Color Superconductivity

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May 2006

Abstract

Color superconductivity is a phenomenon predicted to occur in quark matter if the baryon density is sufficiently high (well above nuclear density) and the temperature is not too high (well below 10^{12} kelvins). Color superconducting phases are to be contrasted with the normal phase of quark matter, which is just a weakly-interacting Fermi liquid of quarks.

In theoretical terms, a color superconducting phase is a state in which the quarks near the Fermi surface become correlated in Cooper pairs, which condense. In phenomenological terms, a color superconducting phase breaks some of the symmetries of the underlying theory, and has a very different spectrum of excitations and very different transport properties from the normal phase.

1 Introduction

Quantum Chromodynamics(QCD) has been the best theory of the strong interaction responsible for binding together protons and neutrons within the atomic nucleus for over 25 years. Nucleons are not treated as the fundamental objects but as the composite states of more elementary particles called *quarks* and *gluons* of size roughly 1fm. Quarks have spin-$\frac{1}{2}$. Gluons are vector gauge bosons that mediate strong color charge interactions of quarks in QCD.

First, I should explore the consequences of the quark picture for the thermodynamics of strongly-interacting matter, i.e., the behavior as conditions such as temperature and density are varied. Fig. 1 shows a proposed phase diagram for QCD. The names of the various phases are shown in green, and the environment in which they might be found in black. Phase coexistence lines are shown as solid lines, critical points as filled circles, and crossovers by shaded regions. The control parameters are temperature T and baryon chemical potential μ. There is no process within QCD which can change the number of baryons N_B minus the number of anti-baryons $N_{\bar{B}}$; in other words we can identify a conserved quantum number $B = N_B - N_{\bar{B}}$ called baryon number. Quarks and anti-quarks carry $B = \pm\frac{1}{3}$ respectively. Now, for systems in which baryon number is allowed to vary, the most convenient thermodynamic potential to consider is the *grand potential* $\Omega(T, V, \mu) = E - TS - \mu B$. Thermodynamic equilibrium is reached when Ω is minimised, and for a system in equilibrium we recognise μ as the increase in E whenever B increases by one. When systems are analysed using the grand canonical ensemble μ is kept as a control parameter, and the baryon density $n_B = B/V$ is a derived quantity whose value depends on the details of the equation of state $n_B = n_B(T, \mu)$.

In the bottom left-hand corner of the phase diagram Fig. 1 where T and μ are both small, the thermodynamic behavior can be described in terms of a vapour of hadrons, which is the composite bound states of quarks and/or antiquarks. As T is raised, eventually there comes a point where phase tranisition occurs to a phase where the dominant components are no longer hadrons but the quarks together with gluons. Since quarks and gluons play similar roles in QCD to the electrons and photons of QED, the phase is called the *quark-gluon plasma* (QGP). Along the μ-axis, the nuclear matter is at $\mu_o \simeq 922$MeV, given by the nucleon rest mass minus the binding energy per nucleon, which is estimated from empirical models of nuclei such as the liquid drop model. Keeping raising μ, there is believed to be a phase transition at a larger value of μ to a phase in which quarks rather than nucleons are the dominant degrees of freedom. Such quark matter may conceivably be found at the cores of compact astrophysical objects such as neutron stars. It has been speculated that Fermi surface phenomena analogous to the Bardeen-Cooper-

