Gravity as an Emergent Phenomenon

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Abstract

While general relativity explains gravitational interactions well, it only answers the question of the nature of gravity by telling us that the geometry of space-time is gravity. Some physicists theorize that gravity is not fundamental, but emergent. Two of the theories that this paper will examine propose gravity as an emergent phenomenon rely on entropy as the origin of gravity or propose that it is a result of examining a quantum field theory on a Lorentz manifold. While these new theories of gravity are certainly attractive, experimental results refute some theories of emergent gravity, and there exist theoretical arguments against the general idea of emergent gravity.

1   Introduction

Four fundamental interactions hold the universe together. The strong force binds protons and neutrons in nuclei and holds quarks in various particles. The weak interaction is responsible for the radioactive decay of subatomic particles. Electromagnetism directs interactions between charged particles. Lastly, gravity, the weakest of all interactions, is the attractive interaction between particles with mass. In addition to being the weakest of the four fundamental interactions, gravity has no quantum mechanical explanation verified by experiment.

Currently, there exist several potential solutions to reconcile gravity with quantum mechanics, or at least discover the fundamental nature of gravity. The first and most intuitive solution for physicists is to quantize gravity just like how the electromagnetic fields have been. If the other forces were successfully integrated into a quantum mechanical theory, then it only makes sense that gravity should be as well.

Some lines of thought claim that gravity is an emergent phenomenon. An emergent phenomenon by definition is a property that is a result of simple interactions that work cooperatively to create a more complex interaction. Physically, the simple interactions occur at a microscopic level, and the collective result can be observed at a macroscopic level. This is seen in various phenomena such as superconductivity and superfluidity, to name two of many. In a mathematical theory of the physics of an emergent phenomenon, the emergence may show up as a result of some mathematical operation or transformation. The common theme in the contention that gravity is an emergent phenomenon is that gravity may not be fundamental. Perhaps gravity is just a result of microscopic interactions which appear to average out on a macroscopic scale and give us gravity as we recognize it. This is completely unlike the picture in which gravity, like the other forces, is mediated by a particle called the "graviton." If a particle does not mediate gravity, then that raises the question of the mechanism behind gravitational interaction.

This paper will examine two versions of emergent gravity – gravity as a result of entropic considerations, and gravity as a result of developing a quantum field on a Lorentz manifold. Specifically, a consideration of the entropy vis-à-vis the holographic principle will allow us to recover the familiar inverse square law and the Einstein field equations. Additionally, the quantum fluctuations on a classical Lorentz manifold will yield expressions to calculate constants such as the cosmological and gravitational constants.

2   Modern gravity

2.1   Theory

For any theory to be successful, it must, of course, accurately describe and explain any observation that we have made. Additionally, new theories must be able to recover any old theories that describe the same phenomenon. For example, if there existed a new theory of electromagnetism, one should expect that in some limit, that new theory would recover Maxwell's equations. Likewise, the emergent theories of gravity that are about to be presented must, at the very least, recover Einstein's field equations. The reason is that

Einstein's field equations tell us everything there is to know about gravity. Einstein's field equations are presented in this simple form:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + g_{\mu\nu}\Lambda = \frac{8\pi G}{c^4}T_{\mu\nu} \tag{1}$$

Recovery of this equation must be one of the criteria for an emergent theory to be considered successful.

2.2 Experiment

Currently, physicists are still working on verifying the gravitational inverse square law in the range of millimeters and below. However, it should be noted that general relativisitic effects such as gravitational lensing and frame-dragging have been verified experimentally. Additionally, values for the gravitational constant and the cosmological constant have been measured.

Any candidate theory of emergent gravity should be able to predict experimental observations, of course. If one of the new theories can recover Einstein's field equations, then it should, in principle, be able to successfully predict anything general relativity can. However, even though this may be true, a new theory of emergent gravity may contain a set of assumptions or constraints which limit its predictive power. If the theory can successfully predict any experimental observation, then that would automatically be sufficient for one to consider it "successful" (of course, it can still be refuted by future experiments). Unfortunately, no explicit calculations were made in any of the theories examined (and it is outside of the author's ability, seeing as it requires an in-depth knowledge of general relativity and quantum field theory), so it is difficult to evaluate exactly how successful each emergent gravity theory is in predicting physical observations.

