Physics 598 ESM Term Paper

Giant vortices in rapidly rotating Bose-Einstein condensates

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1 Introduction

Quantum liquids with superfluidity are able to exhibit the formation of quantized vortices. Dilute gaseous Bose-Einstein condensates provide a good example to investigate superfluidity in a rapid rotation regime. A Bose-Einstein condensate at highly rotational speed is dynamically found to generate a giant vortex, which is a low-density region around the center of rotation. In this paper, we will talk about two different ways in experiments to produce a giant vortex, and discuss the features of each kind of giant vortex separately. Besides, we also give the results and predictions from numerical simulations, comparing them with experiments.

2 The giant vortex created by removing atoms

2.1 The way to remove atoms

Consider an atom with magnetic moment μ in the magnetic field with magnitude B. The energy of the atom due to the magnetic field is $-\mu B$. If μ is positive, there is a force to drive the atom toward the region of higher B to minimize the energy. If μ is negative, instead, the atom tends to move to the lower-field region. Suppose there is a magnetic field, with a local minimum in time average, to trap the atoms with a negative magnetic moment. And we use a specific laser beam to flip the magnetic moment of the atoms. Now the atoms are going to seek a higher magnetic field and then escape from the trap.

2.2 Experimental results

In the experiment, a rotating condensate is prepared in the anisotropic potential trap

$$V_{trap} = \frac{1}{2}m{\omega_\rho}^2\rho^2 + \frac{1}{2}m{\omega_z}^2z^2 \tag{1}$$

where $\rho^2 = x^2 + y^2$, ω_ρ is the transverse trapping frequency($8.3\times 2\pi$Hz), and ω_z is the vertical trapping frequency($5.4\times 2\pi$Hz). The condensate consists of 3×10^6 ^{87}Rb atoms, rotates at the speed of $0.95\omega_\rho$ around z-axis, and contains a lattice of 190 vortices.[1](see Fig.1(a))

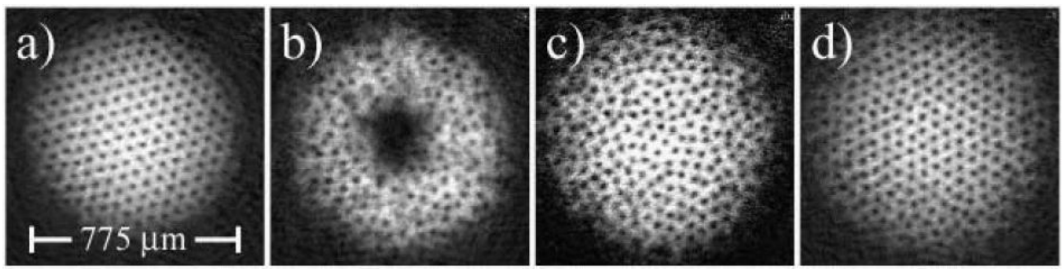

Figure 1: (from ref.[1]) Lattice reforming after giant vortex formation. (a) BEC after evaporative spin-up. (b) Effect of a 60 fW, 2.5 s laser pulse. (c),(d) Same as (b), but additional 10 s (c) and 20 s (d) in-trap evolution time after the end of the laser pulse. Images taken after a sixfold expansion of the BEC.

Then the atom-removal laser of 60 fW is applied through the condensate on the z-axis for 2.5 seconds, creating a core of the condensate (see Fig.1(b)). After enough long time to re-equilibrate, the giant vortex fills in and recovers the well-ordered vortices lattice again (see Fig.1(c) and (d)). By integrating the density of the number of atoms and the density of angular momentum over the volume of the core, we can get the lost of the number of particles and the lost of angular momentum. Because the total number of vertices is proportional to the angular momentum per atom, we can also calculate the number of vortices in the final state. In the experiment, 35% atoms and 10% angular momentum are removed. We can expect the number of vortices should increase from 190 to 266, in good agreement with 260 vortices observed in Fig.1(d).