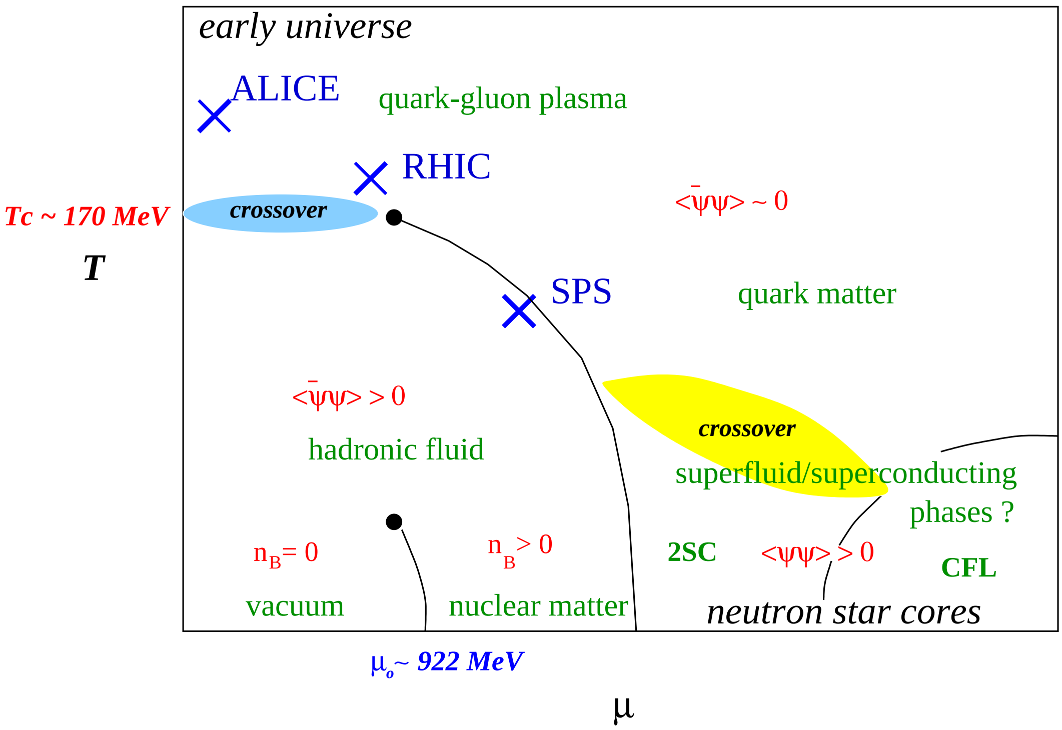

Figure 1: Proposed phase diagram for QCD. SPS, RHIC and ALICE are the names of relativistic heavy-ion collision experiments. 2SC and CFL refer to the diquark condensates).

Schrieffer (BCS) instability, responsible for superconductivity in metals and superfluidity in liquid ^{3}He at low temperatures, may play an important role. In the lower-right region of the phase diagram, therefore, QCD becomes a branch of condensed matter physics.

2 History

The first physicists to realize that QCD implies color superconductivity of quark matter at high density were Stephen Frautschi, a professor at Caltech, and his graduate student Bertrand Barrois. Barrois was able to get part of his work published in the journal Nuclear Physics [2], but the referees of that journal rejected the longer manuscript based on his thesis, which impressively anticipated later results such as the exp(-1/g) dependence of the quark condensate on the QCD coupling g. Barrois then left academic physics.

In the early 1980s color superconductivity received renewed attention from David Bailin and Alexander Love at Sussex University, who studied various pairing patterns in detail, but did not give much attention to the phenomenology of color superconductivity in real-world quark matter [3].

Apart from papers by M. Iwaskai and T. Iwado of Kochi University in 1995 [4], there was little activity until 1998, when there was a major upsurge of interest in dense quark matter and color superconductivity, sparked by the simultaneously published work of two groups, one at the Institute for Advanced Study in Princeton [6] and the other at SUNY Stonybrook [7]. These physicists pointed out that the strength of the strong interaction makes the phenomenon much more significant than had previously been suggested. These and other groups went on to investigate the complexity of the many possible phases of color superconducting quark matter, and perform accurate calculations in the well-controlled limit of infinite density. Since then, interest in the topic has steadily grown, with current research focussing on the detailed mapping of a plausible phase diagram for dense quark matter, and the search for observable signatures of the occurrence of these forms of matter in compact stars.