3 Entropic Gravity

3.1 The holographic principle

Verlinde postulates the emergence of gravity from entropic considerations by using the holographic principle as a framework. The holographic principle states that the information that describes some d-dimensional volume is encoded in a lower-dimensional boundary that encloses that volume. Verlinde motivates the use of the holographic principle by making an analogy between the physics near a black hole and the physics in a flat, non-relativisitic space.

Bekenstein argues that if a particle of mass m were lowered towards a black hole, it can be considered a part of the black hole when it is one Compton wavelength (that is, $\Delta x = \frac{\hbar}{mc}$) away from the horizon [1]. When the particle is a Compton wavelength away, then both the mass and the surface area of the black hole increase.

Instead of using a black hole as the boundary that a particle approaches, we will consider a simple holographic screen. The reason is to consider a flat space-time instead of the extremely curved space-time near a black hole. The particle exists on the side in which space-time has emerged, and the holographic boundary separates the volume of emerged

space-time from the volume whose physics is determined by some microscopic degrees of freedom. Suppose that the particle moves towards the screen as in Figure 1. If it crosses the holographic screen to merge with the microscopic degrees of freedom, then there will be a change in entropy. The change in entropy is more specifically given by the following:

$$\Delta S = 2\pi k_B = 2\pi k_B \frac{mc}{\hbar} \Delta x \tag{2}$$

The right side of the equality comes from the definition of the Compton wavelength and could be thought of as a multiplication of "1". The physical justification for that factor is that by increasing the mass of an object, one might also expect an increase in the microscopic degrees of freedom, and hence a change in entropy. That operation enables us to write the change in entropy as a function of the change in spatial position.

3.2 Entropic forces

The second law of thermodynamics states that the entropy of a system constantly seeks to maximize itself. To do that, the system will seek to rearrange itself into a state whose entropy is maximized. This process is the idea behind the "entropic force." In elementary statistical mechanics, students usually encounter the random walk problem (a cartoon for which has been drawn in Figure 2) in which a polymer may be stretched to some thermal equilibrium
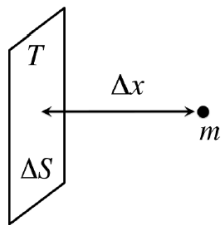

Figure 1: A massive particle exists on the side of the holographic screen in which space-time has emerged. Encoded on the screen is information about that particle. Additionally, the screen itself contains some thermodynamic properties such as temperature and entropy.
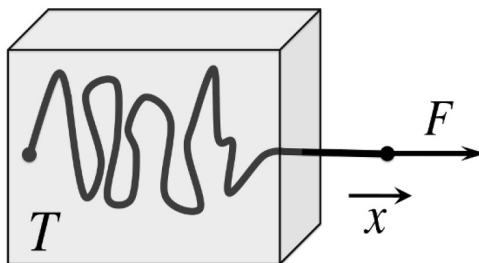

Figure 2: On a macroscopic level, a polymer might stretch or compress in order to reach thermal equilibrium.

length. Considering the entropy, $S(E,x) = k_B log \Omega(E,x)$, one can use the relation

$$\frac{1}{T} = \frac{\partial S}{\partial E} \tag{3}$$

to show that the force F has the form $F \propto -k_B T < x >$ by noting that the work $E = F \cdot x$.

3.3 Derivation of the inverse square law

Motivated by the fact that there exists an entropic force, we might guess that we could formulate gravitational (and perhaps other) forces using entropy. Before we write down any equations or relations, it is best to understand the strategy using simple words. The main idea behind the holographic principle is that information is what drives physical phenomena. To be clear, information is defined as a property such as position that we can assign to a particle. All the information will be split into "bits," like in computer terminology. At the smallest level, the information itself will have its own degree of freedom, which we can relate to the total energy of the system using equipartition.

