Patterns of Collective Behaviour in Ballroom Dance

Katie Bolan

Abstract

On a typical ballroom dance floor, there are many couples on the floor, moving according to the prescribed rules of the dance. Though each couple typically has their own sequence of steps that they try to dance, their motion is constantly being affected by that of the other couples on the floor. Due to these interactions and the characteristics of a given dance, patterns of collective motion arise from the various individual routines. These types of patterns are not unique to competitive ballroom dance floors, but can also be observed in the motion of other groups of individuals. The mathematical and computer models describing these other systems can therefore also be useful in describing the collective motion of couples on a ballroom dance floor.

Introduction

Communities of animals are composed of many individuals, each constantly being affected by and responding to a wide variety of environmental and internal stimuli. Each individual receives a different set of inputs, and there are a variety of possible responses to each of these inputs. Yet, from these individual actions and interactions, patterns form on a scale much larger than the individual organisms [1]. The prototypical examples of such collective phenomena are flocks of birds and schools of fish. These are not, however, the only groups of animals which exhibit such collective behaviour. There are several situations in which patterns of collective motion can be seen in groups of humans, such as pedestrians on a sidewalk [2] and groups of attendees at a concert [3]. Though the individuals involved differ widely in each context, they can often form similar overall patterns. These similarities suggest that the same underlying mechanisms are present in these diverse systems, which can then be described by the same mathematical models.

Overall patterns of movement are often used by groups of dancers or choreographers to achieve artistic or expressive goals in a dance performance. This is most often a result of choreography that dictates every position and movement of each person, rather than arising from individuals' interactions with each other and their surroundings. However, studies looking at dance improvisation have found that similarly artistic patterns can form as a result of the dancers following a simple set of rules [4], [5]. In these experiments, all the dancers are subject to a set of rules, but are free to interpret the rules and improvise within them. In a standard ballroom competition, each couple is also subject to a set of rules for each dance, but there are a variety of choices they can make that satisfy these rules. Each couple typically acts independently of most of the other dancers on the floor; they must be certain to avoid collisions with other nearby couples, though. Given these similarities with other improvisational types of dance and with systems in which small scale interactions cause large scale patterns, models for the collective patterns that emerge on a ballroom dance floor can be developed.

Previous Dance Experiments

The Flock Logic project gave a group of dancers a set of rules inspired by models of flocking in animals: cohesion rules to stay within a certain distance of other dancers, repulsion rules to not let others get too close, and a rule to avoid travelling backwards [4]. The dancers were not initially given further instruction or told how to respond to situations in which the rules conflicted with each other. The experimenters filmed the dancers moving around a room according to the rules and tracked their trajectories. Using these trajectories, they were able to find the sensing graph for the dancers, which showed to whom each dancer was cohering. Based on the connectivity of this graph, individual influence for each dancer could be quantified to look at the effects of individual variation. By comparing their experimental data with simulations of identical particles, they were able to conclude that leadership emerges as a result of human bias among the dancers.

In [5], the author explores methods of producing a complex system, through what he refers to as "emergent choreography." He achieves this through the application of variety of types of rules, each of them intended to increase the likelihood of patterns spontaneously forming. Several types of spatial and temporal patterns are observed and classified into equivalence classes based on their artistic effect. This approach allows both the emergence of patterns from the self-organization of dancers or the creation of a consistent choreography.

Standard Dance Background

The patterns of collective motion that form in a given situation depend upon the rules of motion that individuals must follow [4]. This paper will focus on types of motion that can be observed in the standard ballroom style of dance at the syllabus levels as an effect of the rules associated with them in collegiate competitions. In this style, each couple is in a closed standard frame, as seen in Figure 1, for the entire time they are dancing; each couple can then be treated as a single entity since the lead and follow will always move together and stay in the same relative position. On a standard ballroom dance floor, the couples progress down line of dance, meaning they travel around the floor in a counter-clockwise direction, as shown in Figure 2. Ideally, the dancers stay in motion for the duration of an entire round, which typically lasts between 90 and 120 seconds. Occasionally, a couple may stop moving for a few seconds, in which case they effectively act as an obstacle to other couples.