3 The properties of QCD

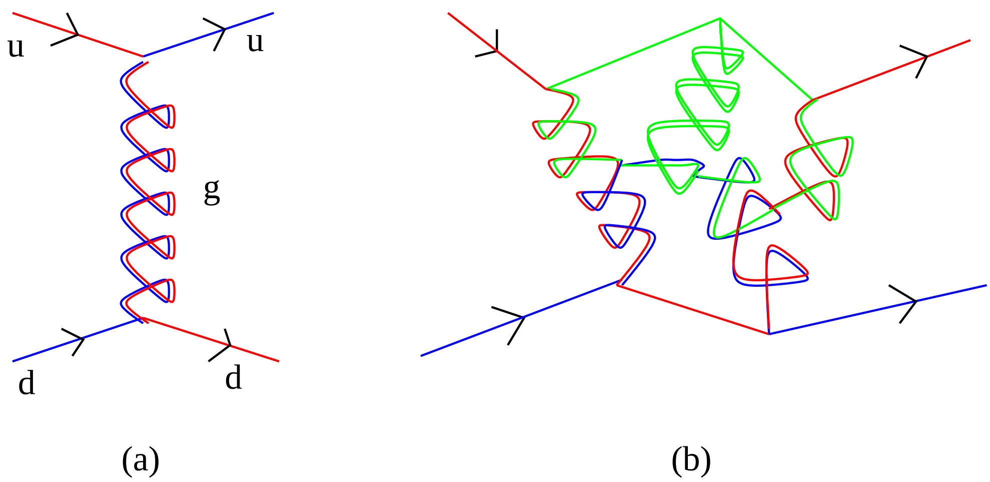

Figure 2: Gluon exchange between quarks. For small inter-quark separation, single gluon exchange (a) dominates. For larger separations gluon self-interactions (b) are also important.

The fundamental interaction between quarks in QCD arises from the exchange of spin-1 particles called *gluons* g, shown schematically in Fig. 2. Gluons are present inside hadrons and are thus also partons, but carry zero baryon number. In QCD the corresponding quantity is called *chromoelectric flux*, and the gluon is the quantum of the chromoelectric and chromomagnetic fields. By contrast with QED, however, single gluon exchange in QCD only gives an accurate description of the force between quarks at very small distances. At larger separation, things become much more complicated because as well as interacting with q and $\bar{q}$, gluons can interact with themselves, in contrast to photons which are electrically neutral and hence do not self-interact. Most of our quantitative knowledge about the quark-gluon interaction at distance scales $\gtrsim O(0.5\text{fm})$ comes from formulating the equations of QCD on a discrete mesh of spacetime points and modelling quantum fluctuations of the q, $\bar{q}$ and g fields by numerical simulation, as known as *lattice gauge theory*. It turns out that the potential between a $q\bar{q}$ pair at separation r is

$$V(r) = -\frac{A(r)}{r} + Kr. \tag{1}$$

For small r the first term in (1) dominates, and describes an attractive Coulomb-like interaction. It is important to note, however, that the coefficient A itself has a mild scale-dependence due to quantum effects[1]. Detailed analysis reveals that $A(r) \propto 1/\ln(r^{-1})$, implying that the interaction between quarks gets weaker as their separation decreases. In the limit $r \to 0$ the quarks can be considered non-interacting, a property known as *asymptotic freedom*. Asymptotic freedom enables inelastic electron-proton collisions to be interpreted in terms of scattering of high-momentum virtual photons off almost-free partons; its theoretical discovery thus played a pivotal role in establishing QCD as the theory of the strong interaction.

As r increases the second term in (1) takes over, implying that the $q\bar{q}$ potential rises linearly with separation. This can be understood by considering Fig 3, which shows lines of chromoelectric flux between a quark and anti-quark. By contrast with QED, the field lines do not spread out in space to form a dipole pattern, but remain localised within a narrow region of diameter $\sim 0.7\text{fm}$ between the sources known as a *fluxtube*. Within the tube the chromoelectric field strength is roughly uniform; therefore the energy of the system is proportional to the length of the fluxtube, in agreement with Eq.(1).

Let us develop the analogy with QED a little further. The QCD quantity corresponding to electric charge is called *color*, and comes in three forms: "red", "blue". and "green". Both quarks and gluons have color quantum

[1] A is proportional to strong force coupling constant α_S.