Let us assume that there exist some N bits of information on a boundary that completely encloses some distribution whose total mass is M. Under the framework of the holographic principle, it follows that $N \propto A$, where A is the area of the boundary. Arbitrarily define the number of bits N as

$$N = \frac{Ac^3}{G\hbar} \tag{4}$$

where G is a constant of proportionality, but is chosen so as to turn into the gravitational constant. Now assume that each bit holds some energy. The energy that each bit has is assumed to be given by equipartition. It follows that the total energy is $N = \frac{1}{2}Nk_BT = Mc^2$. Suppose now that the mass is enclosed in a spherical holographic screen of radius R. Combining (2), (3) and (4) together, the very familiar inverse square law for gravity emerges:

$$F = -G\frac{Mm}{R^2} \tag{5}$$

Aside from the fact that factors of c and $\hbar$ were put in to force the equation to take the familiar form, it is actually quite impressive that we could get this far through a simple consideration of entropy.

3.4 Obtaining Einstein's field equations

Consider a static background that has a global Killing vector ξ^a. Recall that mathematically, the Killing vector field preserves the space-time metric. That is, if an object were to live in space-time with the Killing vector field, then physically moving every point on that object the same distance in the direction of the vector field will not distort the object.

The four-velocity is $u^b = e^{-\phi}\xi^b$ and the four-acceleration is $a^b = e^{-2\phi}\xi^a\nabla_a\xi^b$ ($\phi \equiv \frac{1}{2}\log(-\xi^a\xi_a)$; e^ϕ represents the redshift factor, and ϕ is the generalized gravitational potential). Using the Killing equation $\nabla_a\xi_b + \nabla_b\xi_a = 0$, the acceleration can be rewritten as $a^b = -\nabla^b\phi$.

Considering again a holographic boundary with an entropy S and temperature T, we can write the temperature of the screen as

$$T = \frac{\hbar}{2\pi} e^{\phi} N^b \nabla_b \phi \tag{6}$$

Like before, we may write the entropy as a function of displacement. By considering small changes in the entropy and displacement, we can write an expression for the entropy gradient.

$$\nabla_a S = -2\pi \frac{m}{\hbar} N_a \tag{7}$$

Again, like before, statistical mechanics show us that the force F can be written $F = T\nabla_a S$, which means that the entropic force in this relativisitic case looks like

$$F = -me^{\phi}\nabla_a \phi \tag{8}$$

With only the holographic principle in consideration, it seems as though we obtain a formulation for the force. It seems consistent with our current understanding since the force is the gradient of some potential.

Verlinde next presents a *sketch* of how to reach Einstein's field equations. First, consider a small element of information on the holographic screen. Let it be denoted by $dN = \frac{dA}{G\hbar}$. Presumably, G is a constant of proportionality, but it should turn into the familiar gravitational constant. Each bit of information carries a mass unit $\frac{1}{2}T$. The total mass M is
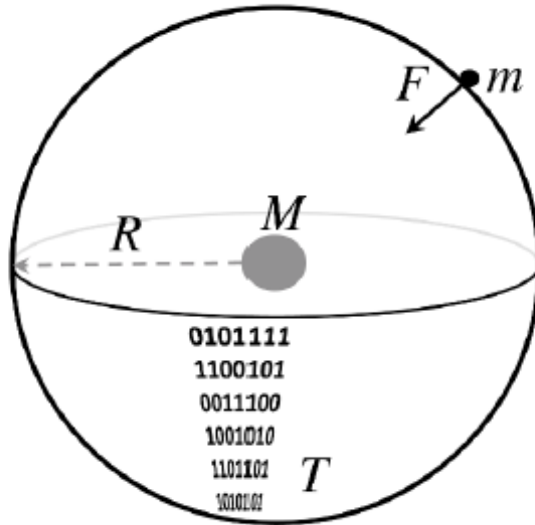

Figure 3: A holographic screen totally encloses a distribution of mass. On the screen is information about everything inside the boundary.

$$M = \frac{1}{2} \int_S T \, dN \qquad (9)$$

Rewriting T and dN as specified above, rewriting the potential ϕ in terms of the Killing vector, and finally applying Stokes' Theorem and the relation $\nabla^a \nabla_b = -R^b{}_a \xi^a$, the mass turns into

$$M = \frac{1}{4\pi G} \int_\Sigma R_{ab} n^a \xi^b \, dV = 2 \int_\Sigma \left(T_{ab} - \frac{1}{2} T g_{ab} \right) n^a \xi^b \, dV \qquad (10)$$

The right hand side of (10) comes from rewriting the mass M as an integral over the volume denoted by Σ, which is bounded by the holographic screen S. T_{ab} are components of the stress-energy tensor from general relativity. Keeping in mind that the integrands are equal, we can obtain the Einstein field equations. A full derivation is not provided, as the term proportional to the cosmological constant Λ is not present.

