Studying Collective Phenomena Using Ultracold Atoms in Optical Cavities

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Abstract

The extreme precision and tunability of ultracold atom experiments allows condensed matter problems that are often intractable to the traditional condensed matter physicist to be studied in the laboratory. Atoms in optical cavities display emergent self-organizing behavior, opening the door toward studying problems such as crystal dynamics and supersolidity, which cannot be simulated with externally determined lattice potentials. These systems exhibit spontaneous symmetry breaking of both discrete and continuous natures. Both single-mode and multi-mode cavities have been studied, with rich applications of the latter to the study of defects, frustration, and glassiness.

I. Introduction

Despite its relative infancy as an avenue of research, the use of cold atom experiments to model condensed matter systems has attained much attention and success. The reasons for this success are many-fold, but among them are the extreme precision and tunability that can be achieved, which allows the researcher great freedom to explore the parameters of a given condensed matter problem. This comes in contrast with actual condensed matter experiments in which materials often prove difficult to work with and measure, or a material with the desired properties simply has not been discovered yet. Possibly the most intriguing example so far of the power ultracold atom experiments is the observation of a superfluid-Mott insulator transition in trapped rubidium [1].

However, ultracold atom experiments are still severely limited in the scope of phenomena they can simulate. In fact many of the most interesting problems in condensed matter rely on spontaneous breaks in symmetry and emergent, many-body phenomena, which are not easily translatable to an ultracold atom setting. In particular, with atoms confined to a lattice rigidly determined by laser interference patterns it is impossible to study the dynamics of crystal formation or the host of non-crystalline phases that are found throughout nature. The problem has been that the lattice potential serves merely as a backdrop against which the atoms move, instead of a dynamic field which is influenced by collective effects of the atoms.

Recent theoretical and experimental work involving the use of resonant cavities has shown that atoms in these systems will self-organize, providing a great window into the realm of crystal dynamics, topological defects, and many other interesting phenomena. Indeed, this work shows a promising way toward extending the applicability of ultracold atom experiments to problems in soft condensed matter and supersolidity [3]. This document will seek to provide an overview of the theoretical and experimental work on the applications of using ultracold atoms in resonant cavities to simulate emergent condensed matter phenomena. We will begin with some exposition on the background theory and experimental methods involved. Then we will discuss the work done on single- and multi-mode cavities and with thermal or Bose-Einstein condensed atoms.

II. Background

The basis of nearly all ultracold atom trapping experiments is the fact that two counterpropogating lasers create an electromagnetic standing wave with periodic areas of high and low field. If polarizable particles are placed in this field it will induce a dipole moment in the particles and cause them to be attracted to the areas of highest intensity. In effect, the particles experience a periodic potential with lattice spacing determined by the wavelength of laser light used. The key to trapping particles in such an optical lattice is that they must be extremely cold or else simple thermal motion will cause them to hop between lattice sites or escape the trap altogether. Though in principle any polarizable particle can be trapped in such a manner, in practice this has only been possible for a very small number or atom elements. Thus in most of the discussions below trapped particles will simply be referred to as atoms.

Ultracold atom experimentalists are able to create lattices of varying configuration in one, two, and three dimensions which are essentially unaffected by the presence or motion of the trapped atoms. Figure 1 shows how two- and three-dimensional lattices can be formed using two or three sets of counterpropogating lasers. In many cases an externally determined lattice structure is desirable: the uniform lattice potential is exactly what makes ultracold atom systems akin to crystalline solids. Yet if one desires to study pattern formation or the host of non-crystalline phenomena, this rigid lattice system is not suitable. What is required is a setup in which the atoms influence their own confining potential. Such a situation can be achieved through the use of resonant cavities.

