Flocking Behavior

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Abstract

Flocking is the phenomenon in which self-propelled individuals, using only limited environmental information and simple rules, organize into an ordered motion. This ordering even occurs at two dimensions where ordering is not possible equilibrium systems. This paper will explain the biological and physical motivation for studying flocking, and the computational, theoretical, and experimental efforts undertaken to explore this phenomenon.

Introduction

A large flock of birds flying is a delightful sight to children and adults alike. A diver's view of an underwater school of fish is even more awe-inspiring. Both are examples of collective motion: individuals moving together as one. Specifically, the phenomenon is termed *flocking*, though *schooling* is often used to describe the same phenomenon in fish. Flocking is the phenomenon that individuals all move with approximately the same velocity, so that they remain together as a group. Animals that exhibit flocking range in size from buffalo to bacteria.

Biological Basis

Biologists were the first scientists to investigate flocking. One of their questions was, "Why do animals flock together?" For this behavior to be favored in natural selection, it must provide survival or reproductive benefits for the population. Many such mechanisms have been postulated and investigated. However, this question is unfortunately not relevant to a physicist's interest in flocking, so it will not be further addressed in this paper.

Another more relevant question is, "What are the properties of a flock?" B. L. Partridge says that a school of fish can consist of a minimum of three members. For instance, in a group of two fish, their behavior shows one to be the leader and the other the follower. However, a group of three fish behaves as if none of the fish are leading, but rather each fish has an influence on the whole [1]. E. Shaw describes this as "a truly egalitarian form of organization" [2]. Furthermore, it was found that while fish in a school have preferences for nearest neighbor distance and even a preference for a specific positional angle with respect to their neighbors, this was only expressible as a statistical average over a long period of time. As a result, fish maintain no rigid "crystal structure" in a school. As they move, an individual fish's position in a school can change from front to back and edge to interior as the school moves and turns [1]. Flocks of birds and schools of fish are quite similar in this way. Birds also maintain no rigid structure, although there is evidence for influence toward such a rigid spatial structure. At the very least, there are preferences for where a bird positions itself with respect to nearest neighboring birds. In particular, there is an aversion to having a nearest neighbor in front of and below itself [3]. This would seem to be explained by a bird's desire to have a view of the ground. There is, however, no evidence for any preference as to the positioning of second-nearest neighbors. Thus, the approximate spatial ordering only extends to first-nearest neighbors and also constantly changing with time.

Another relevant question is, "How do animals maintain the order of a flock?" The first serious investigator of this question was A. E. Parr. According to E. Shaw, Parr sought to debunk the idea that the collective behavior of fish schools required them to have a collective mind or communicate through a supernatural long-range force [2]. He found that blinded fish lost their ability to school, thus showing that vision was the most important sense involved in fish schooling. Furthermore, fish in total darkness do not school, though schooling can occur in extremely dim light. However, it was later discovered that blinded fish can indeed school. This is due to an extra sense exhibited in fish and not in land animals. Fish have a line of hair cells that run lengthwise down their bodies called lateral lines. These hair cells are similar to those found in the human inner ear in that they are of the same cell type and sense the flow of fluid around the fish. Using this sense, fish can feel nearby fish movement and swim together with them. Regardless of these additional details, Parr's original point is clear: fish apparently school together using only biological senses. Furthermore, these biological senses are naturally limited in range. Under the most favorable conditions, sight would decrease in sensitivity like the inverse square of the distance (the area subtended by diverging light rays), and lateral line sense like the inverse cube of the distance (the volume of water displaced).

Insight from Computer Graphics

While biological arguments provide strong support for the idea that the limited ranged of senses is the cause of flocking behavior, further support for this came from a very different field: computer graphics. C. W. Reynolds wanted to create an easy way for animators to create realistic-looking animations of flocks of birds. According to [4], in order to animate a flock of birds or a similar group, traditionally, an animator would have to direct the path of each bird individually. Not only would it be difficult to make the flock appear to be moving naturally to a human audience, but it would be prohibitive to make changes after planning each original flight path. He envisioned a program that would generate realistic-looking flight paths for a flock of birds using no central authority on the flock and only local rules for the interactions between birds, or *boids*, as he called them (short for "birdoids").