The derivation of (10) relies specifically upon the consideration of null holographic screens. A more complete derivation, Verlinde points out, relies on the use of time-like screens and the consideration of small volumes of space-time. Examining small volumes of space-time allows one to consider each of those elements as Minkowski space, which inevitably allows him or her to choose appropriate Killing vectors.

4 Gravity from quantum field theories

4.1 Action of a Lorentz manifold

The framework behind what is known as "induced gravity" contains the following assumptions:

1. The geometry in which the massive particles live is a Lorentzian manifold. That is, the signature of the metric in the 4-dimensional manifold is $(3, 1)$ (or $(1, 3)$, depending on convention).

2. The manifold must have no constraints. In other words, assume no boundary conditions or that the geometry itself is quantized, to name two constraints that someone might assume.

Given this simple framework, we can consider one-loop quantum field theory on the manifold. The one-loop contribution to the effective action of a scalar field will have the form

$$S_g = -\frac{1}{2} \log \det(\Delta_g + m^2 + \xi R) = -\frac{1}{2} \mathrm{Tr} \log(\Delta_g + m^2 + \xi R) \qquad (11)$$

By examining the difference between that one-loop contribution and the contribution from another effective action with two different metrics g and g_0 that are defined on the same manifold, one actually obtains a divergent integral. However, this is easily dealt with

by introducing a cutoff length. The cutoff length would be at some small length (or large wavenumber) so as to prevent ultraviolet divergence. Several more calculations can be seen in Ref. [8]. To avoid taking up space, the final result will be quoted here:

$$\mathcal{S} = \int d^4x \sqrt{-g}\Big\{ -\Lambda - \frac{R(g)}{16\pi G} + K\, O(R^2) + \mathcal{L}_{matter} \Big\} \tag{12}$$

Hence, expressions for the various constants in the Einstein-Hilbert action are obtained by matching powers in R. The expressions below are the general results of the calculations:

$$\Lambda = \Lambda_0 - \frac{1}{32\pi^2}\,\mathrm{str}\Big[\frac{\kappa^4}{2} - m^2\kappa^2 + \frac{m^4}{2}\log\Big(\frac{\kappa^2}{m^2}\Big)\Big] + \mathrm{UV\ finite} \tag{13}$$

$$\frac{1}{G} = \frac{1}{G_0} - \frac{1}{2\pi}\,\mathrm{str}\Big[k_1\kappa^2 - k_1 m^2\log\Big(\frac{\kappa^2}{m^2}\Big)\Big] + \mathrm{UV\ finite} \tag{14}$$

$$K_2 = (K_2)_0 + \frac{1}{32\pi^2}\,\mathrm{str}\Big[k_2'\log\Big(\frac{\kappa^2}{m^2}\Big)\Big] + \mathrm{UV\ finite} \tag{15}$$

$$K_4 = (K_4)_0 + \frac{1}{32\pi^2}\,\mathrm{str}\Big[k_4'\log\Big(\frac{\kappa^2}{m^2}\Big)\Big] + \mathrm{UV\ finite} \tag{16}$$

Λ and G are the cosmological constant and gravitational constant from general relativity. K_4 is the K in equation (12), but K_2 is not explicitly there. To fix that, the K_2 term should be added in, and will be multiplied by a factor of $C_{abcd}C^{abcd}$.

4.2 Discussion

Visser notes that Sarkharov, Pauli, Frolov and Fursaev and renormalization group theory all succeed in that they manage to obtain values for Λ, G and K as defined in the action in equation (12) through one-loop calculations. Despite that, the four different calculations actually emphasize different aspects of the expressions for the constants in equation (12). Sarkharov demands one-loop dominance, Pauli demands one-loop finiteness and Frolov and Fursaev demand both dominance and finiteness.