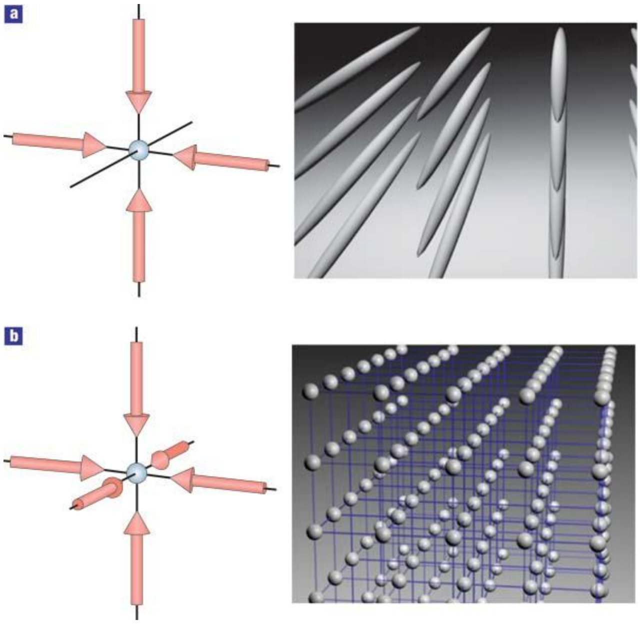

Figure 1: Schematic drawings of (a) two- and (b) three-dimensional optical lattices. In (b) lines are included to highlight the simple cubic lattice structure. [2]

Before self-organizing behavior was even theorized for these systems, resonant cavities were investigated in relation to the search for new methods of laser cooling and methods that can be applied to a wider range of subjects. In 2000 it was shown that an atom in an optical cavity will coherently scatter light into the cavity's resonant mode standing waves when illuminated by a low-saturating laser beam that is far detuned from any atomic resonances [4]. This was confirmed in experiments [5,6] and while it is potentially interesting to the problem of laser cooling, it is still effectively a single atom process with no collective effects other than those which decrease cooling rate. However Domokos and Ritsch proposed a new method which specifically takes advantage of cavity-mediated inter-atomic coupling to produce cooperative cooling [7]. The key to this cooperative cooling is that the new setup results in an emergent self-organization of the atoms into a periodic lattice that effects greatly enhanced, "superradiant" scattering into the resonant modes.

Domokos and Ritsch worked out the case for a one-dimensional system of thermal atoms in a single-mode cavity, but the real wealth of possibilities for these types of systems came when subsequent researchers considered multi-mode cavities and Bose-Einstein condensates (BEC). In these areas there is some very interesting experimental work, but in general theoretical research is much farther along. The following sections will discuss self-organization and the prospects for research into emergent phenomena in single-mode classical systems, single-mode BEC systems, and multi-mode systems.

III. Single-Mode Thermal System

Theory

The simplest system is that proposed Domokos and Ritsch in which thermal two-level atoms occupy a single-mode resonant cavity. The general setup for this system is shown in Figure 2. The subtle, but important, change from previous work which underlies the entire phenomena of self-organization in a cavity is that the pump laser is introduced perpendicular to the cavity axis instead of parallel.

According to the theory, the amplitude of the light scattered from the atoms is position dependent. Those atoms at nodes of the cavity standing wave do not scatter at all while atoms at the antinodes scatter maximally. This leads to the phase of the coherently scattered light field to be position dependent as well; scattering from atoms positioned half a wavelength apart will destructively interfere [7]. Also of importance is the fact that since the inter-atomic interactions are mediated by the field and since cavity photons move much faster than the atoms, the atoms can be thought of as having infinitely long range interactions with equal strength.

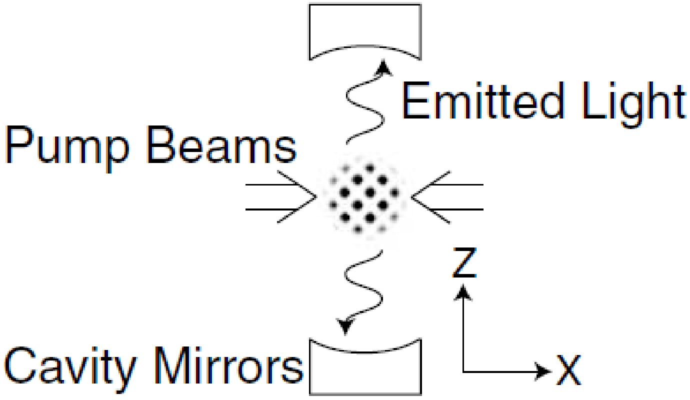

Figure 2: Setup for single-mode cavity cooling and self-organization. Note that the pattern shown in the center reflects the only quasi-one-dimensional geometry. [9]

