Eric A. Petersen
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Gravitation and the Emergence of Space-time

Abstract:

The concept of space-time is fundamental to the theory of General Relativity. However, when developing theories of gravity which incorporate quantum mechanics, problems arise on the scales of the Planck length and Planck time. Several solutions have been proposed where the classical idea of space-time as the fundamental backdrop upon which the laws of physics guide the evolution of the universe is rejected. Instead, it is argued, space-time is an emergent property of the more fundamental notion of physical interactions or events. Some of these solutions arise from casting gravity as an entropic force. Others involve imposing certain commutation relations on non-commutative geometries or using Monte Carlo methods on a sub-Planck scale grid and solving for the path integral. Some physicists, however, remain unconvinced that an emergent notion of space-time is necessary and their objections will also be noted. The broad idea of emergent gravity cannot, as of yet, be dismissed, however, certain models can be excluded.

1 Introduction:

General Relativity is a very successful theory in the domains in which it has been tested. The description provided by the theory allows for calculations which are much more precise than the older Newtonian theory and predicts many new areas of physics such as gravitational redshifts, deflection of light beams around massive objects, and even completely new objects - black holes. The fundamental aspect of the theory is that matter distorts space-time. This is generally accepted as "just how the universe is" but recently, some researchers have begun to examine the possibility that this may actually be an emergent phenomenon.

1.1 Gravity:

The first successful theory of gravity was Newton's famous Law of Universal Gravitation. The equation $\vec{F}_G = \frac{GMm}{r^2}\hat{r}$ was able to explain all terrestrial experiments for centuries and was so good at describing the solar system that two independent astronomers made extremely precise predictions about the location and mass of Neptune before it had been observationally discovered[1]. In 1915, Einstein published his theory of General Relativity, which did much more than simply correcting Newton's theory to a few more decimal places. New effects which have been observed are gravitational redshifts, the precession of Mercury, gravitational lensing, and, dramatically, black holes. All of these have been observed with remarkable precision and, though many alternative theories have been proposed, GR remains the most satisfactory (for both simplicity and accuracy) model yet proposed. However, in the strong field regime (near a horizon or in the early universe) observational evidence is scant, leaving room for other theories[2].

One of the key players of General Relativity is the metric. The metric is a rank 2 symmetric tensor which contains much information regarding the geometry of the system in question. By generalizing the familiar dot product, it not only calculates the invariant distance, ds^2, but it also shows causality (past and future), determines the trajectories of test particles (geodesics), and identifies locally inertial frames[3]. Mathematically, the metric is used to take inner products and to raise/lower indices (change to contravariant or covariant form). Derivatives of the metric give rise to connection coefficients (called Christoffel symbols in a coordinate basis) which allow one to examine parallel transport on a curved manifold. These symbols are given by equation 1.

$$\Gamma_{\alpha\beta\gamma} = \frac{1}{2}\left(\partial_\gamma g_{\alpha\beta} + \partial_\beta g_{\alpha\gamma} - \partial_\alpha g_{\beta\gamma}\right) = \frac{1}{2}\left(g_{\alpha\beta,\gamma} + g_{\alpha\gamma,\beta} - g_{\beta\gamma,\alpha}\right) \tag{1}$$

Taking derivatives of these symbols gives rise the Riemann Curvature Tensor (a rank 4 tensor), which can be contracted to form the Ricci Tensor (a rank 2 tensor), given by equations 2 and 3.