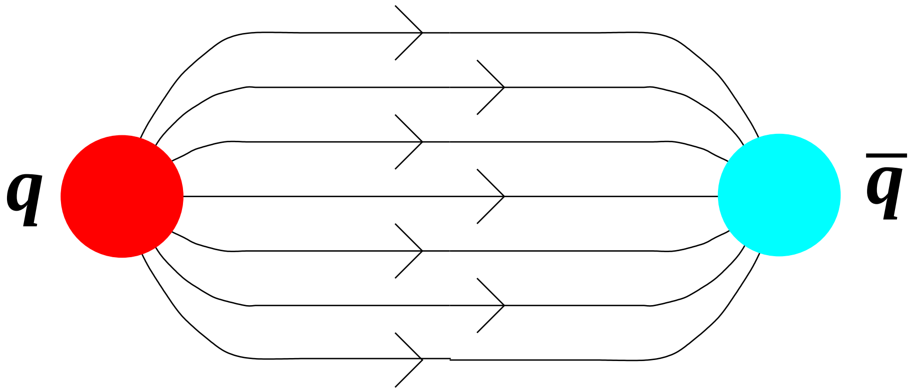

Figure 3: Chromoelectric flux between a $q\bar{q}$ pair.

numbers. Anti-quarks have complementary colors "anti-red", "anti-blue" and "anti-green". The gluon exchange responsible for QCD forces carries color between different particles; hence the colored nature of the gluons can be regarded as the origin of their self-interaction. The only stable finite-energy systems are those formed from complementary combinations of color, such as $q_g\bar{q}_{\bar{g}}$ mesons or $q_r q_b q_g$ baryons. The $q\bar{q}$ pair formed in string-breaking is produced with exactly the right color combination to maintain this overall color-neutrality. Forces between color-neutral objects such as nucleons within the nucleus can be viewed as a second-order effect akin to Van der Waals forces between neutral atoms.[2] From our viewpoint the most important aspect is that colored objects such as isolated quarks or gluons are never observed. This property of QCD, supported by all the theoretical considerations of the previous paragraphs, and to date not contradicted by experiment, is called *color confinement*.

The two crucial ingredients we have identified, asymptotic freedom and color confinement, are built into a much simpler description of the strong interaction known as the *Bag model*, in which massless quarks move freely within a spherical hadron of radius R, but are prevented from travelling further by an inwards-acting pressure due to the confining nature of the bulk vacuum.

We now come to another important aspect of QCD dynamics. When a particle with spin $\vec{s}$ propagates, it is possible to define a quantity called *helicity* $h = \vec{s}.\vec{k}/|k|$, which is the projection of the spin axis along the direction

[2]In fact inter-nucleon forces can be modelled by the exchange of color-neutral mesons such as pions. The dimension of a nucleus is comparable with the Compton wavelength of a pion.

of the particle's motion, defined by the momentum $\vec{k}$. For a spin-$\frac{1}{2}$ particle like the quark, there are two possible helicity eigenstates $h = \pm\frac{1}{2}$, usually referred to as left- and right-handed states since they are related by a mirror reflection. A quark's helicity is not altered by either emission or absorption of a gluon; hence in the absence of any other effect one might deduce that the numbers of left- and right-handed quarks are separately conserved in QCD, leading to two good quantum numbers B_L and B_R. A moment's thought, however, shows that this can only be the case if quarks have zero mass and hence travel at the speed of light. Otherwise, it is possible to Lorentz boost to a frame in which the quark's momentum has the opposite sign; since angular momentum along the boost axis is not changed, helicity in the new frame must also have the opposite sign. We conclude that in a relativistically covariant treatment massive quarks must be described as a superposition of helicity eigenstates, the mass m parametrising the overlap between them and hence effectively the rate of $L \leftrightarrow R$ transitions. Since in this case only $B = B_L + B_R$ remains as a good quantum number, we say that the *chiral symmetry* relating left and right-handed quarks and enabling them to be thought of as independent particles is broken by the quark mass.