One may notice that with a simple assumption of uniformly distributed atoms, the above actually leads to a total destruction of the coherent field, which is the opposite of what is desired. However, any real atom cloud will have some density fluctuations. With the system no longer uniform, the destructive interference is not perfect and a coherent field will indeed form. As mentioned before, for the proper detuning conditions a standing wave field will induce polarization in the atoms and they will be attracted towards the areas of highest field. Just as in the phenomena of Bragg scattering, developing periodic grouping of atoms actually enhances coherent scattering and further deepens the potential wells. The result should be “runaway” process that ends only when the condensation process is equalized by heating or interatomic repulsion. But recall that the intensity maxima of the scattered light field are periodic in half a wavelength which is exactly the condition described above for destructive interference. Again it seems that this should prevent any emergent lattice structure, but again consideration of fluctuations provides the key to a resolution. The authors of [7] show that any instantaneous fluctuation in lattice population causes a spontaneous symmetry break so that every *other* well is occupied. Now separated by a full wavelength, scattering from the atoms interfered constructively and the runaway process is truly manifested.

It may be useful now to take a step back and review this process as it is fundamentally the same in multi-mode cavities and with BECs. Atoms beginning in a nearly uniform spatial distribution coherently scatter pump light into a resonant cavity. Density fluctuations in the initial spatial distribution result in a net growth of the standing wave field which in turn influences the atoms to move toward the field intensity maxima (potential wells). Superradiant scattering ensues due to the developing periodic arrangement, accelerating the process. A population imbalance between "even" and "odd" wells is self-feeding until all the atoms have organized into one of the two degenerate sub-lattices. This condensation halts when heating or other forces stabilize the potential well growth.

Pursuant to this work Nagy, Asboth, Domokos, and Ritsch studied self-organization in a ring cavity and derived a full phase diagram for the system via a mean field approach [13]. Though this geometry actually supports two counter-propagating resonant modes, the results are qualitatively applicable to the geometry described above. The phase diagram is displayed in Figure 3 for a fixed cavity detuning similar to those in experiments. There are four general phases: the spatially uniform phase, two separate, but related, self-organized phases, and two regions of instability. The pump amplitude is normalized by the cavity decay rate κ and the collective coupling ζ is related to the atom density and therefore the atom number for fixed cavity dimensions.

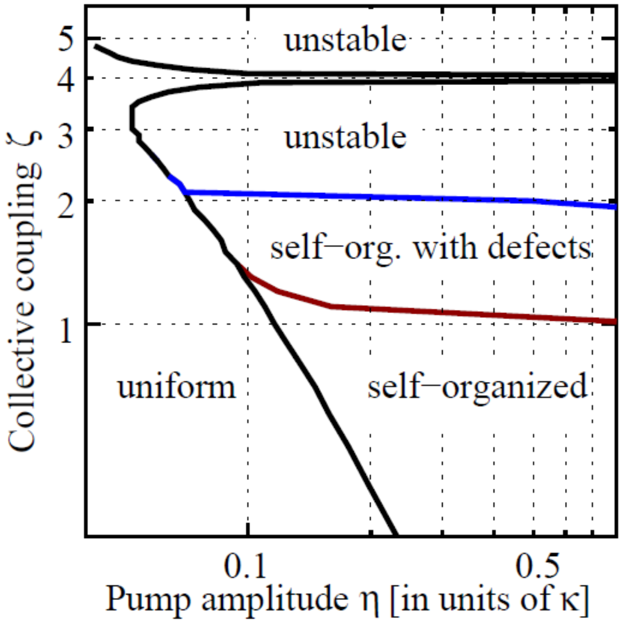

Figure 3: Phase diagram for thermal atoms confined to a one-dimensional single-mode cavity. The units of both axes are arbitrary. [13]

The square root relationship between the critical pump amplitude and the collective coupling holds true except for around ζ = 4 where the pump is no longer detuned from the cavity and the critical pumping amplitude diverges. The other interesting piece of the diagram is the phase which exhibits stable defects along with self-organization. In fact, these defects are atoms occupying points in the opposite sub-lattice at which secondary potential minima have formed due to the increased atom number.