$$R^{\alpha}_{\beta\gamma\delta} = \Gamma^{\alpha}_{\delta\beta,\gamma} - \Gamma^{\alpha}_{\gamma\beta,\delta} + \Gamma^{\alpha}_{\gamma\lambda}\Gamma^{\lambda}_{\delta\beta} - \Gamma^{\alpha}_{\delta\lambda}\Gamma^{\lambda}_{\gamma\beta} \tag{2}$$

$$R_{\mu\nu} = R^{\lambda}_{\mu\lambda\nu} \tag{3}$$

A useful quantity for understanding the degree to which spacetime is curved is the Ricci scalar (also called the curvature scalar). This is formed by a further contraction on the Ricci tensor, $R = R^{\mu}_{\mu}$. These three quantities are great for describing the curvature imposed by the metric, but one combination of them, the Einstein tensor, is of particular value to General Relativity. The Einstein tensor, $G_{\mu\nu}$ is formed by equation 4 and is connected, through equation 5 (where the second equality holds with geometerized units: G=1=c), to the energy distribution (symbolized by the Energy-Momentum tensor, $T_{\mu\nu}$) of the universe[4]. The presence of the constant, Λ, in the equation has an interesting history, but nowadays it is usually only included for cosmological purposes[3].

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{mu\nu} \tag{4}$$

$$G_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu} + \Lambda g_{\mu\nu} \approx 8\pi T_{\mu\nu} \tag{5}$$

1.2 Emergence:

Emergence is a property of matter in which a collection of microscopic objects interact to create a macroscopic system with qualitatively different properties than the microscopic system. A simple, yet illustrative, example could be dissipative forces (friction, drag, etc.) which arise from many interactions between molecules in the two objects (or fluids) transferring the kinetic energy of the system as a whole to smaller scales, resulting in an increase in temperature. In this example, it could be said that the force of friction *emerges* from the underlying molecular interactions. An important aspect of this emergence is that the friction force can be understood ($F_s = \mu_s N$) without making reference to the (complicated) interaction properties of the molecules from which it arises. In fact, many well established fields of science (chemical bonding, thermodynamics) turn out to emerge from some much newer field of study (quantum mechanics, atomic theory). Thus, the claim that gravity, often quoted as one of four fundamental forces, may be emergent should not be quite as shocking as it may, at first, appear.

A major concept in emergence is the idea of spontaneous symmetry breaking. This is done when the governing principles (say, the Hamiltonian) of a system obey some symmetry (say, translation invariance or axisymmetry). A simple example would be placing a pencil on its tip (all forces are symmetric with respect to azimuthal angle) which then falls (and points in only one direction). A more interesting example would be a string of interacting spins, initially randomly aligned, at finite temperature being cooled to a lower temperature. In the absence of an external magnetic field, there is no a priori reason to believe that the spins will align one way or the other (symmetry) but, past a critical temperature, the spins will start to have an overall magnetization (broken symmetry).

Another concept which is important when examining the macroscopic emergence of a system from microphysical constituents is granularity. This deals with how precisely the system is observed. For instance, the ideas of pressure and density in a fluid are coarse-grained in that they have meaning (quite important meaning for fluid dynamics) on the scale of the whole fluid but, when "zooming in" to the scale of the individual molecules of the fluid (angstroms, picoseconds) the terms loose meaning (the space around the atoms is mostly empty, so the density would essentially be a sea of delta functions on a background density of zero). An experimenter whose data is averaging over a large region (coarse-grained) will see the emergent behavior; an experimenter whose data focuses on a very small scale (fine-grained) will only see the underlying microphysics. For example, if one were to probe a bar magnet with extremely high precision, one would just see a jumble of spins, however, if one looks at the magnet as a whole one would see that there is a preferred direction of magnetization.

2 Gravity and Quantum Mechanics

Two of the most successful and fundamental theories in physics are General Relativity (described above) and Quantum Mechanics (not discussed in this paper). These theories match very well with experiment and observation when confined to their own domains (large distance scales for GR, short ones for QM). Quantum Mechanics is king from the subatomic world to molecular dynamics while GR describes mostly astrophysical systems from cosmological scales down to the GPS satellites in Earth's orbit. Because both theories reduce to familiar Classical Mechanics when length, time, mass, and energy scales approach what an average human might encounter in their every day lives, one might naively assume that this arrangement could be "good enough" to describe the universe.