The three rules of interaction were 1) collision avoidance, 2) velocity matching, and 3) attraction toward boids within a short radius. These rules follow the constraints that each boid has a maximum velocity and acceleration. Remarkably, these three simple rules, when properly applied, produce realistic-looking flocking behavior. The phrase "properly applied" requires further explanation. Each of the three rules generates an influence on the boid, so some intelligence is required to determine the boid's acceleration in the face of conflicting influences. Simply adding the influences as force vectors is not ideal, because if forces are nearly equal and opposite, the tendency is to remain unchanged in direction, but indecision can be more costly than simply choosing to follow one influence over another. Thus he ranked them in the order they are listed above. For example, if collision avoidance is critical, it will take precedence over the other rules, but if there is little force from the collision avoidance rule, then the others can apply fully or in part. More specifically, the boid is only given a force quota to spread among the three rules. Force is given to the highest priority rule, and then to the lower priority rules until the total force required by these rules matches the force quota. Such a system reduces computational effort, because each rule does not always need to be computed. This would also reduce the mental effort requirement for a flocking animal. Such a hierarchical decision tree was implied in [4] when B. L. Partridge showed that in a conflict between following the visual sense or the lateral line sense, schooling fish followed the visual sense. Reynolds also added an obstacle avoidance rule, based on long-range vision, and an external goal-seeking rule so that animators could have a way to direct the flock in a way that would not disturb the natural behavior of the boids.

The First Physical Model

Reynolds' program is a useful tool for animators, but as a model, it is still too complex to be useful to physicists. The physics community took an interest in flocking after a paper by

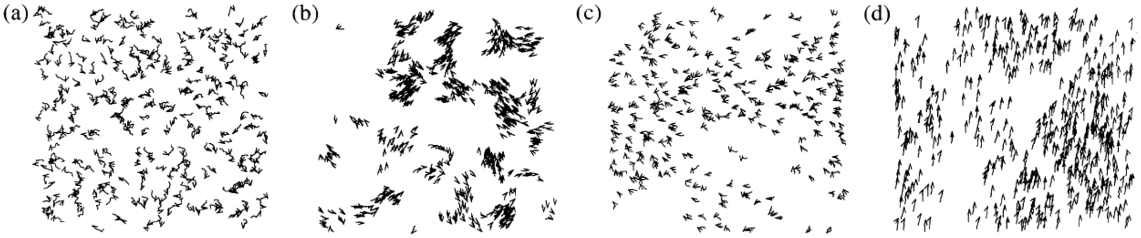
T. Vicsek [5]. He proposed a very simple simulation model and demonstrated that it exhibited flocking. His model is paraphrased as follows:

In a 2-dimensional box of side length L with periodic boundary conditions, at $t = 0$, N particles were distributed in random positions $\mathbf{x}_i$. Velocity for each particle was constrained to a constant value v, and initial directions θ_i for each particle's velocity were randomized. At each time step, position and direction followed

$$\mathbf{x}_i(t+1) = \mathbf{x}_i(t) + \mathbf{v}_i(t)\Delta t \qquad \text{and} \qquad \theta_i(t+1) = \left\langle \theta_i(t) \right\rangle_r + \Delta\theta$$