Experiment

The work of Domokos and Ritsch provides several pointers towards experimental testing. First of all, they predict a quadratic dependence of resonant field intensity on atomic number. Because the cavity is inherently leaky (and necessarily so for the cooling process) one may simply measure the light output of the cavity as a measure of the field intensity within. As stated before, organization into the predicted structure should induce superradiant scattering into the cavity field and this will serve as a hallmark of spatial organization. Additionally, because there are two degenerate sub-lattices there should be some signature to tell which the system chose to occupy. In 2003, Black, Chan, and Vuletic performed the experiment proposed by Domokos and Ritsch and confirmed their theoretical predictions [9].

The experiment used a 7.5cm optical resonator, a retroreflected pump laser, and cesium atoms. Because cooling was not the primary focus of the experiment, the cesium atoms were cooled and prepared in a magneto-optical trap (MOT) prior to release into this system. Black *et al.* found that above a threshold pump intensity emission from the atoms into the cavity (as detected by a photodiode) was greatly enhanced. They found a cavity scattering to free space scattering ratio of 200, far above the expected ratios (of order unity or less) for non-collective phenomena such as random scattering or Raman lasing. Because this phenomenon is a collective effect, one may expect that the intensity threshold should decrease with atom number and increase with temperature and this is exactly what was observed. One should also note that because the experiment was not truly one-dimensional, the lattice configuration observed was actually a two-dimensional square lattice with sub-lattices being offset from each other. This slight variation was considered by Domokos and Ritsch and was shown not to affect their predictions [7].

The authors also confirmed the symmetry breaking prediction of Domokos and Ritsch. They measured the relative phase between the emitted light and pump laser both in steady state configuration and with periodic pulses of randomizing MOT light. For the steady state experiment very small phase drifts were measured even over long timescales, exactly as is expected if the atoms are simply trapped in one sub-lattice. If light pulses that randomize the atom positions are applied, we expect to see periodic phase jumps of π radians corresponding to the situation when opposite sub-lattices are selected upon reorganization. Figure 4 shows the experimental results. The MOT light pulses are applied around 1ms and 3ms in the plot. As can be seen in Figure 4(b), the phase of emitted light is constant until the MOT light pulses destroy the spatial ordering. When ordering is reestablished the opposite sub-lattice is occupied and we see a π phase shift. The sharp decrease in emission occurs because when the atoms are not spatially ordered the coherent scattering is no longer superradiant. Note that while the figure shows a phase jump each at each MOT light pulse, this need not be the case as the two sub-lattice configurations are equally probable.

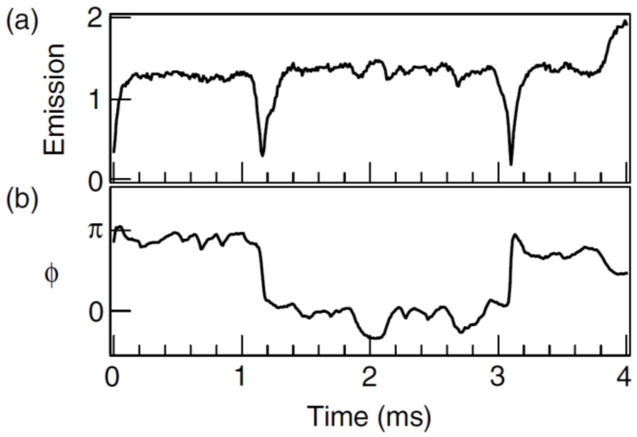

Figure 4: (a) Intensity of the emitted cavity light in arbitrary units. (b) Phase of the emitted light as compared to the pump laser. [9]

Black *et al.* also investigated the correlation between observing the atoms organize into the same sub-lattice after MOT light pulses versus the separation between pulses. For short pulse separations the same sub-lattice was more likely to be selected. As pulse separation increased both sub-lattices became equally likely. The decay from preferential sub-lattice choice to equal likelihood was observed to be exponential with a time constant of 11±3 μs.

