [8,9].

The study consisted of taking pictures from two vantage points that could be used to triangulate the location of each bird in the flock. The pictures were taken at a rate of 10 frames per second for up to 8 seconds depending on flock position and movement [8,9]. With the triangulated position of each bird, a computer reconstruction of the flock can be generated in virtual space. This medium provides easy to analyze data of flocks of birds in nature. For the paper, 10 flocks of birds ranging in number from 448 to 2631 birds were selected from over 500 series of images taken [9].



Images (a) and (b) are images of the same flock of birds. Immages (c) and (d) show the two renderings generated from the pictures taken. The digital versions of the pictures are used to electronically triangulate the position of each bird. Insets (e) and (f) demonstrate the way that the flock can be rotated in the computer using the digital renderings of the flock [9].

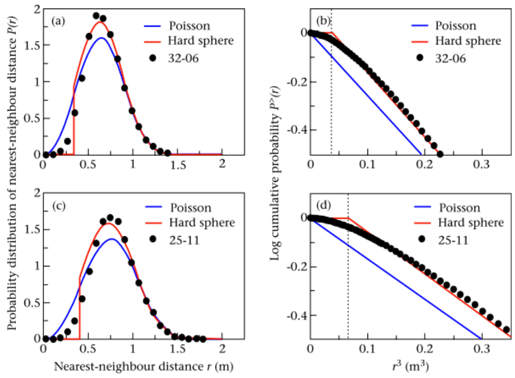


The black bars are the dot product of the short axis of the flock shape (I_1) with gravity and velocity with gravity. There is a clear trend that the flock is horizontally flat and also travels in that horizontal plane [9].

Analysis of Virtual Flocks

Some of the global properties that the group analyzed were the flock shape, flock orientation with respect to gravity, the relationship between flock orientation and changes in the polarization in direction. Their results show that the flock shapes generally have their shortest dimension (I_1) in the plane perpendicular to gravity. In addition to this, the polarization was consistently perpendicular to gravity. There was one observed flock that changed its polarization inside one of the clips. The flock orientation did not change in this flock despite a significant change in the polarization of the flock [9].

To have a better understanding of the interactions between the birds, the probability of a bird having a nearest neighbor a certain distance away was calculated. The decay of the probability function gives insight into the competition between a Poisson random distribution and a hard shell repulsion model [9]. The two flocks analyzed here have hard sphere radiuses of .17m and .2m respectively. These distances correspond to half the wingspan of the average starling [8].

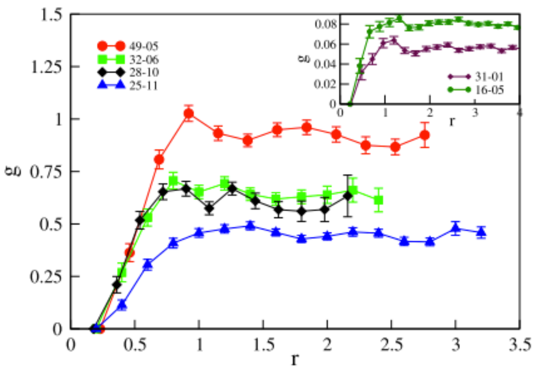


The probability of a bird having a nearest neighbor a certain distance away in two flocks. There is a clear competition between hard sphere repulsion and a Poisson random distribution around the average mean spacing [9].

In 2010 the group from Rome expanded their analysis to include more flocks of birds observed and different quantities transferred over from condensed matter physics. The main quantity that was added to the analysis in the second paper is the pair correlation function (Γ). The pair correlation function describes how correlated the movement of two particles separated by a distance r is and is defined as

$$g(r) = \frac{1}{4\pi r^2}\frac{1}{n_c}\sum_{i=1}^{n_c}\sum_{i\neq j}\delta(r - r_{ij}) \tag{10}$$

The results of the pair distribution function show that the flocks of birds are more organized than gaseous fluids but less organized than fluids [8].



The figure shows the pair correlation function of 4 flocks of birds from the Rome study. In the plots there is clearly a first peak, and peaks that form after that. This shows that there is some order to the system. Since the peaks are not very sharp, there I still significant randomization in the flocks [8].

The problem of quantifying the flocking behavior commonly found in nature is one of deep physical significance. There have been several iterations of models with analysis of the polarization in the flock as an order parameter. The use of concepts originally used in condensed matter physics has been shown to describe the phase transition between an ordered state and disorder. Despite the quantified description of the transition between the two phases, there is still much work to be done in understanding the specific form of the interaction between the birds and the evolutionary development of the behavior. The intent of this paper was to be and informative introduction to the topic of flocking. Hopefully it inspired deeper though when looking at flocks in the real world.

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