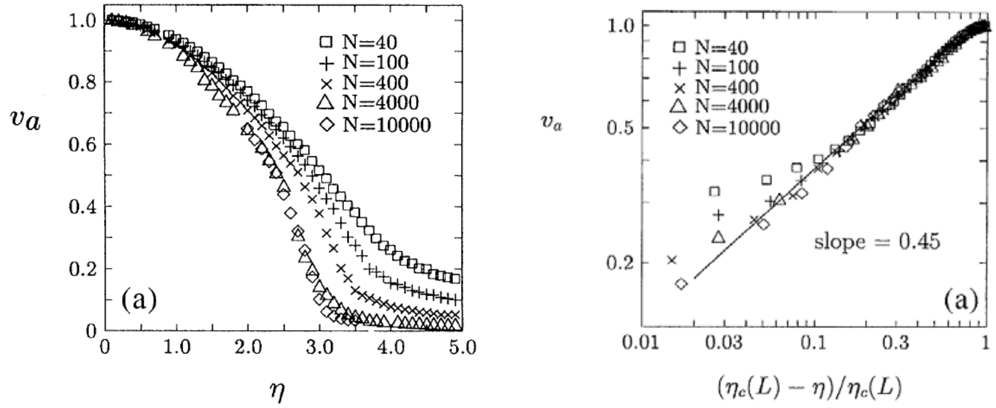
where $\left\langle \theta_i(t) \right\rangle_r$ is the average direction of the velocities of particles within a distance r of the ith particle. Particles further away have no influence on the given particle. $\Delta\theta$ is a random number uniformly chosen between $-\eta/2$ and $+\eta/2$. This creates fluctuations in velocity direction, representing the effect of finite temperature.

Viscek made an analogy between his model and a model of a ferromagnet. The velocities of the particles are analogous to the spins and the interactions are local. When $v = 0$, the model exactly corresponds to the 2D XY model, albeit not on a regular lattice. When simulated with different parameters for fluctuations and density, he found four different behaviors [5]:



a) High noise, low density: particles moved independently
b) Low noise, low density: particles formed groups that were independent
c) High noise, high density: particles moved with some correlations
d) Low noise, high density: all particles moved in the same direction

The last behavior is physically interesting because it is analogous to a ferromagnetic state: rotational symmetry is spontaneously broken. The average velocity is zero except in the last case, so average velocity is a suitable order parameter. He measured the critical exponents and also found that the size of the scaling region increased as the system size was increased, strong signs that this is a true phase transition. This was something that the physics community could be excited about: a very simple model that produces a "novel type of phase transition" as the title of Vicsek's paper proclaims [5].



Above are two plots from [5] showing the increase of the scaling region as *N* is increased. v_a is the average velocity, the order parameter.

A Paradox?

This transition is especially novel because the transition happens in a system of 2 dimensions. According to the Mermin-Wagner theorem, no ferromagnetic state exists in a system of 2 or fewer dimensions [6]. The theorem generally states that there can be no spontaneous breaking of a continuous symmetry in 2 or fewer dimensions. The heuristic argument is that in low dimensions, fluctuations become too strong for long range order to develop. This theorem, however, is not a fundamental principle, but rather an exact mathematical result that holds for a huge class of Hamiltonians [7]. In particular, it requires the system to be in thermal equilibrium. Therefore, an apparent violation of this theorem in a non-equilibrium system such as this one is not surprising. Clearly, the movement of the particles is the cause, because at $v = 0$, the system is merely the 2D XY model in thermal equilibrium. However, a full understanding of the physical origin of this violation and its consequences requires an analysis of the model behind the simulation.

A Theoretical Approach

J. Toner and Y. Tu performed such an analysis in [8]. First, they reasoned that physics should only be interested in the long length-scale properties of a flock. In this regime, the flock can be modeled as a continuous hydrodynamic model. In such a top-down approach, universal properties could be found for all flocks obeying certain basic symmetries. As they explained, a flock has only two symmetries: conservation of mass and rotational invariance. In particular, momentum is not conserved due to a lack of Galilean invariance. To understand why flocks are not invariant under Galilean transformations, consider a flock of birds moving coherently, each with the approximately same velocity. Taking a Galilean transformation by subtracting the average velocity of the flock from each bird leaves a flock of roughly stationary birds. This is clearly a different physical situation.