Discussion

Black *et al.* have convincingly confirmed many of the predictions of Domokos and Ritsch. However, one important question that is left unanswered is the scaling of resonant mode intensity with atom number. Domokos and Ritsch predicted a quadratic relation as evidence of collective phenomena, but in the experiments the atom number was never systematically varied or measured, perhaps because it was not possible to do so.

This work was on the simplest system which would show self-organization phenomena, and is still very akin to the traditional ultracold atom simulations of condensed matter. For example the lattice sites are still rigidly set by the cavity geometry and the spontaneous symmetry breaking is only of a discrete nature, selecting which of two degenerate sub-lattices is occupied. On the other hand this system does represent a great step forward toward being able to study the dynamics of lattice formation and all the interesting problems that go along with it. And cooperatve, collective effects are definitely present as evidenced by the superradiant scattering and theorized quadratic dependence of scattering on atom number. Overall though, this system does not live up to the emergent phenomena enthusiast’s desire to be able to simulate things like crystal formation and supersolidity. For those problems we must move onto more complicated setups, specifically, resonant cavities more complicated geometry and quantum mechanical atoms.

IV. Single-Mode Quantum System

Theory

As a brief, but interesting, interlude before moving on to multi-mode cavities we will consider what happens when the setup described above is filled not with thermal atoms, but with a BEC. Because this system is quantum mechanical in nature a much more careful and detailed approach must be taken. This was done by Nagy, Szirmai, and Domokos who studied the problem as a case of coupled matter and radiation fields in the mean field approximation [10]. To complicate this more, the leaky nature of the cavity meant that a coherent Hamiltonian dynamics approach could not be used. The authors treated the system as a steady-state of a driven open system far from equilibrium and considered perturbations about that steady-state.

After developing the model, calculating the phase transitions, deriving the low lying excitations, investigating defects, and studying defect states the authors conclude that the self-organization behavior of a BEC is the same as for thermal atoms. This is interesting first because it is a quantum phase transition that moves smoothly into a classical one. Furthermore, thinking about the two level atoms as spins and recalling that they are all equally coupled, we realize an experiment realizing an effective Dicke model Hamiltonian is possible [11].

Experiment

Baumann *et al.* conducted this experiment and observed both the quantum phase transition theorized for the Dicke model and the self-organization predicted by Nagy *et al.* [12]. Their experimental setup is essentially a more refined version of that shown in Figure 2. Rubidium BECs were prepared in a crossed-beam dipole trap, and single-photon counting modules were used to measure the leaking cavity light. Baumann *et al.*'s fundamental results are shown in Figure 5. The lower graph shows the cavity light intensity as a function of time as the pump laser intensity is increased. As expected there is a dramatic increase in cavity intensity when the self-organized BEC atoms scatter superradiantly. The top parts of Figure 5 show snapshots of the trapped BEC before and after the transition. The small occupations seen in Figure 5(c) are attributed to the fact that the atoms were loaded into the one-dimensional standing wave of the pump laser.

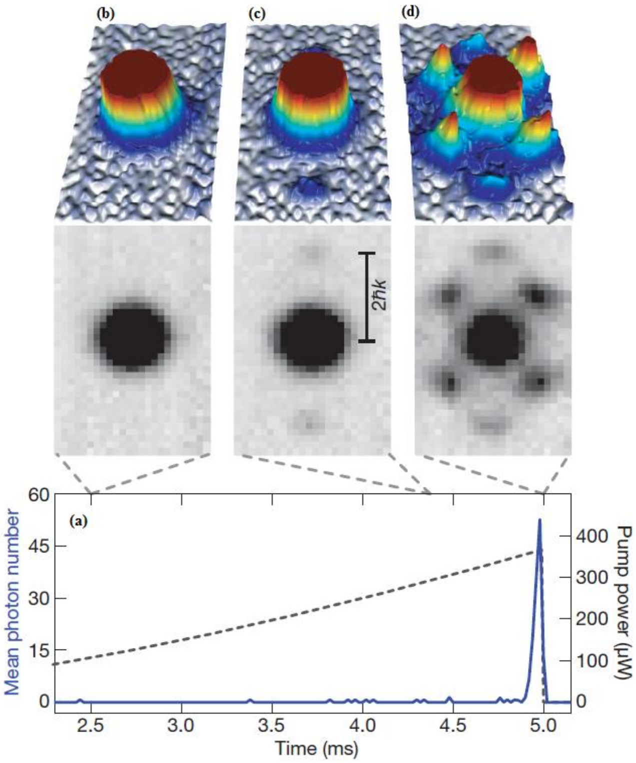

Figure 5: (a) Plot showing the cavity light intensity (solid line) as the pump power (dashed line) is increased. (b)-(d) Snapshots of the BEC before and after it has transitioned to a self-organized lattice. [12]