There do, however, exist several instances in which extreme amounts of curvature (a GR effect) happen on microscopic scales (the Quantum domain). One example would be the theoretical (as of yet, unobserved) micro-black holes. These subatomic particles would greatly affect the local spacetime. Another, less hypothetical example deals with cosmogony, the birth of the universe. According to the widely accepted inflationary Big Bang model, the extremely early universe had incredibly high temperatures and density, yet the spatial extent of the universe went down to the Planck scale (one Planck length, $\ell_P \sim 10^{-35}m$; one Plank time, $t_P \sim 10^{-43}s$). In some quantum theories, the other three fundamental forces (Strong, Weak, and Electromagnetic) would have unified to form one force (with the Strong force emerging before the Electro-Weak force splits). Aesthetically, it would be nice if the same theory could describe Gravity as well. More importantly, on this scale quantum effects would cause energy-momentum fluctuations which would, through GR, cause spacetime to devolve into an ugly quagmire of infinite values for certain parameters[3]. The extremely turbulent, non-smooth nature of spacetime at this scale has been called "spacetime foam" and it makes any actual calculations impossible; see figure 1 for an illustration in the form of drawings of spacetime curvature at increasingly fine resolution.

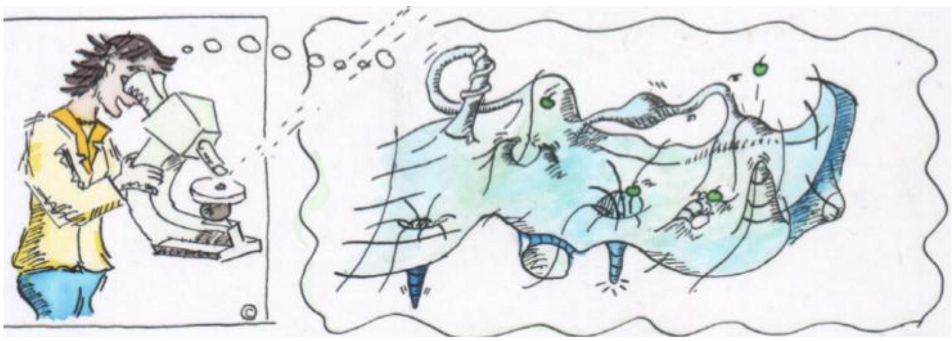

Figure 1: This cartoon illustrates a physicist examining the microscopic spacetime foam through an improbably powerful microscope. This image was originally published in [5], with credit to E Rijke.

Another connection between GR and QM is the concept of black hole entropy. The no-hair theorem states that a black hole is characterized entirely by its mass, spin and charge; any other information regarding the matter that originally made up the black hole is lost. A black hole's event horizon increases whenever it accretes matter and, during a collision with another black hole, the final area of the event horizon is greater than the sum of the two initial horizons. Between these two facts, one might begin to suspect that the area of the event horizon might be tied to the entropy of the black hole. It turns out that the two are proportional. This leads to the notion that the black hole should come to equilibrium with a temperature bath of photons. At first, this seems impossible, given that the black hole should be incapable of emitting any blackbody radiation. The solution is that a kind of tunneling takes place when virtual pairs of (anti-)particles erupt into existence near the horizon. One (anti-)particle may fall in while the other is ejected to infinity; this is known as "Hawking radiation". Hawking radiation reduces the mass of the black hole, leading to evaporation, and allows it to (on a cosmological timescale) come to equilibrium with a temperature bath[2].

3 Entropic Gravity:

3.1 The Model:

Certain forces can be derived from entropic considerations. Osmotic forces and elastic polymer forces are examples. Take the case of the latter; the polymer molecule entropically favors a short jumble of monomers to a long, thin chain because there are many more microstates available to it. Statistical mechanics can be used to show that the resulting force is identical to Hooke's law (assuming constant temperature): $\vec{F} \propto -T\Delta\vec{x}$. Two characteristics of this law are important for defining an entropic force: (1) the force points toward higher entropy (shorter macroscopic length) and (2) the force is proportional to temperature.