Chiral symmetry breaking (χSB), like that of other symmetries in many-body or quantum field theory, can occur via the theory's own dynamics. We have seen that QCD is responsible for a strong attractive interaction between q and $\bar{q}$. The force is so strong, in fact, that the state usually considered as the ground state or *vacuum*, namely that of no particle present, is actually unstable with respect to formation of a *condensate* of tightly bound $q\bar{q}$ pairs, much as the ground state of superfluid helium is a Bose condensate of He atoms in the lowest quantum state. Let us denote the vacuum by the ket $|0\rangle$, and the field operators which create or destroy a quark when acting on a ketas $\bar{\psi}, \psi$ respectively. A χSB vacuum is then given by

$$\langle\bar{\psi}\psi\rangle \equiv \langle 0|\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L|0\rangle \neq 0. \tag{2}$$

Since neither $|0\rangle$ is annihilated by ψ, nor $\langle 0|$ by $\bar{\psi}$, the vacuum must contain $q\bar{q}$ pairs. In QCD it is believed that the value of $\langle\bar{\psi}\psi\rangle \simeq (250\text{MeV})^3$, which can be interpreted as the number of such pairs per unit volume.

As Eq.(2) implies, the condensate pairs ψ_L with $\bar{\psi}_R$, and *vice versa*. Since $\bar{\psi}\psi$ leaves $B_L + B_R$ invariant but changes $B_L - B_R$ by two units, a non-vanishing condensate implies that the latter quantity has no definite value in the vacuum and only B remains as a good quantum number. A left-handed quark propagating through such a vacuum can be annihilated by ψ_L, leaving $\bar{\psi}_R$ to create a right-handed quark with the same momemtum. The quark will thus flip its helicity at a rate proportional to $\langle\bar{\psi}\psi\rangle$ – in other words, it will propagate just as if it had a mass. Spontaneous χSB in QCD is thus the agency by which the nucleon acquires most of its mass. It is also a natural consequence of quark confinement. Consider the bag model description of massless quarks moving back and forth within a small volume. At the surface of the bag the quarks must reverse their direction of travel, but not their angular momentum, which is always conserved. Therefore the interaction with the bag wall changes their helicity. Since this cannot be achieved though any process involving gluon exchange between the quarks in the bag, it must arise because the QCD vacuum in the volume outside the bag contains a non-vanishing density of $q\bar{q}$ pairs with which the bag quarks can exchange helicity. Confinement implies χSB.

4 Quark Matter and Color Superconductivity

I now consider the behaviour of QCD as a function of chemical potential μ. For T strictly zero as μ increases, the ground state is initially the state with no particle present, ie. the vacuum. This situation persists until μ reaches the value of the nucleon rest mass minus the binding energy per nucleon in nuclear matter, when it becomes energetically the ground state with a bound nucleon fluid. Ignoring Coulomb repulsive forces between protons, this energy can be estimated from nuclear physics as 16MeV/nucleon; therefore we can identify an *onset* value $\mu_o \simeq 922\text{MeV}$ at which point baryon density n_B jumps from zero to nuclear density $n_{B0} \simeq 0.16\text{fm}^{-3}$. Since the vacuum and nuclear matter coexist at this point, the value $\mu = \mu_o$ corresponds to the "room chemical potential" that would be measured should we ever be able to construct a suitable potentiometer! Because $n_B = -\frac{1}{V}\frac{\partial\Omega}{\partial\mu}$, the discontinuity implies a first order phase transition. We expect the transition to persist for $T \neq 0$ on grounds of continuity, and therefore show it as a coexistence line emerging from $\mu = \mu_o$ in Fig. 1. As it now separates a phase in which baryons can be present but are dilute from one in which they are condensed, it is known as the *nuclear liquid-vapour transition*. It is anticipated that the line ends at a critical point with $T_c \simeq O(10\text{MeV})$; it is possible that critical opalescence has been detected near this point in the form of the broad distribution of fragment sizes observed in medium-energy nuclear collisions.