Figure 1: A couple in standard frame. They will maintain this relative position between their upper bodies throughout the dance. Photo from Krista Fogle, "Find your frame," *Dance Spirit*.

Each couple must also maintain enough distance between themselves and other couples to avoid collisions; there is no upper bound on how far apart they may be, as long as they remain on the dance floor, in contrast to the flocking and cohesion rules typically applied in experiments [4] and simulations [1]. The ability to move around the floor, while avoiding running into other couples and remaining within the bounds of what is allowed in each dance, is known as floorcraft.

Each dance also has its own necessary features and movements that characterize it, to which the dancers must conform. Further rules regarding the alignment and direction of travel are applied to specific figures within each dance that a couple may choose to use. These figures must come from the syllabus for the level at which the couple is dancing; figures that are not in the syllabus or that are listed at higher levels than that at which the couple is competing are not allowed in competitions.

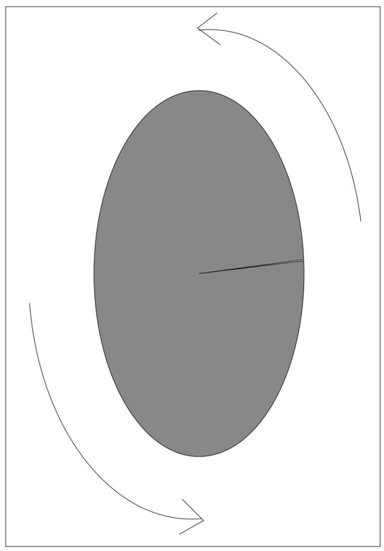

Figure 2: As seen from above, couples travel around a rectangular floor in a counter-clockwise direction. The shaded region (not to scale) in the center of the floor is forbidden; couples may not dance through the center. Most choreography is developed to work well with the long side, followed by a corner, the short side, and a second corner.

Mathematical Description

Let N denote the number of couples on the floor. To simplify the equations, the short side of the dance floor will be stretched to create a square floor, around which the couples travel in a circle of radius R. Then, all of the dancers are going to try to travel in the positive $\hat{\phi}$-direction, using a polar coordinate system centered on the floor. If we classify couples based on "types of motion" as in [2], based completely on the intended direction of travel, all couples have the same classification. The factors that change between couples are the magnitude of the intended velocity v_0, and both the magnitude and direction of the actual velocity $\vec{v}$.

Based on the approach used in [2], four factors affect the density $\rho(x,t)$ of couples in a given location x at time t. These factors are the actual velocity of dancers approaching their intended velocity, the interactions between dancers, changes in intended direction, and the entrance or departure of dancers from the system. The third of these can be ignored because the intended direction is constant for all dancers in this system. Considering only a single round, in almost all cases, the same N couples remain on the floor the entire time, so the last factor can also be ignored.

The first factor can be described in terms of a propulsion force on the i^{th} couple as

$$\vec{F}_{i,prop} = \tau(v_{0,i} - v_i)\hat{\phi}$$

where τ is the strength of the force [3]. This force, in the absence of other interactions, results in $\vec{v}_i$ approaching $\vec{v}_{0,i}$ with time constant $\frac{1}{\tau}$. A single couple on the dance floor would fairly easily reach their intended velocity after beginning to dance; having only one couple on the floor almost never happens in the syllabus levels of collegiate competitions, though. Interactions are inevitable at the densities of dancers typically present in these competitions.

The pairwise interaction rate $\frac{1}{\tau_{ij}}$ of couples i and j will be proportional to the magnitude of their relative velocities, to the average distance that a couple moves for each figure l (which is analogous to the stride length in pedestrian motion), and to the average density of couples, so

$$\frac{1}{\tau_{ij}} = \langle\rho\rangle l |v_i - v_j|$$

[2]. Then when an interaction does occur, three possible outcomes are described. Suppose couple i is approaching couple j. Couple i can try to avoid couple j by going around them to the right or left; couple i could match the velocity of couple j, and continue behind them; or couple i could stop moving to prevent a collision with couple j. The end result of these three options can be encapsulated in a probability function of each of them occurring,

$$P_{ij}(v_i, v_j, v_i^*) = \sum_{k=1}^{3} P_{ij}^k(v_i, v_j, v_i^*)$$

and resulting in an altered velocity v_i^* of couple i due to the interaction [2]. Ceasing to move does not typically result in favourable marks from the judges, so that response to interaction should be given the lowest probability. In a similar vein, if couple i can get past couple j, the risk of collision is reduced, and they have more freedom to approach their preferred velocity; this response should therefore be given the highest likelihood. While exact values will depend on the individuals dancing, in general

$$P_{ij}^1 > P_{ij}^2 > P_{ij}^3$$

indexed in their order of description above.