To create an entropic model of gravity, imagine that nearby points are separated by "screens". These screens act like event horizons in that their area stores information and is thus related to entropy, though they are not actual horizons around any singularity. This model is called the holographic principle and has its roots with the AdS/CFT correspondence (which relates quantum conformal field theory to relativistic anti-DeSitter space). On one side of the screen our usual notions of spacetime have already emerged, while on the other the unknown microphysics (about which we make no assumptions aside from time reversal symmetry which leads to energy conservation and temperature) is used. We will take the screen to be a sphere surrounding a massive body with the "normal" emergent laws taking place in the outside region, as in figure 2.

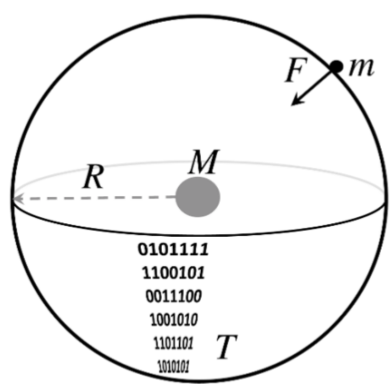

Figure 2: This schematic represents the thought experiment. The sphere of radius R is the screen; the mass M inside distributes information across the surface, leading to an effective temperature T. The outer emergent region has has a test particle of mass m impinging on the screen and experienceing a force F. Image originally published in [6].

As a particle of mass m approaches to within a Compton wavelength of the screen, the entropy increases (just as for an accreting black hole) by an amount

$\Delta S = 2\pi k_B$ for $\Delta x = \hbar/mc$. This leads to $\Delta x = \frac{2\pi k_B mc}{\hbar}\Delta x$. The usual form for an entropic force, $F\Delta x = T\Delta S$ can be used to relate the force to the temperature as in equation 6.

$$F = \frac{2\pi mc}{\hbar} k_B T \tag{6}$$

Now the crucial element is the energy within the screen. Assuming equipartition, there is a simple relationship between the temperature that will determine the force in our holographic model and the energy within the screen; there is also a simple relationship between the energy and the mass that used in Newtonian gravity. These relationships are given by equation 7,

$$Mc^2 = E = \frac{1}{2} N k_B T, \tag{7}$$

where N is the number of "bits" on the surface. By analogy with the event horizon, we know that $N \propto A$, and we will use G in the proportionality constant; it will later be shown to be the familiar G from Newtonian theory. Using $N = \frac{Ac^3}{G\hbar}$ and equation 7 we can solve for the temperature of the screen in terms of the mass we are supposing is inside of it, the radius of the screen, and physical constants as in equation 8.

$$T = \frac{2\hbar GM}{4\pi R^2 k_B c} \tag{8}$$

Combining equations 8 with equation 6 gives equation 9, which is exactly Newton's law of gravitation. Similar, but more lengthy arguments can give rise to Einstein's equations[6].

$$F = \frac{GMm}{R^2} \tag{9}$$

3.2 Experimental Rejection:

One test of this method comes from experimentation with ultra-cold neutrons in a gravitational well. The Schrödinger for this setup is given by equation 10, with the boundary condition that $\psi(z=0) = 0$, indicating that the bottom surface is a perfect mirror.

$$\left(\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial z^2} + V(z)\right)\psi_n = E_n\psi_n \quad | \quad V(z) = \begin{cases} m_n g z & \text{if } z \geq 0 \\ \infty & \text{if } z < 0 \end{cases} \tag{10}$$

The solution to this equation is given by 11, where the length scale is given by $\ell = (\hbar^2/2gm^2)^(1/3) \approx 5.9\mu m$, x_n is the n^{th} zero of the Airy function, and N_n is a normalization constant (in all cases, n¿0).

$$\psi_n(z) \;=\; N_n Ai\left(\frac{z}{\ell} \,+\, x_n\right) \tag{11}$$

The energy levels, $E_n = m_n g z_n$, give rise to predicted, discrete heights above the mirror which the neutrons are allowed to occupy; $z_n \;=\; -x_n\ell$ and for n=1, the value is $z_1^pred = 13.7\mu m$. Experiments have observed the discrete pattern and have identified the ground state height as $z_1^{exper} = 12.2 \pm 0.7_{stat} \pm 1.8_{sys}\,\mu m$, a good agreement with the predictions.