What happens as μ, and hence n_B, increase? Unfortunately the lattice gauge theory simulations become ineffective once applied to QCD with $\mu \neq 0$. For densities up to 2 - $3n_{B0}$ we can extrapolate from our current knowledge of nuclear physics. Beyond that we are forced to rely on approximate treatments such as the bag model. As n_B increases we again expect a transition from a phase in which matter exists in the form of nucleons to a higher entropy phase where the dominant degrees of freedom are quarks. Naively this should occur at densities of the order of a billion tonnes per teaspoonful where the volume per baryon equals the baryon volume, and the bag surfaces just touch. For degenerate neutron matter at this critical density we have

$$n_{Bc} \simeq 2\int_0^{k_F} \frac{4\pi k^2}{(2\pi)^3} dk = \frac{k_F^3}{3\pi^2} \simeq \frac{(\mu_c^2 - M^2)^{\frac{3}{2}}}{3\pi^2} \simeq \frac{1}{R^3} \simeq 1\mathrm{fm}^{-3}, \tag{3}$$

giving $\mu_c \sim 1200$MeV. Various model estimates yield $\mu_c \sim 1100$ - 1500MeV, and the jump in density at the transition $\Delta n_B \sim 2$ - $5 n_{B0}$.

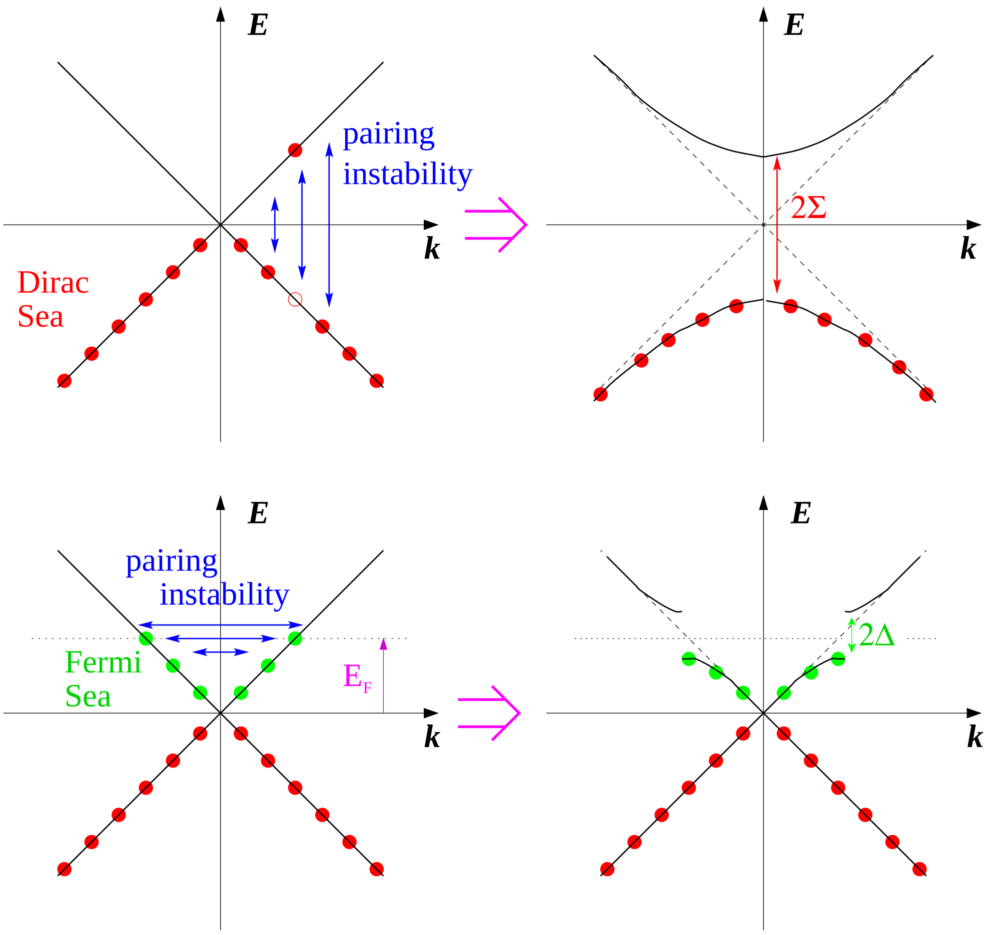

Figure 4: Pairing instabilities leading to chiral symmetry breaking (top), and superconductivity (bottom)