Observed Collective Phenomena

Net Momentum around Floor

The simplest collective property of the dancers' motion, and that of the largest scale, is their non-zero net momentum. This characteristic of standard dance is not seen in all types of dance; Latin dances, such as cha cha and rumba, are notable examples with no overall direction. Simulations of schools of fish have resulted in the group of fish either moving together or in a disorganized swarm, as the result of varying how the fish interact with neighbouring fish [1]. In this model, each individual has three "zones" around it. In order of increasing size and decreasing importance, the zones are repulsion, orientation, and attraction. In the simulations, these zones of attraction and repulsion were held constant, and the extent of the zone of alignment, in which neighbouring fish travel in the same direction, was varied. As the zone of alignment increased in size, the motion changed from swarming, to circling in a torus, to parallel movement in one direction [1].

Similar zones can be applied to dancers, who need to maintain a safe distance from each other, but also need to remain on the same dance floor. The relative importance placed on each zone also corresponds to the order of priorities of floorcraft; it is imperative to avoid collisions with other couples. If no collisions are imminent, the couple can focus on progressing down line of dance. The size r_0 of the standard frame of a couple puts a lower bound on the size of the zone of repulsion. Because there is little reason to try to be close to other couples, the zone of attraction is effectively the size of the entire dance floor; it is only when one is at the edge of the floor that they are compelled to move towards the others. A couple's zone of alignment needs to be sufficiently large that it overlaps with that of couples with nearby angular positions, but does not coincide with those of couples on the other side of the floor. This choice of parameters would reproduce the circling motion around the center of the floor. Choosing the radii of the zones appropriately then, a two-dimensional version of the school of fish simulation could model the largest scale of motion of the couples on a dance floor, given some initial configuration.

Density Waves

As the beginning of a round, as the dancers take their starting positions on the floor, they tend to distribute themselves evenly around the floor. Once the music starts and the couples begin to move, they will start at slightly different times and move at different velocities, introducing perturbations to the homogeneous distribution of couples. When this happens in a crowd of pedestrians, [2] found that density waves could propagate through the crowd. This sort of wave is noticeable in heat 2 of the foxtrot in [6] with a period of about 30 seconds. There is a region of high density dancers that maintains its relatively high density while travelling around the floor.

Translation Symmetry

One of the most readily apparent forms of collective motion among ballroom dancers is multiple couples dancing the same figure synchronously. This creates a translation symmetry around the floor. A striking example of this effect can be seen in [7] between seconds 25 and 40 when over half the couples begin their foxtrot with a feather step followed by a reverse turn, which are among the most common standard foxtrot figures. Throughout the video, one can see this synchronization continue to occur with the simultaneous rise and fall of couples who may be dancing different figures, or on a smaller scale, with two or three couples performing identical steps at the same time. This kind of synchronization is also seen in [5], as a result of the rules imposed on the dancers, regarding sequences and vertical heights of various movements.

All of the couples on the floor at the same time have the same set of figures they may use (the syllabus for the dance and level), and based on the alignments of each of the figures, they can only be preceded and followed by a subset of the total number of allowed figures for a given dance and level. As mentioned in Figure 2, there are also some sequences that allow greater ease of travel around different parts of the floor and therefore commonly used together. The combination of these factors results in many of the couples having similar "rehearsed movement sequences" as described in [5] for regions of the dance floor. The music dictates the timing of the steps of all couples, often resulting in two or more couples in similar surroundings on the floor being perfectly synchronized. As the couples move around the floor, this can also cause a ripple effect, also described in [5]. Dancers may follow each other, using the same set of figures for the same region of the floor, shifted in time by an integer number of measures of the music.