This can be compared to the entropic predictions. As the neutron approaches the screen by an amount Δz, the entropy of the screen changes by an amount ΔS. The patch of the coares-grained screen representing the neutron is denoted by a subscript $\mathbb{N}$, leading to equation 12, where λ_n is the neutron Compton wavelength.

$$S_{\mathbb{N}}(z+\Delta z) \;-\; S_{\mathbb{N}}(z) \;=\; \Delta S_S(z) \;=\; 2\pi k_B \frac{\Delta z}{\lambda_n} \tag{12}$$

Equation 12 implies that, as a neutron in an originally pure state changes height, it must evolve into a mixed state. This means that the vertical translation operator, U_z is not unitary which, in turn, implies that its generator, P_z is not Hermitian. By assuming that the screen is at maximum entropy it can be shown that the new momentum operator, $\tilde{p}_z$, works out to $\tilde{p}_z \;=\; -\imath\hbar\partial_z \;-\; 2\pi\imath mc$. The corresponding wavefunction, $\tilde{\psi}$, is given by equation 13, while the energy, $\tilde{E}$, works out to be $\tilde{E} = mgz_n + 2\pi^2 mc^2$.

$$\frac{\partial^2\tilde{\psi}_m}{\partial z^2} \;-\; \frac{4\pi mc}{\hbar}\frac{\partial\tilde{\psi}_n}{\partial z} \;=\; \frac{2m}{\hbar^2}\left(V(z) - E_n - 2\pi^2 mc^2\right)\tilde{\psi}_n \tag{13}$$

The energy shift in $\tilde{E}$ is not observed, but the real nail in the coffin comes from wavefunction. The solution to equation 13 is an exponentially decaying analog of 11, as seen in equation 14.

$$\tilde{\psi}_n(z) \;=\; \tilde{N}_n e^{-2\pi(z/\lambda)}\psi_n(z) \tag{14}$$

The exponential in this wave function is very important in that it means that the probability of detection should be about one for heights less than the neutron Compton wavelength and about zero for any height greater than that. The means that the slit should have been transparent to neutrons with $h < z_1$, which is completely the opposite of the observation of the slit being opaque in that region. Thus, no matter how well the entropic theory of gravity works in duplicating classical formulas, it must be dismissed as inconsistent with experiment[7].

4 Other Models:

4.1 Non-covariant Gravitons:

Of course, there are many hypothetical means for gravity to be seen as an emergent property. In order to be simultaneously true and usefull, these models must produce different results than GR in some unexplored regime, yet reduce to GR (to the appropriate accuracy) in every case where GR has already been tested. By parameterizing the differences and taking into account uncertainties in the current observations, one can constrain the parameters of the new model. This example shows how stringent these constraints can be, leading one to wonder how much could be gained by replacing the old theory with the new.

It can be shown[10] that a graviton (massless, spin 2 boson) cannot undergo proper Lorentz transformations in quantum field theory. This means that the graviton can either obey gauge symmetries or be relativistic, but not both. If a nonrelativistic graviton were to exist, then it would not only violate Lorentz covariance, but also the equivalence principle. Experiments show that the equivalence principle is valid to one part in about $\sim 10^{13}$. One way around this is to assume that the graviton does not travel at the speed of light. If this were the case than high energy cosmic rays would release a gravitational analog of Cherenkov radiation; experiment constrains the difference between the two speeds to be less than one part in $\sim 10^{15}$.