As this is a top-down approach rather than deriving equations of motion through approximations of individual interactions, they are written as the most general equations of motion consistent with the symmetries. All of the details about individual properties are handled by a small number of coefficients. Following [9], which has a more complete derivation, the equations of motion are

$$\begin{aligned}
&\partial_t \vec{v} + \lambda_1 \left(\vec{v}\cdot\vec{\nabla}\right)\vec{v} + \lambda_2 \left(\vec{\nabla}\cdot\vec{v}\right)\vec{v} + \lambda_3 \vec{\nabla}\left(\left|\vec{v}\right|^2\right) \\
&\quad = \alpha\vec{v} - \beta\left|\vec{v}\right|^2 \vec{v} - \vec{\nabla}P + D_B \vec{\nabla}\left(\vec{\nabla}\cdot\vec{v}\right) + D_T \nabla^2 \vec{v} + D_2 \left(\vec{v}\cdot\vec{\nabla}\right)^2 \vec{v} + \vec{f}, \\
&P = P(\rho) = \sum_{n=1}^{\infty} \sigma_n \left(\rho - \rho_0\right)^n, \\
&\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot\left(\vec{v}\rho\right) = 0.
\end{aligned}$$

The first equation has extra convective terms $\lambda_2\left(\vec{\nabla}\cdot\vec{v}\right)\vec{v}$ and $\lambda_3\vec{\nabla}\left(\left|\vec{v}\right|^2\right)$ due to the lack of Galilean invariance, and an adjustable parameter λ_1, which would simply be 1 in the usual Navier-Stokes equations. The terms $\alpha\vec{v} - \beta\left|\vec{v}\right|^2\vec{v}$ give the birds a preferred speed. The three D_i terms represent diffusion or viscosity. They are all present because they are consistent with rotational invariance and are third order or less in v. The σ_n are phenomenological parameters representing attraction and repulsion effects between birds in an attempt to maintain an ideal density ρ_0. The last equation is simply conservation of mass.

Through a full dynamical renormalization group analysis, the authors concluded that the linear approximation to the hydrodynamic equations of motion become invalid for $d < 4$. The renormalization group analysis is very difficult and could not explicitly solved for the critical exponents in $2 < d < 4$. In 2 dimensions, however, due to the fact that there is only one dimension for the Goldstone mode, the problem is tractable. This is fortunate because 2 dimensions is where the existence of a phase transition is surprising. The exact critical exponents in $d = 2$ are

$$\zeta = \frac{3}{5}, \qquad z = \frac{6}{5}, \qquad \chi = -\frac{1}{5}$$

Defined by the scaling

$$\begin{aligned}
&\vec{x}_\perp \to b\vec{x}_\perp, \\
&x_\parallel \to b^\zeta x_\parallel, \\
&t \to b^z t, \\
&\vec{v}_\perp \to b^\chi \vec{v}_\perp \\
&\delta\rho \to b^{\chi_\rho} \delta\rho
\end{aligned}$$

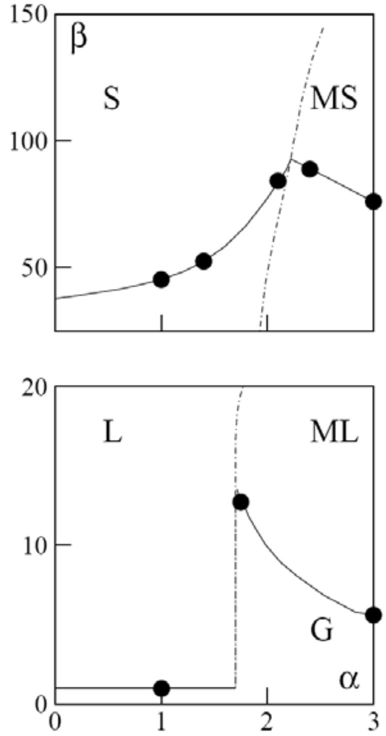
Since the scaling exponent for transverse velocity, χ, is negative, velocity fluctuations at long length scales are suppressed; thus, long range order is present in 2 dimensions. The authors continue to show that while this result also holds when the dimensionality is greater than 2, exactly 2 dimensions are favored for movement while the others are disfavored. This is, of course, the situation with birds, because while they are free to fly in 3 dimensions, flight is principally in 2 dimensions.