The authors also varied the pump-cavity detuning and the pump power to map out the phase diagram of the system and found that it agreed very well with theoretical predictions calculated in a mean field approximation. They observed that the threshold pump power scaled linearly with the effective cavity frequency as predicted by the Dicke model. Also the phase boundary scaled with recoil energy instead of with temperature, showing that the transition was indeed quantum mechanical and not classical [13]. Finally, the authors point out that in the self-organized state the crystal has superfluid phase coherence and may thus be interpreted as a supersolid.

Discussion

Though the subjects under investigation are now condensates and the analogies and interpretations are much more grand, the essential emergent phenomena have not changed from what was discussed previously. What is still happening is that atoms originally in some relatively uninteresting spatial configuration self-organize into a periodic structure because of after the original state has become unstable to (now quantum) fluctuations. That being said, the observation of these phenomena in a superfluid condensate and the experimental realization of both an effective Dicke model Hamiltonian and especially a supersolid are important advancements in condensed matter. Furthermore the experimental and theoretical prospects of using BECs are much richer so continuing research into multi-mode cavities has mainly focused on them.

V. Multi-Mode Systems

Theory

The advantages of single-mode cavity systems are that they are easily realizable in experiment and the phenomena are simple enough that they can be detected by techniques that are not prohibitively complicated. The tradeoff is that they are relatively well understood now and their applications to more complex condensed matter problems are limited. Multi-mode cavities, on the other hand, show promise in the study of things like topological defects, crystallization dynamics, frustration, and glassiness. This difference is due mainly to the fact that the many degenerate or nearly degenerate modes in a multi-mode cavity mean that organization represents a continuous symmetry breaking instead of the discrete symmetry broken in the single-mode case. Since there has so far been no experimental work done on BECs in multi-mode cavities, this discussion will contain only theoretical results and experimental considerations.

Gopalakrishnan, Lev, and Goldbart published a series of papers taking an in-depth look at emergent crystallinity, quantum phase transitions, supersolidity, frustration, and spin-glassiness of BECs in multi-mode cavity systems [3,14-15]. In general the authors restrict their analysis to spherical concentric cavities, a schematic for which is shown in Figure 6(b), because of the simple geometry. The first point to be made about the new system is that many of the cavity modes are degenerate or nearly degenerate. Thus the selection of lattice sites is not rigidly fixed by the geometry of the cavity and the symmetry broken by self-organization is continuous instead of discrete. This results in truly crystalline behavior with emergent rigidity to lattice deformations.

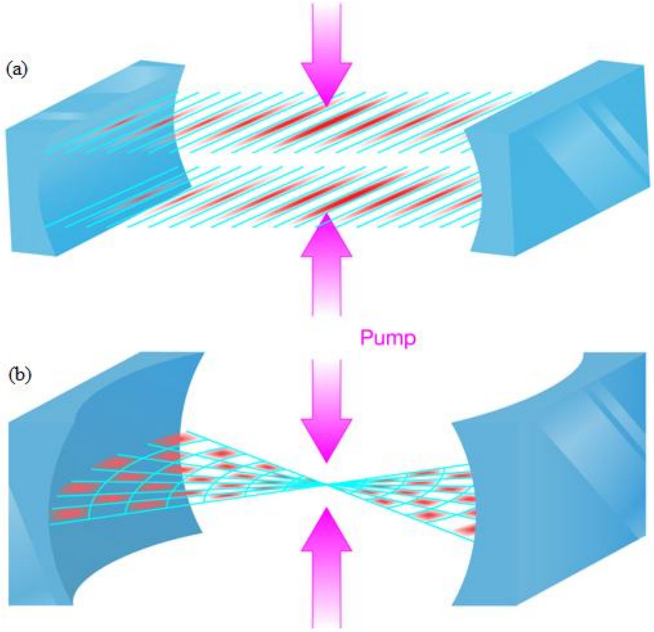

Figure 6: The geometries of (a) a linear single-mode linear cavity and (b) a multi-mode spherical concentric cavity are compared. [8]