4.2 Causal Dynamic Triangulations:

By using nonperturbative lattice methods, specifically Causal Dynamic Triangulations (CDT), it has been shown that familiar spacetime can emerge from an underlying lattice. This is done by evaluating the path integral, Z, as described by equation 15 through a Monte Carlo scheme of N verticies with grid spacing, a, below the Planck length.

$$Z(G_N, \Lambda) = \int_{g \in \mathbb{G}} \mathbb{D}g e^{\frac{\imath}{G_N} \int d^4x \sqrt{|g|}\,(R - 2\Lambda)} \to \lim_{N \to \infty} \sum_{g \in \mathbb{G}} \frac{1}{C_g} e^{\imath S_{Regee}(g)} \tag{15}$$

The righthand side of equation 15 shows the CDT method of numerically evaluating the integral. The quantity S_{Regee} is a coordinate free alternative to the usual action Einstein-Hilbert action, S_{EH}. This is where the triangular approach comes in. Curved, higher dimensional surfaces are approximated by a series of flat triangles, with a spacing of a. This is represented in figure 3.

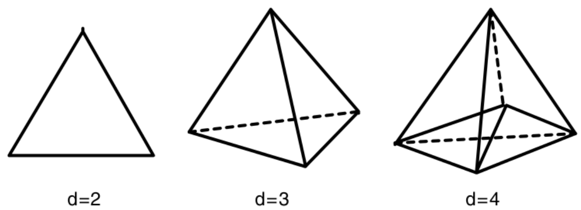

Figure 3: Triangels used to build figures in progressively higher dimensionallity, as done in CDT. Image originally published in [5].

With these conditions, simulations have been performed which show the emergence of spacetime quite well. With a properly tuned Cosmological Constant, Λ, this spacetime shows the proper 4-dimensional, smoothly curved spacetime of cosmology; spatial representations along cosmological proper time even show an evolution as described by the Friedmann equations. On much smaller scales, the 2-dimensional spacetime foam is observed at the Planck scale and quickly disappears at larger scales. These results are illustrated in figure 4. One weakness of this theory is its inability (at least so far) to reproduce Newton's law of gravitation between two distinct massive bodies[5].

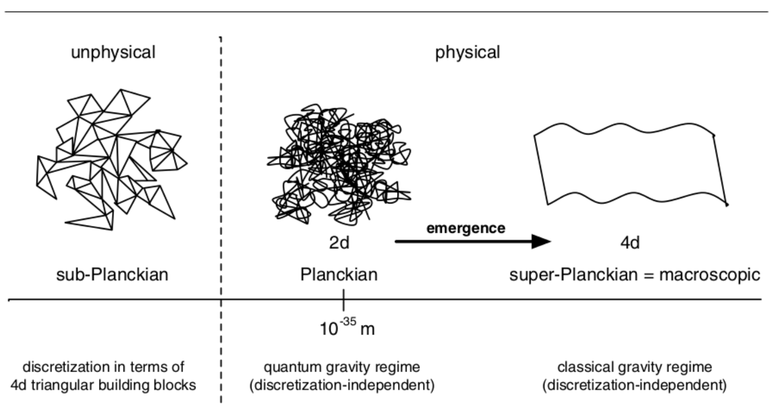

Figure 4: From the numerical (not physical) grid scale (to the left of the dashed line) a physical description of spacetime emerges (to the right). At the Planck scale the spacetime foam is evident, but quickly decays to the familiar, smooth spacetime of GR. Image originally published in [5].

4.3 Noncommutative Spacetime:

Another model uses Electromagnetism and noncommutative geometry to show an emergence of gravity. In this model, spacetime is viewed as noncommutative, like in quantum mechanics. This geometry is symplectic, meaning that, unlike the standard Riemannian geometry, there is no well defined curvature. It can be shown that, in such a geometry, electromagnetism is a diffeomorphism of a symmetry which can be spontaneously broken by a symplectic two-form of the background geometry. Out of this symmetry breaking, gravitation emerges[8].