Let's discuss the nature of this *quark matter* (QM) phase. In the bag model, the QM phase corresponds to the bag interior in which chiral symmetry is restored and quarks are light. That this might be so on more general grounds is illustrated in the upper part of Fig. 4, where χSB is shown as due to a pairing instability between quarks and anti-quarks of equal and opposite momenta; the $\bar{q}$ are here interpreted as holes in the *Dirac Sea* of negative energy states. χSB occurs if the binding energy of the $q\bar{q}$ pair exceeds the energy needed to excite them: the result is the modified single-particle excitation spectrum shown at upper right, with an energy gap 2Σ between the highest occupied and lowest empty states. Now when $n_B > 0$, some positive energy states are also occupied, as shown at bottom left; we refer to these as belonging to the *Fermi Sea*. It is impossible to excite $q\bar{q}$ pairs if the q state has momentum $k < k_F$ and is hence already in the Fermi Sea; such pairs are *Pauli-blocked*. At some point, therefore, available $q\bar{q}$ states require so much energy to excite that it is preferable to revert to a chirally symmetric ground state. Since $k_F \gtrsim \mu_c/3 \gg m_{u,d}$, we deduce the quarks near the Fermi surface which participate in QM's interaction with other forms of matter are highly relativistic.

Much recent interest has been aroused by the idea that QM might have richer properties than those of a simple relativistic fermi liquid. Consider the lower panel of Fig. 4: if the qq interaction is even weakly attractive at the Fermi surface, then another pairing instability, the so-called *BCS instability*, is expected between quark pairs at antipodal points, leading to a ground state with a non-vanishing *diquark condensate* $\langle qq \rangle \neq 0$. Analogously to χSB, the instability leads to an energy gap 2Δ between highest occupied and lowest vacant one-particle states, the distinction being that this time the gap is located at the Fermi surface. In metals at temperatures of a few kelvin, a BCS instability can arise between *Cooper pairs* of electrons due to an attractive force arising from interaction with vibrations of the underlying crystal lattice of positively-charged ions. The Cooper pair condensate leads to the phenomenon of *superconductivity*, signalled by electric current flowing without resistance in a narrow layer close to the sample surface; the screening effect of this supercurrent results in the *Meissner effect*, namely the complete exclusion of magnetic field from the sample. A BCS instability between neutral helium atoms in liquid ^{3}He at milli-kelvin temperatures, on the other hand, leads to frictionless flow and quantisation of vorticity, a phenomenon known as *superfluidity*.

In QCD the force between two quarks due to single gluon exchange is attractive (unlike single photon exchange between two electrons), implying that a weak BCS instability should be present in QM. More recent calculations which attempt to model realistic strong interactions in the regime

$\mu \gtrsim \mu_c$ predict a much bigger effect, with Δ as large as 10 - 100MeV. A crucial consideration, not applicable to χSB, is that the qq wavefunction is constrained by the Pauli Exclusion Principle. As a result the ground state is sensitive to the flavor composition of the available quarks. Suppose $\mu/3$ is not much greater than m_s; in this case $k_{Fs} \ll k_{Fu,d}$ and pairing is effectively restricted to the two light flavors. The diquark condensate which thus forms is

$$\langle qq\rangle_{2SC} = \epsilon^{\alpha\beta 3}\epsilon_{ab}\langle\psi_a^\alpha(k,\uparrow)\psi_b^\beta(-k,\downarrow)\rangle \neq 0. \tag{4}$$

The quark spins are combined in an antisymmetric singlet state; the overall antisymmetry of (4) under quark exchange is then enforced by the alternating ϵ tensors acting on flavor $a, b = 1, 2$ and color $\alpha, \beta = 1, \ldots, 3$ indices. Now, since the qq_{2SC} wavefunction is not color-neutral, we infer by analogy with the electrically-charged Cooper pair that the ground state is *color superconducting*. The most immediate consequence is that out of the eight gluons, the five which carry color #3 acquire a mass of $O(\Delta)$ and hence cannot penetrate QM over distances much greater than a screening length $\sim \Delta^{-1}$, in direct analogy with the Meissner effect in metallic superconductors.[3] The three gluons left massless carry combinations of only the first two colors. The qq_{2SC} wavefunction however, like the QGP, respects chiral symmetry; for this reason although the superconducting description will be the more natural for $T \lesssim \Delta$, there may well be no true transition separating 2SC and QGP phases, and we have shown them separated by a crossover in Fig. 1.