Synchronized activity has also been observed to occur in contexts other than dance, such as an audience applauding after a performance and the flashing of fireflies [1]. These situations are typically modelled by a Kuramoto model for coupled oscillators. If there is a small enough initial variation in the frequencies of the coupled oscillators, they will synchronize with each other. Concentrating on the rise and fall of the dancers, rather than each individual step, they can be thought of as oscillators, coupled by the music, with a period of one measure of the music.

Lane Formation

It is common for dance instructors to describe line of dance as a three-lane highway, as opposed to a single line. There is room on each side of the floor for more than one couple to occupy any given angular position. Depending upon the number of couples on the floor, it may not even be possible for all couples to fit in a single line. The necessity of multiple lanes can be clearly seen from 8:44 to 8:54 during the quickstep on the side of the floor closest to the camera [8]. As the density of couples on this side of the floor increases, they separate into a slow moving lane near the center of the floor and another lane of couples passing them on the outer edge. Beyond this time, the density on this side of the floor decreases, and the separate lanes disappear, as a result.

Formation of separate lanes for different types of motion has also been seen in models of pedestrian motion when there is a sufficiently high density of pedestrians on a sidewalk, and when they have an "asymmetrical avoiding probability" [2]. This feature of the motion depends both on the interaction term and the responses to interactions described above. To observe lanes, there needs to be a high enough rate of interactions between couples or pedestrians; for interactions to occur at all, there needs to be a relative velocity between individuals. This pattern was found for groups of pedestrians travelling in opposite directions. However, by changing to an appropriate reference frame, this situation becomes identical to groups travelling in the same direction at different speeds. The rate will also increase as a result of an increased density. Given a sufficiently high rate of interaction, lanes can form when the response to these interactions was avoidance with a preference for movement to the left over the right, or vice versa. In a given dance, the allowed figures typically tend more to one side than the other, creating such an asymmetry.

In the video, lanes formed exactly when these conditions were met: there was a region of increased density and a velocity difference. They also disappeared when the conditions changed, indicating that the pedestrian model can accurately predict the same effect in a standard dance. This model also showed that when lanes form, it "has the advantage as well as the purpose to reduce the total rate ... of interactions" [2]. This benefit could be observed in the video as well. Each of the lanes moved independently of the other. None of the dancers had to drastically change their pace, yet there were no collisions despite the difference in velocities.

Traffic Jams and Collisions

As mentioned above, in an ideal competition round, all dancers would remain in motion for the duration of the round. Unfortunately, there are times when a couple may need to stop to regain their footing or to avoid a collision. This couple then becomes an obstacle that other couples must either avoid by going around them or stop behind. The effect can sometimes grow from there. This same behaviour is commonly seen with cars on highways, and has also been observed in pedestrian traffic. According to the analysis in [2], it is differences in velocity that cause such jams. This reason explains why these problems tend to be more common in the faster dances, such as quickstep and Viennese waltz; when one couple stops, the speed difference is much larger for these dances.

Collisions often occur in conjunction with such traffic jams. The effect of the collisions on the collective motion depends upon the time scale on which they occur. This can be approximated by

$$\tau_{coll} = \frac{(2r_0\rho)^{-1}}{v_0}$$

the mean free path, divided by the intended velocity [3]. This model for mosh pits assumes a flocking force, which causes individuals to travel in the same direction; to use this model for standard dance, this force is assumed to have a very high strength and to act on a small time scale τ_{flock}. The collisions cause disorder and randomize the direction of motion, but as long as $\tau_{flock} \ll \tau_{coll}$, an ordered state with large angular momentum forms [3]. This model does not take into account any sort of floorcraft or attempts to avoid collisions. The time scale on an actual standard floor would then likely be much higher. For dances with a sufficiently low intended velocity, it may even be longer than the length of a round. For the higher velocity dances, it is common for the deck captains to divide it into a greater number of heats, reducing the density of dancers on the floor at one time, again lessening the likelihood of collisions. Therefore, while these collisions are unpleasant, their long time scales indicate that they pose little risk to the overall collective motion of all couples.