Sarkharov assumes one-loop dominance and obtains an expression for G [2]. Of the terms that diverge, he only accepts the ones that are the most dominant (e.g. he takes out logarithmically divergent terms). While Sarkharov does not explicitly calculate the other constants, it could be done under his assumption. However, one of the problems is that the smallness of the cosmological constant Λ, Visser notes, is put in by hand. On the other hand, the constants K_2 and K_4 are of order unity, which is actually compatible with experiment. Pauli focused mostly on the cosmological constant Λ because his calculations focused in a flat space-time. Instead of assuming that one-loop physics was dominant, Pauli demanded that the results of one-loop physics be finite. However, the constraints that Pauli enforces implicitly require physics beyond the standard model, meaning that his calculations are not experimentally testable yet. Frolov and Fursaev combine Sarkharov's and Pauli's assumptions and constraints [3]. Additionally, Frolov and Fursaev completely ignore terms of order R^2 because of their divergence. Through renormalization group theory, one-loop divergences are absorbed into zero-loop physics. It also demands that as the quantum field

theory that was developed on the manifold undergoes a phase transition, there are particular quantities that must be invariant.

It may be difficult to support these theories. The different sets of calculations require various constraints, which either require verification of physics beyond experimental scope, or may even increase the complexity of the theories to the point where it could create internal inconsistencies. In any case, it seems as though this is an avenue in emergent gravity that could be worthy to pursue in the future with the correct tools. Starting from only a quantum field theory on a Lorentz manifold, it seems as though gravity shows up. One last important point to make is that this theory presupposes that there is a Lorentz manifold. This is completely unlike Verlinde's entropic gravity, which only assumes the existence of some massive particles and the validity of the holographic principle, but never assumes a specific geometry.

5   Criticisms

This section will balance the discussion by presenting criticisms. One criticism claims that entropic gravity is incorrect due to an experimental result. The second criticism attacks the underlying principle of emergent gravity – the idea that gravity is not fundamental.

5.1   Slow neutron experiment

The main objection to Verlinde's entropic gravity concerns the compatibility between the implied microscopic theory and experimental observations. Kobakhidze cites an experiment concerning the gravitational bound states of a neutron [4, 5]. The experiment guided slow neutrons, which moved partly due to Earth's gravitational field, through a horizontal slit between a mirror and absorber. Figure 4 shows the experimental setup. The result of the experiment was that the neutrons could not transmit through the experimental setup if
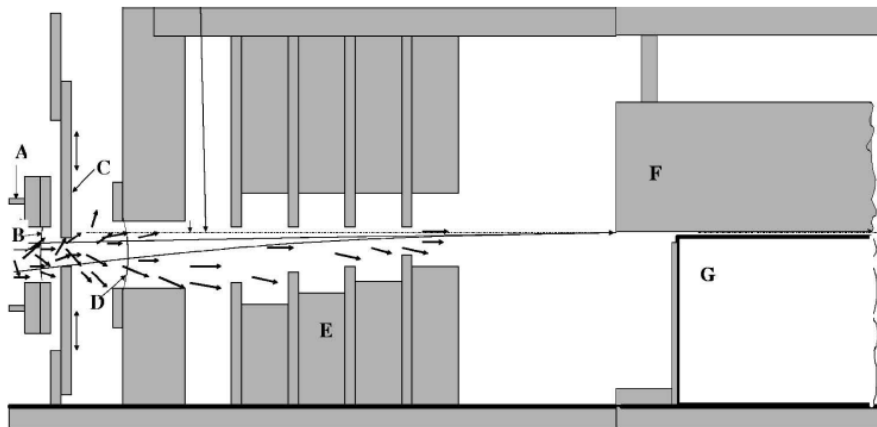

Figure 4: Experimental setup of the slow neutron experiment. Neutrons start at A and move into the apparatus towards the absorber F. Picture courtesy [4]

the height of the absorber (F in the figure) above the bottom mirror (G in the figure) was smaller than the smallest height of the least energetic stationary state with a gravitational potential. Kobakhidze noticed that using Verlinde's line of reasoning, entropic gravity would have predicted that the neutrons would have transmitted even if the height of the absorber relative to the mirror was smaller than the smallest height for the stationary states.