When a stronger laser pulse is applied and removed suddenly, the core area exhibits a damped oscillation. Both oscillation frequency and amplitude are observed to increase with decreasing the initial rotation speed of the condensate. (see Fig.2)

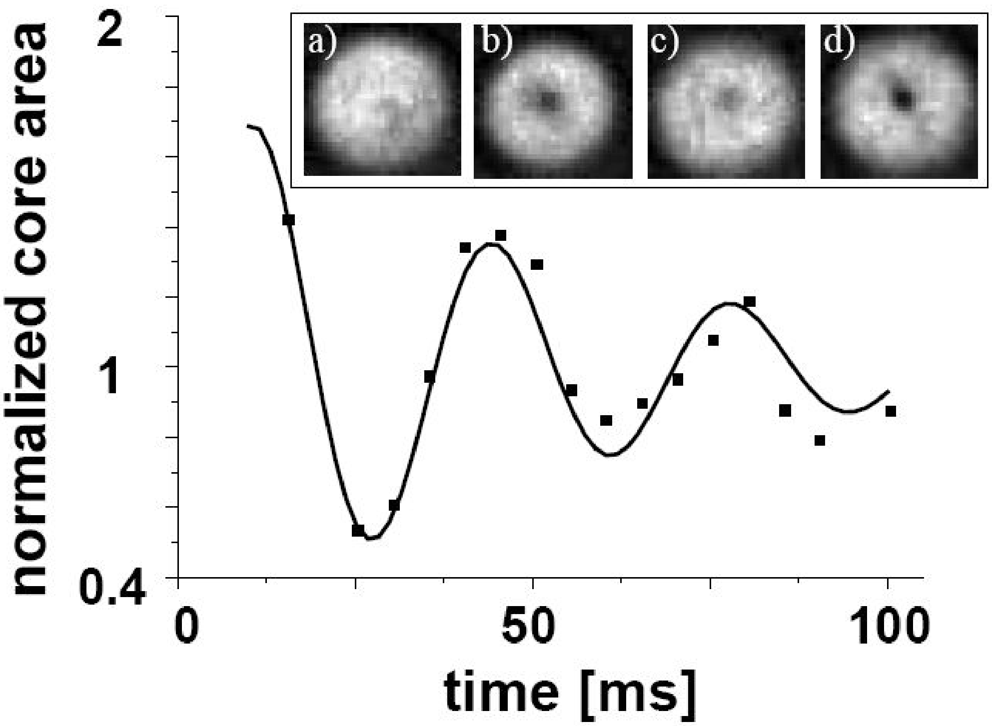

Figure 2: (from ref.[1]) Oscillation of core area after an 8 pW, 5 ms laser pulse. Starting conditions are 2.5×10^6 atoms with rotation rate $0.9\omega_\rho$. Time given is the in-trap evolution time after the end of the pulse. Core area is normalized to mean of all data points. Inset: Initial condition 3×10^6 atoms with rotation rate $0.78\omega_\rho$; in-trap images taken after a 14 pW, 5 ms laser pulse followed by evolution time of (a) 20 ms, (b) 40 ms, (c) 60 ms, and (d) 80 ms.