This model can be used to make testable predictions about cosmological properties which have already been observed. For instance, without any fine-tuning of the cosmological constant or invoking dark energy, the age of the universe works out to be about $13.9 \times 10^9 yrs$, which compares favorably to the observed value of $13.7 \times 10^9 yrs$. Another victory for the theory is that it successfully predicts an inflationary epoch, which has become an important aspect of modern cosmology for its explanation of the flatness problem (the seemingly improbable coincidence that space, as a whole, is largely Euclidean). One interesting departure from the standard cosmological model (ΛCDM) is that the universe evolves in a "big bounce" rather than the more widely believed "big bang". Figure 5 shows the evolution of the scale factor (roughly, the size of the universe). This theory does depend on certain aspects of Supersymmetry which, while having not been entirely ruled out, are far from certain at this point[9].

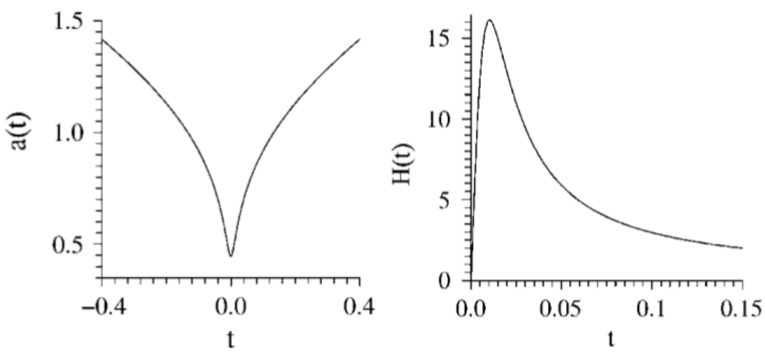

Figure 5: The image on the left shows the evolution of the scale factor as a function of time in a "big bounce" universe. In a "big bang" model, the universe would erupt from a singularity at t=0. The image on the right shows evolution of the Hubble parameter (normalized to the current value). Image originally published in [9].

5 Conclusions:

The most popular emergent theory of gravity, the entropic theory proposed by Verlinde, is very thought-provoking. Through relatively simple arguments, gravity can be shown to be a manifestation of entropy by invoking the well known correspondence of entropy and event horizon area. However, experiments involving cold neutrons trapped in a gravitational well support the more traditional view of gravity, rather than the entropic one. This failure causes one to wonder if all the extra machinery built up for this theory (the relatively untested holographic principle) is really a worthwhile investment or if this is all just a “neat trick” with little physical basis.

Other theories also run into problems when attempting to match observations. For example, one theory which admits for gravitons to break Lorentz symmetry leads to a violation of the equivalence principle, which is verified to one part in 10^{13}. A more successful theory involves numerically solving for the gravitational path integral on a Monte Carlo grid with spacing less than the Planck length. This is very good at producing the spacetime foam and smoothly varying large scale structure of a vacuum spacetime, but has trouble (perhaps more of a temporary technical difficulty than an inherent shortcoming) getting distinct massive objects to attract according to Newton’s Law. A final theory uses symplectic manifolds and noncommutative geometry to allow gravity to emerge from electro-magnetism. This method is extremely successful in cosmological tests; it accurately predicts the age of the universe, has an inflationary period, and does not require fine-tuning of the cosmological constant. However, this theory, too, could be in hot water if the properties of supersymmetry are not found to match certain requirements.

One common problem which plagues all of these theories, and which also explains why there are so many, is the lack of robust experiments. Of the theories which have yet to be ruled out, none of them have an overwhelming body of supporting evidence. Some what annoyingly for these theories, GR is a very good theory, at least in every range in which it is feasible to test. This means that the observations need to be made with extremely high gravity (eg. near a horizon or in the early universe), which is hard to observe, or on very small scales (like in the experiment referenced by Kobakhidze), where gravity is an extremely weak force in comparison to other effects. Thus, the debate about the emergence, or not, of gravity will likely be along for a long time.

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