In light of the physical evidence presented, it seems as though the use of entropy to show that gravity is emergent is likely not the correct route. However, even if gravity is not an entropic force, it may still be emergent through other means. As we will see, there can still be life in the idea that gravity is an emergent phenomenon by considering a quantum field theory on a Lorentz manifold.

5.2   Space-time discreteness

We have already seen that there is an objection to emergent, entropic gravity on the grounds of empirical observation. While that directly refutes Verlinde's entropic gravity, there exists an objection to the general idea that gravity is not fundamental. Shan Gao attempts to show that gravity is fundamental by examining the implications of an assumed discrete space-time geometry [6]. The assumption of discrete space-time geometry also implies that the uncertainty of the position of a particle ought to have a minimum value $L_U \leq \Delta x$. By adding in this extra constraint, the Heisenberg uncertainty principle should contain an additional term:

$$\Delta x = \frac{\hbar}{2\Delta p} + \frac{L_U^2 \Delta p}{2\hbar} \tag{17}$$

The energy $E = pc$ contained in a region of size L will change the proper size of the region by the amount

$$\Delta L \approx \frac{L_U T_U E}{2\hbar} \tag{18}$$

where $T_U = \frac{L_U}{c}$. Considering Einstein's field equations (written without the cosmological constant and denoting the gravitational constant by κ)

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa T_{\mu\nu} \tag{19}$$

κ may be determined by considering discrete space-time. Considering first a spherical region S of radius R whose metric in the region S is the same as on its boundary and second the region outside of S which has the Schwarzchild metric, the proper size of the region S can be calculated to be

$$L \approx 2R + \frac{\kappa E}{4\pi} \tag{20}$$

Because $2R$ is the original size of the region, we can combine what we know to find the gravitational constant:

$$\kappa = 2\pi \frac{L_U T_U}{\hbar} \tag{21}$$

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Examining the limits of κ, we can see that if either L_U or T_U approach zero, then κ also must go to zero. Physically, what that means is that if the minimum size of space-time approaches zero, then the gravitational interactions will be killed as a result. The reason that this refutes gravity as an emergent phenomenon is that these relationships hold true for both macroscopic and microscopic objects. Should the phenomenon emerge at a macroscopic level (e.g. entropic forces), then it would be inherently emergent.

Testing for the discreteness of space-time will, of course, be difficult in the current state of experimental physics. It would require collider energies significantly higher than whatever is the most powerful right now.

6   Conclusions

The question of the nature of gravity has been asked over and over again. While it has been difficult to find a testable theory, there have been alternate interpretations of gravity, namely gravity as an emergent phenomenon. One of the theories starts with the concept of the holographic principle and develops it to show that the gravitational force we feel could actually be an entropic force. Additionally, from mere entropic considerations, we can obtain the Einstein field equations that are known in general relativity. The other theory presented considers a quantum field theory developed on a Lorentz manifold. The effective action can be shown to be analogous to the Einstein-Hilbert action in general relativity, and from there the gravitational and cosmological constants may be calculated. There even exist models which argue that gravity is emergent in a "world crystal," explicitly using condensed matter as a way to explain gravitational interactions [9].

Of course, these theories are not entirely bulletproof. Verlinde's entropic gravity seems to predict the transmission of neutrons when in reality it does not happen. While no direct refutation of Sarkharov's idea of induced gravity has not been found (as far as the author is aware), the idea of emergent gravity in general may be under attack by the idea advanced by Gao, who argues that a discrete space-time could prove that gravity is fundamental.

Many of these ideas indeed seem grand and noble. Unfortunately, at this point in time, experiments still need to be thought up and proposed to test various ideas of gravity. One way that we could approach this is to create theories that use assumptions that have been thoroughly tested such that we can take them as actual theory instead of hypothesis. As an example, the holographic principle, while attractive, elegant and useful as a model, empirically might not even be true. It is, of course, refreshing to see that physicists still keep the problem of gravity in the back of their minds. Without theorists such as Verlinde or Sakharov or Pauli, alternate modes of thinking about gravity may not have existed for a long time.

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