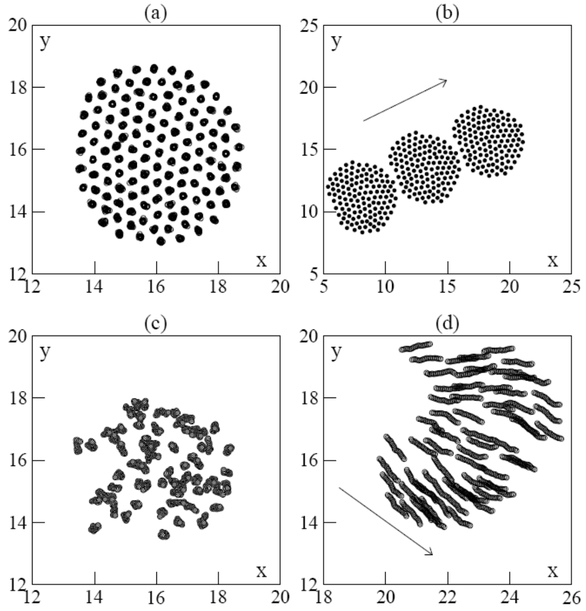
Besides this impressive renormalization group result, Toner and Tu also calculated sound speeds, which show a complex, directionally dependent structure and correlation functions, which can be measured in experiments of real flocks.

Further Results from Simulations

The hydrodynamic model of flocks gives universal behavior and it can be generalized into other situations, such as allowing flock members to be born and die in the dynamics so that mass is no longer conserved, or having a flock of several different species with different properties [9]. However, many questions cannot be solved analytically, so new models and simulations are still important tools to investigate properties of flocks.

One such property is that birds of a flock group together, not merely align their directions, although even in Vicsek's original model, which has no attractions, flocks of independent birds are observed. When Vicsek's model [5] is taken to its low density limit, simulating birds in wide open space, they disperse because there is no attraction to pull them back together once they escape each other's critical radius *r*. More precisely, for any fixed level of noise, decreasing the density will disperse all flocks, but localized flocking will occur at all densities for sufficiently low noise [10]. An extension of the model, found in [11], includes both a Lennard-Jones type interaction and an alignment interaction. Because of this attraction, five phases appeared in the zero-density limit.





Above: Depictions of four phases:
a) solid b) moving solid c) liquid d) moving liquid

To the left is an approximate phase diagram in the zero density limit. The four phases pictured are present as well as the gas phase. Parameters α and *β* are the strengths of the alignment interaction and the Lennard-Jones interaction, respectively. Both figures are from [11].

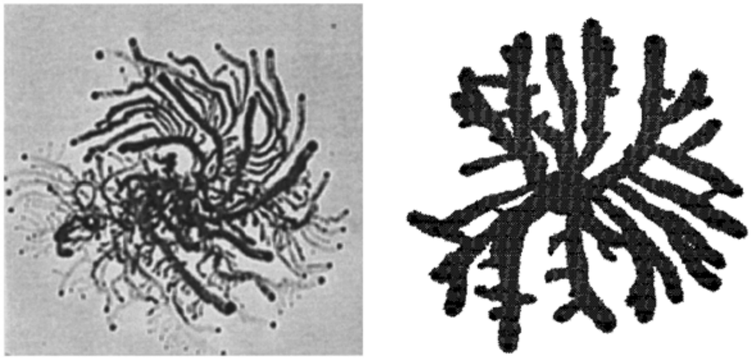
Another Kind of Collective Behavior

Besides moving together straight ahead, fish, birds, and other animals also exhibit another collective behavior: vortex motion. Fish and birds sometimes circle coherently while feeding or looking for food, i.e. vultures. But, perhaps the most familiar sight would be of a swarm of insects, such as gnats around you head on a hot day or moths around a light at night. Indeed, this collective behavior is sometimes called *swarming*, although that tends to mean simply an aggregation of insects, rather than a group with coherent motion.

In Vicsek's model, vortices were thought to be unstable for any nonzero v_0 in [10], but V. L. Kulinskii and collaborators in [12] have recently shown mathematically that Vicsek's model allows for vortical solutions in 2D. Furthermore, long-lived vortices were observed in a simulation of Vicsek's model with interactions in [13]. Nevertheless, vortices in nature often form by boundary conditions, external fields, or exotic interactions.