For larger μ, k_{Fs} should increase up to the point where strange quarks can participate in the pairing. In this case a more symmetric "color-flavor-locked" (CFL) condensate can form:

$$\langle qq\rangle_{CFL} = \sum_i \epsilon^{\alpha\beta i}\epsilon_{abi}\langle\psi_a^\alpha(k,\uparrow)\psi_b^\beta(-k,\downarrow)\rangle \neq 0. \tag{5}$$

If anything the CFL state is still more exotic; all 8 gluons are rendered massive implying color superconductivity, and chiral symmetry is also broken. Moreover, since the qq_{CFL} operator either creates or annihilates two units of baryon number, B is no longer a good quantum number – it can be shown that this B-violation leads to superfluidity. The CFL state is currently believed to be the stable ground state of matter as $\mu \to \infty$, and is shown as a distinct phase in Fig. 1.

[3]In weak interaction physics the identical effect, now known as the *Higgs phenomenon*, arises as a consequence of the condensation of a scalar *Higgs field* and gives $W^\pm$ and Z^0 bosons masses of $O(100\text{GeV})$, restricting the effective range of the weak interaction to 10^{-18}m.

5 Neutron Star

Where might QM be found? To reach the higher densities needed for QM an external pressure is needed; the most likely source is the gravitational binding in the compact astrophysical objects of mass $O(10^{30}\text{kg})$ and radius $O(10\text{km})$ known as *neutron stars*. Neutron star central densities are estimated as lying in the range 5 - $10n_{B0}$. It may well turn out that our best experimental probe of QM will come through careful observation of the known neutron star population, focussing on quantities such as the rate of change of angular momentum, cooling rate, and magnetic field. One interesting possibility is that once $\mu/3 \gtrsim m_s$ the ground state of matter includes an appreciable fraction of strange quarks, and hence a composition $\sim uds$; neutron stars may actually be made of such *strange quark matter* (SQM). It is even conceivable, though unlikely, that SQM is the ground state for $P = 0$, and nuclear matter therefore only metastable. Observational and experimental evidence for these fascinating new phases may be difficult to acquire. Since neutron star temperatures are typically $\lesssim 10$MeV, it may seem that they are a natural place to look. However, two factors work against us; even if a neutron star has a QM core its bulk is probably normal nuclear matter, and color superconductivity is a phenomenon confined to the vicinity of the Fermi surface rather than the whole Fermi Sea and therefore has relatively little impact on the equation of state. Both imply that any effect of color superconductivity is likely to be quite subtle. One prediction is that a color superconducting core would stabilise neutron star magnetic fields and prevent them from decaying over cosmologically significant timescales. Another speculation concerns the formation of a neutron star by core collapse of a very massive star during a supernova explosion. During this violent event the star cools from $\sim$50MeV to $\sim$10MeV over a timescale of a few seconds by emission of massless weakly-interacting particles called *neutrinos* ν, $\bar{\nu}$. [4] If a transition to superconducting QM occurs in this period, ν - q scattering with $k \lesssim \Delta$ will be Pauli-blocked, with the effect of making the core effectivelytransparent to neutrinos. They may emerge from the collapsing core in a sudden burst, rather than steadily diffusing out over 10-20 seconds as in standard collapse scenarios; it may be feasible to detect such a burst in terrestrial neutrino detectors [8].

[4] An anti-neutrino is produced in the archetypal weak interaction, β-decay of the neutron, via $d \to ue^-\bar{\nu}$.

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