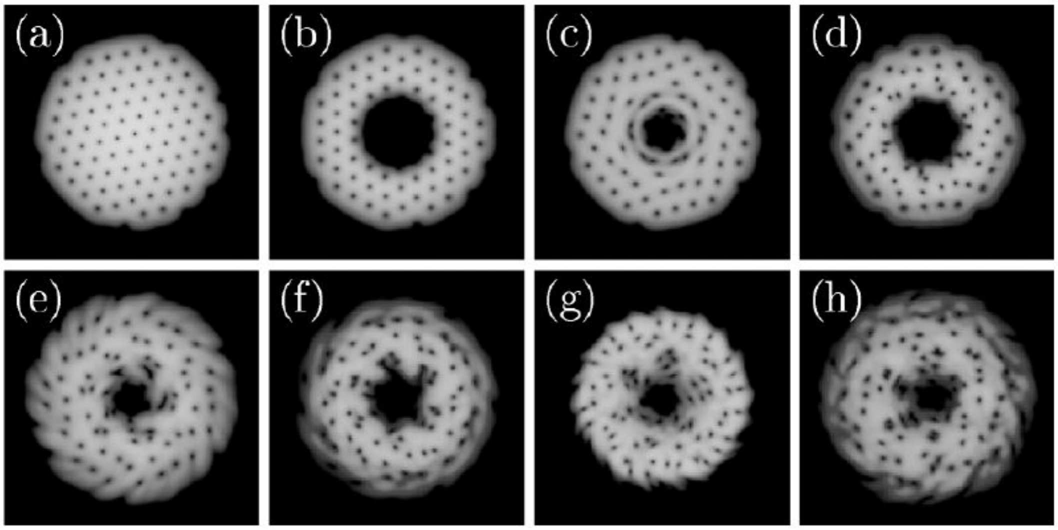

Figure 3: (from ref.[2])Oscillation of the giant vortex core area. The initial equilibrated vortex lattice rotates at angular frequency $\Omega = 0.9\omega_\rho$, and has $\mu = 12\hbar\omega_\rho$. Condensate density is locally suppressed from the vicinity of the trap center as described in the text. In effect, the giant vortex core area begins to oscillate followed by the excitation of the overall breathing mode of the condensate. The snapshots (a)-(h) are taken at times $t = (0; 5; 19; 33; 60; 70; 81100) \times 10^{-2}T$. The sides of each picture are $40\tilde{a}_0$ wide.

2.3 Numerically simulated results

The wave function of the condensate can be described dynamically by time-dependent Gross-Pitaevskii equation

$$i\hbar\frac{\partial\psi(\mathbf{r},t)}{\partial t} = \mathcal{H}(\mathbf{r},t)\psi(\mathbf{r},t) \tag{2}$$

in a rotational frame with a an angular velosity $\mathbf{\Omega}$. The Hamiltonian operator

$$\mathcal{H} = -\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r},t) + g|\psi(\mathbf{r},t)|^2 - \mathbf{\Omega}\cdot\mathbf{L} \tag{3}$$

where g is the interaction strength determined by mean-field approximation, and $\mathbf{L}$ is the angular momentum operator. For given initial number of particles, we can find the chemical potential $\mu = \langle \mathcal{H} \rangle$ satisfying the normalization condition of the wave function. The potential $V(\mathbf{r}, t)$ is of the form (1), plus another term $V_{beam}(\mathbf{r}, t)$ due to the external Laser beam. We choose the incoming Laser beam such that

$$V_{beam}(A, \rho_0) = Ae^{-4\rho^2/\rho_0^2} \tag{4}$$

We can see if A is pure imaginary, V_{beam} can lead to a density decay of the condensate with time. In this simulation, $g = 1000\hbar\omega_\rho \tilde{a}_0^2$, where $\tilde{a}_0 = \sqrt{\hbar/2m\omega_\rho}$, $\mathbf{\Omega} = 0.9\omega_\rho$, and $V_{beam}(-100i\hbar\omega_\rho, 7\tilde{a}_0)$ is applied at the center of the trap, during the time interval $t = [0, 0.05T]$, where $T = 2\pi/\omega_\rho$[2]. Chronological pictures of the evolution of the condensate are shown in Fig.3.