Other Styles

Dancesport is made up of four different styles: standard, smooth, rhythm, and Latin. There are, of course, some similarities between the styles, but somewhat different rules are applied to the couples in each case.

Smooth

Smooth has many of the same characteristics as standard; all the couples travel down line of dance, and much of the standard syllabus is allowed in smooth as well. The standard frame is also present in smooth. However, it is not necessary to remain in closed frame the entire time. Several smooth figures require an open frame, where the lead and follow partially separate from each other. This separation increases the "size" of the couple, leading to a greater frequency of interactions. The couple can also no longer be treated as a single unit; possible interactions within a couple can affect their motion, in addition to those with other couples. The additional presence of anomalous figures that momentarily halt progression, or even move against line of dance, would lead to quite complicated interactions terms for smooth dances.

Rhythm/Latin

The rhythm and Latin dances (except samba), do not have a line of dance or preferred direction of motion. Because couples do not need to align their direction with that of neighbouring couples, the large scale motion resembles the disordered swarming behaviour observed in [1] for schooling fish. Changes of direction are common, and typically desirable to increase a couple's visibility; the term describing changes of intended velocity from [2] can no longer be ignored for these styles. Because of the lack of a large net momentum, the number of dancers in each heat is usually greater. However, these styles are similar to smooth, in that the lead and follow can partially separate from each other, leading to a large effective size of a couple, as shown in Figure 3. This effective size, coupled with expressive arm stylings, make interactions incredibly common, and potentially even dangerous.

Samba is a Latin dance that does have a defined line of dance, down which couples can, and should, progress. It is not necessary to progress the entire time, though, and there are several figures that are stationary. It might be possible to approximate samba as some combination of Latin and smooth.

Figure 3: A New Yorker from cha cha. The lead and follow are side by side with their free arms extended. Photo from Dancesport Kingdom.

Conclusions

Several of the collective patterns that are prevalent in standard ballroom dance are also seen in other interacting groups of organisms. Models of these other interacting systems can therefore be used to capture aspects of standard ballroom dance that it shares with these systems. By looking at such diverse systems that exhibit similar collective phenomena, the essential underlying mechanisms driving the formation of these types of patterns can be deduced.

Little work has been done that focuses directly on this form of dance, though, so there is not a single model that captures all the characteristics of this style. Many videos of collegiate dancesport competitions are publicly available online. Analyzing these videos, and using the aforementioned models as a starting point, a more comprehensive mathematical description of the emergent patterns could be developed. One could also look at the differences between the standard dances, to explain why certain patterns are more prevalent in some dances than in others within this style. Such an approach could also be applied to the other styles comprising dancesport, each of which has its own set of characteristics and rules, which require their own set of descriptive equations.

References

[1] Sumpter, D.J.T. (2006). "The principles of collective animal behaviour." *Philosophical Transactions of the Royal Society B* Vol. 361, Issue 1465.

[2] Helbing, D. (1992) "A fluid dynamic model for the movement of pedestrians." *Complex Systems 6*, 391-415.

[3] Silverberg, J.L. et al. (2013) "Collective motion of humans in mosh and circle pits at heavy metal concerts." *Physical Review Letters* Vol. 101, Issue 22.

[4] Leonard, N.E. et al. (2014) "In the dance studio: an art and engineering exploration of human flocking." In: LaViers A., Egerstedt M. (eds) *Controls and Art*. Springer, Cham, 27-49.

[5] Hagendoorn, I.G. (2002). "Emergent patterns in dance improvisation and choreography." In: Minai, A.A. and Bar-Yam, Y. [Eds.]. *Unifying Themes in Complex Systems* Vol. IV. Proceedings of the Fourth International Conference on Complex Systems, 183-195.

[6] VU Ballroom (12 Nov. 2017) "Purdue 2017 standard bronze TF round 1." *YouTube*.

[7] VU Ballroom. (12 Nov. 2017) "Purdue 2017 standard silver FT round 1 F." *YouTube*.

[8] VU Ballroom (12 Nov. 2017) "Purdue 2017 standard bronze WQ round 1." *YouTube*.