Gopalakrishnan *et al.* use a functional integral quantum field theoretic approach to derive an action for the condensed atoms and reformulate it in terms of an order parameter for self-organization. One of the first main discoveries was that the action derived for BEC crystallization was not the Landau form of crystallization, but more akin to a Brazovskii action which is associated with convective pattern formation and liquid crystals[3]. This leads to the realization that the supersolid created in the experiments of Baumann *et al.* may more accurately be termed a "supersmectic" because it shows both superfluid and *liquid*-crystal order. Regardless of whether it is a supersolid or supersmectic, this setting has the potential to be immensely helpful to the study of supersolidity. In traditional condensed matter settings it is very difficult to establish whether or not a supersolid state exists (or indeed can exist), due in part to the fact that the experimental phenomenology of supersolidity is not well established. But this latter fact is due mainly to the former, and so a vicious circle exists to impede further discovery. The system at hand may help break the cycle because it provides easy means to detect both crystallinity (superradiant scattering) and superfluidity (the host of standard techniques already at the disposal of ultracold atomic physicists) [14].

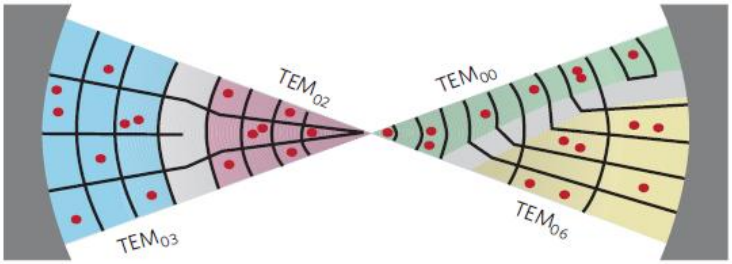

Figure 7: Example of a two-dimensional ordered state, including defects, that possible in a spherical concentric cavity. The lines represent nodes of the cavity light field which separate the even and odd anti-nodes. Dots represent BEC atoms. [3]

Multi-mode cavities also exhibit exciting effects that one normally associates with emergent phenomena like defects and frustration. Defects are easily discussed in the case of a single sheet of the pump laser's periodic potential structure, so they will be discussed first. Figure 7 shows a possible ordered state that contains defects. The black lines show the cavity light field nodes that separate the checkerboard pattern of even and odd antinodes. The dots represent atoms which one can see have (allowing for the defects and domain walls) all fallen into one sub-lattice. On the left side of Figure 7 we see dislocation defect that causes the formation of two different cavity mode domains, the TEM$_{02}$ and TEM$_{03}$. On the right side there is a textural variation defect that separates regions of TEM$_{00}$ and TEM$_{06}$. One should note that though they are shown to be in the figure, different modes may not be exactly degenerate due to cavity finesse and interatomic effects. This could change the possible structures for self-organization, but Gopalakrishnan *et al.* have proposed a simple method accounting for this in their analysis [3].

To investigate the rich possibilities for frustration in this system, we consider two sheets of the pump laser potential which are symmetrically off-center of the cavity. As is generally the case for frustration, we are looking at cases where different local energetics or constraints require organization into locally ordered regions. In Figure 8 this is seen as the radial change from TEM_{2m} to TEM_{1m}. The proximity of the different mode types to the sheets of interest in Figure 8(a) is associated with the preference of the system to organize into that mode type at that point. Clearly the TEM_{2m} modes are favorable near the center of the cavity while the TEM_{1m} modes are preferred at the edges. Defects between the domains can be discommensurations (triangle, left) or dislocations (squares, right). Frustration can also be exhibited between the sheets. The parity of the different cavity modes means that the checkerboard structures of the two sheets should be the same, but defects local to one sheet or the other add different additional considerations.