We can calculate the oscillation frequency by considering the centrifugal potential about the condensate flow around the core area. The velocity of the condensate flow in the boundary of the giant vortex is $v = \hbar l/m\rho$, where l is the total quanta of vorticity included in the giant vortex. Combine the centrifugal potential $\hbar^2 l^2/2m\rho^2$ with the trap potential, and expand it near the equilibrium radius R_0. For small oscillation, we get the frequency

$$\omega_{core} = 2\sqrt{3}l(\frac{\tilde{a}_0}{R_0})^2\omega_\rho \tag{5}$$

Fig.4 shows the relation between ω_{core} and R_0. The predictions of Eq.(5) fits the simulation results well. Besides, Eq.(5) also agrees with the observation in the experiment of 2.2 that lower rotation frequencies of the condensate causes higher oscillation frequencies of the giant vortex, because R_0 is proportional to the healing length $\xi \propto (\omega_\rho^2 - \Omega^2)^{-1/5}$[3]. Another intuitive way is suggested that an increasing Ω will increase the distance between vortices and then decrease the total vorticity quanta l in the giant vortex with constant area. The oscillation of the giant vortex and the breathing mode of the condensate can happen at the same time. The inset of Fig.4 records the simultaneous pattems of them.

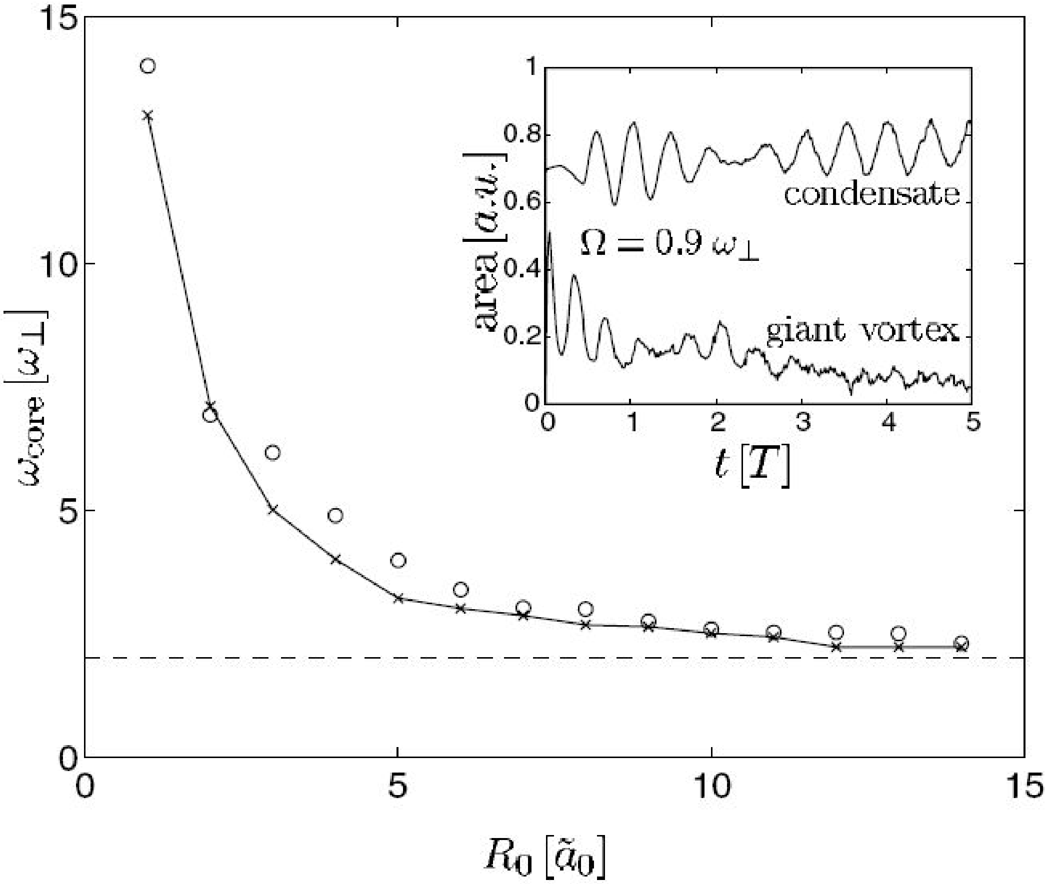

Figure 4: (from ref.[2])Giant vortex core area oscillation frequency as a function of the radius of the giant vortex. Data from our simulations ($\times$), joined by a solid line for clarity, are plotted together with the predictions ($\bigcirc$) from (5). The dashed line is the breathing mode frequency $2\hbar\omega_\rho$ for the 2D system. The inser is described in the text, and the lower curve in it is scaled by a factor 5.