One example of a vortex being formed due to an external field is the phenomenon of *Daphnia* swimming in a circular fashion around a shaft of light, as studied in [14]. The tiny zooplankton, while moving continuously, are attracted to light, so they circulate around the light beam. Such creatures are well studied in the laboratory because of their small size. For *Daphnia* with low density, half of the population circles clockwise and half counterclockwise. For a high enough density, all of the *Daphnia* swim the same direction. This is another example of spontaneous symmetry breaking. However, since the symmetry is discrete, clockwise or counterclockwise, this system is similar to the flocking phenomenon in 1 dimension, which is also extensively studied. A hydrodynamic model was also developed for this system, but it, like Y. Tu's model, is difficult to apply to real situations. The authors in [14] demonstrate that a local model of collision avoidance, based on assumed properties of *Daphnia* visual ability, account for the spontaneous symmetry breaking and create realistic *Daphnia* behavior.

Another mechanism to generate vortices is *chemotaxis*. Bacteria with means of locomotion are often attracted toward specific chemicals. The bacteria studied in [15] create a very interesting structure. Vortices of bacteria spread out from a central source, leaving a trail of bacteria as they move. If a vortex becomes too large, it can break up into smaller vortices, creating diverging branches. As explained in [15], while chemotaxis can simply mean an attraction toward a chemical gradient, these bacteria use chemotaxis for signaling. Essentially, bacteria release chemoattractors into the medium. Bacteria naturally are directed along the gradient of chemoattractors, which would guide bacteria to join a large clump. This interaction is exotic because the chemoattractant remains in the medium for a long time, so a bacterium can be directed to find a clump of bacteria even if the clump has since moved on. This makes the interaction effectively long-ranged. Pictured below are a photograph of a real system of bacteria and a simulation of the new model. The agreement between the two is striking.



Finally, in the case of *Dictyostelium*, a vortex state is formed, but evidence shows that chemical signaling is not needed. When researchers used a strain of non-signaling *Dictyostelium*, vortices still formed, albeit only in higher densities than normal bacteria. If chemical signaling is not what ultimately causes vortex motions, then *Dictyostelium* could be an example of vortex behavior without long-range interactions or external fields [16].

The phenomenon of vortex collective motion is included in this paper because it is another kind of collective motion that can be more easily explored experiments in a laboratory. Also, since causal mechanisms for vortex motion are so diverse, this phenomenon requires more of a bottom-up approach, where models are created based on biological insight and compared to real experiments.

Future Directions

When T. Vicsek's paper [5] first introduced flocking to the physics community, it sparked interest because it provided a simple model that exhibited a new type of order. Then a hydrodynamic equation was developed [8], opening the door for physicists to explore the properties of this universal equation through more elaborate models. Finally, some physicists become interested in creating models more faithful to biological systems themselves, in order to make specific predictions or interpret experiments. These three directions still have room to grow.

Reference [9] introduces a new type of order that can be found in a system of self-propelled particles: that of *apolar nematic order*. This would describe a self-propelled particle that is symmetric front and back, so that it moves forward or backward with equal probability in each time step. The ordered state would thus be one where the axes of particles were aligned, but by definition there would be no net velocity. The authors in [9] express optimism that there are more phases to be discovered because, as they say, "there is every reason to expect that every phase known in equilibrium condensed matter systems has an analog in flocks." That may be too optimistic, but the discovery of another ordered phase would be significant.

There is certainly much more to explore with regard to models based on hydrodynamics. The work in [13] convinces this author of that. It shows that even Vicsek's model, the simplest physical model of flocking, has very interesting phase transition dynamics. If that is the case, then extensions to the model as proposed in [#7] such as multiple component flocks or creation and annihilation of boids should prove to be at least as interesting.

Finally, the connection to biology is no less important. Accurate measurements of flocks can be used to check the limits of validity of the hydrodynamic model, or physicists could apply their insight and tools to make a contribution to theoretical biology.

Conclusion

This is only an introduction to flocks and other collective behavior, from biological motivations to universal results from the most powerful methods of physics. It is also hoped that the work thus far in this field is only just beginning. There is still a lot of interesting physics to explore.

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* = particularly good