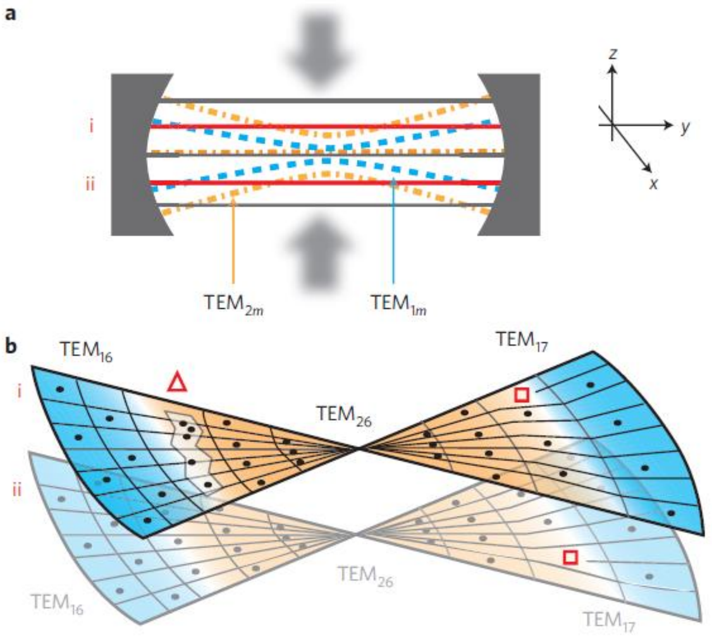

Figure 8: Frustration exhibited the self-organized structure of a spherical concentric cavity. (a) The two sheets of interest are identified by the labels (i) and (ii) and it is shown how they intersect two TEM cavity modes. (b) Details of the two sheets show the frustrated separation into different domains and the presence of defects. [3]

Discussion

Gopalakrishnan *et al.* give a very detailed and insightful investigation into the dynamics and properties of the emergent behavior that is possible with BECs in multi-mode cavities. However this is still a relatively early work and the field of related problems is so wide and varied that they constrain themselves to the simplest geometry and only some of the possible cavity field modes. In their analysis the authors assume all the cavity modes are degenerate which they admit is not quite the case. They do, however, propose a method to account for non-degeneracy. There is also some discussion of how this work translates to other multi-mode cavities, the conclusion of which is that the methods should still be valid and the results qualitatively similar. In general, the use of multi-mode cavities greatly enriches the field of study. Because the spontaneous symmetry breaking is continuous this work is much more applicable to the condensed matter problems of crystallization. Also the possibilities for defects and frustration are much wider.

VI. Conclusion

Optical cavities allow researchers to combine the tunability and precision of ultracold atom experiments with the emergent and compliant behavior that makes some of the most intriguing and exciting condensed matter problems just that. When placed in optical cavities and exposed to a transverse pumping laser, trapped atoms (whether thermal or quantum mechanical) exhibit self-organizing and pattern formation behavior. This can be as simple as selection from two degenerate sub-lattice structures, or as complicated as Bose-Einstein-condensed atoms forming a supersmectic with defects and frustration. We have seen that in single-mode cavities atoms pumped above threshold will self-organize into a periodic structure, breaking a discrete symmetry between sub-lattices and exhibiting superradiant scattering into the cavity field modes. In other cavities such as the spherical concentric geometry a host of degenerate or nearly degenerate cavity modes means that self-organizing atoms break a continuous symmetry in their pattern formation. This is the same action as crystal (or as was discovered liquid-crystal) formation, meaning that multi-mode cavity systems may be useful in expanding the abilities of ultracold atom experimentalists into the realm of crystal dynamics. The ease of identifying and characterizing supersolid behavior in these systems may help resolve the near impasse in supersolid research that exists in the traditional condensed matter setting.

At present the more basic theoretical work has been verified in experiment, but the experiments that approach the desired ability to simulate things like the dynamics of crystal formation and spin-glasses have not been conducted. Experimental work has, however, begun to scratch the surface of the field of quantum phase transitions with the observation of a transition under an effective Dicke Hamiltonian. It is probable that future theoretical and experimental studies stemming from this work will be different from and exceed any expectations one may now have.

VII. References

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