3 The giant vortex in combined quadratic and quartic trap

In recent technology, a blue detuned laser is developed to provide the potential of the form[4]

$$U(\rho) = U_0 exp(-\frac{2\rho^2}{w^2}) \tag{6}$$

Combine $U(\rho)$ and the quadratic potential in Eq.(1). When ρ/w is small enough, we get the trap of the form approximately

$$V_{trap} = \frac{1}{2}m\omega_\perp^2\rho^2 + \frac{2U_0}{w^4}\rho^4 + \frac{1}{2}m{\omega_z}^2z^2 \tag{7}$$

where $\frac{1}{2}m\omega_\perp^2 = (\frac{1}{2}m{\omega_\rho}^2 - \frac{2U_0}{w^2})$ is positive for large w. In the experiment, the power of laser is $1.2mW$, $w = 25\mu m$, $U_0 = k_B \times 90nK$, and $\omega_\perp/2\pi = 64.8Hz$. The condensate is stirred to rotate at stirring frequency Ω_{stir} near $\omega_\perp$.(see Fig.5)

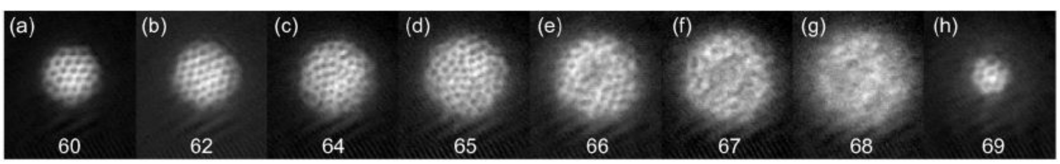

Figure 5: (from ref.[4])Pictures of the rotating gas. We indicate in each picture the stirring frequency Ω_{stir}. The vertical size of each image is $306\mu m$

It is measured that the effective rotation frequency Ω_{eff} increases with increasing Ω_{stir}. When $\Omega_{stir}/2\pi$ grows over $65Hz$, Ω_{eff} is still increasing, but the number density of the vortices near the center of the trap decreases drastically. This phenomenon suggests that vortices near the center merge to form a giant vortex.

Numerically, a highly 3D system is simulated. It is shown in Fig.6 that how a giant vortex appears with rotation frequency growing. First, isolated vertices with unit vorticity form at low Ω[Fig.6(a)]. As Ω increases, the

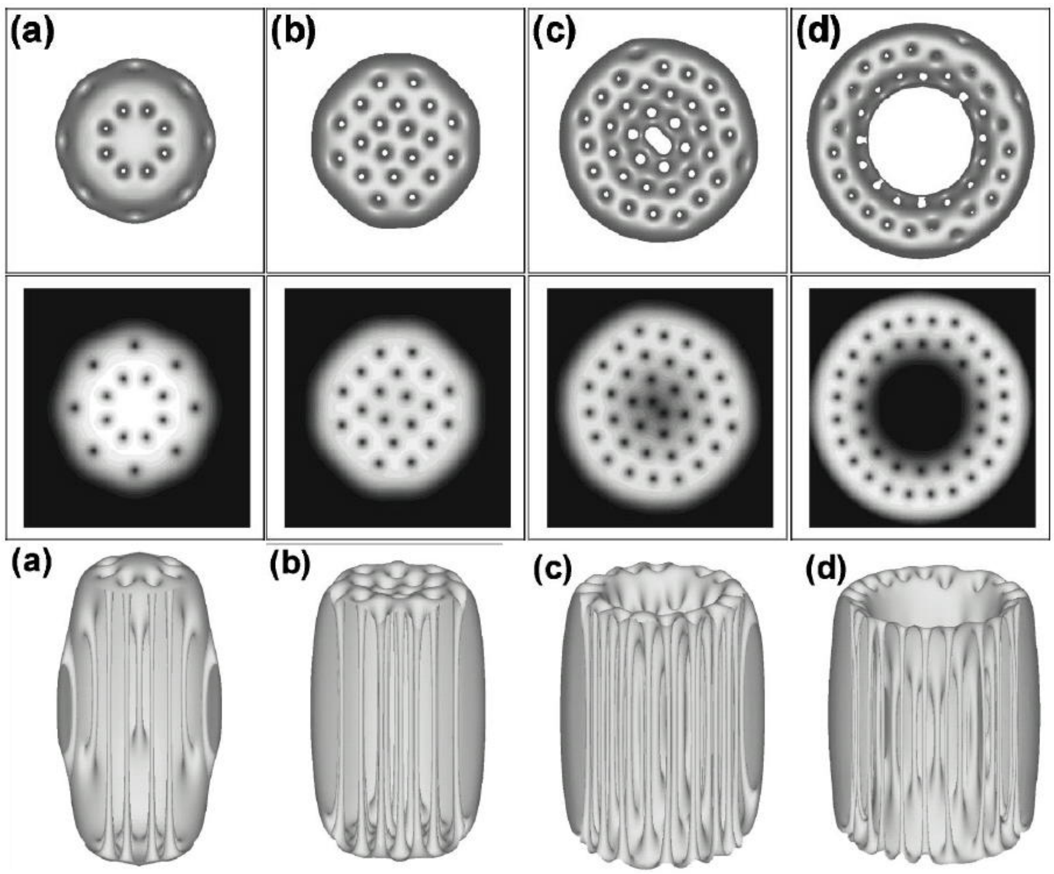

Figure 6: (from ref.[5]) Top view of the isosurface of lowest density (up), density contours in the plane z=0 (middle), and side view (down) for Ω/ω_ρ=0.32(a), 0.4(b), 0.48(c), and 0.56(d).

number of vortices become more [Fig.6(b)]. From the side view of the condensate, we notice that most vortices are straight while some are bending. When $\Omega/\omega_\rho = 0.48$, these is already a giant vortex on the top, and two vortices begin to merge in the plane $z = 0$. A hole begins to form at the center[Fig.6(c)]. For $\Omega/\omega_\rho = 0.56$, a hole is completely formed at the center, with the vortex lattice around the giant vortex concentrically.Ω[Fig.6(d)]

A giant vortex in BECs has been investigated extensively. There are still interesting phenomenon about the giant vortex of the condensate in various potentials. For example, it is found numerically that in a quartic-minus-quadratic trap, there are two kind of vortices with different geometry. And it is allowed to have a pure hole or a giant vortex, which can be distinguished only in the phase diagram. Unfortunately, we can't create such a potential in techniques today. It still needs to be verified by experiments in the future.

References

[1] P. Engels, I. Coddington, P. C. Haljan, V. Schweikhard, and E. A. Cornell, Phys. Rev. Lett. 90, 170405 (2003).

[2] T. P. Simula, A. A. Penckwitt, and R. J. Ballagh, Phys. Rev. Lett. 92, 060401 (2004).

[3] U. R. Fischer and G. Baym, Phys. Rev. Lett. 90, 140402 (2003).

[4] V. Bretin, S. Stock, Y. Seurin, and J. Dalibard, Phys. Rev. Lett. 92, 050403 (2004).

[5] A. Aftalion and I. Danaila, Phys. Rev. A 69, 033608 (2004).