Skyrmions in Chiral Magnets

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Abstract

Skyrmions are topological solitons originally invented by Tony Skyrme as a model for the nucleon, but which have since been realized in a broad range of other physical contexts. In magnetic materials, two dimensional versions of these skyrmions appear both in the bulk or in thin films as topologically stable, 'vortex-like' spin configurations that result from chiral interactions. This paper reviews the properties of magnetic skyrmions, beginning with their topological properties, moving to a theoretical description using a Landau free energy, and then experimental observations of their behavior in a variety of systems.

1 Introduction

A topological soliton is a continuous, localized field configuration which cannot be continuously deformed into the uniform state[5]. Skyrmions are a certain class of topological soliton invented by Tony Skyrme in his famous 1962 paper as a way to create particle-like objects from quantum fields, where this topological stability would represent the fact that fundamental particles do not decay[1]. The topology of a skyrmion is characterized by an integer quantity, the winding number, which cannot be changed without introducing a discontinuity somewhere in the field. Topological protection often leads to a form of energetic stability when introducing a singularity causes a local divergence in the energy density. Such field configurations are then extremely robust against fluctuations, and therefore long-lived.

Analogous structures are also known to appear in certain magnetic materials due to chiral interactions, where the topological field in question is the magnetization. The demonstrated stability of these magnetic skyrmions to thermal fluctuations has prompted numerous studies into their potential applications in new modes of memory and in spintronic devices. To better understand this property, it is necessary to have a short preliminary on topology

2 Topological defects

Rather than provide an exhaustive discussion, the objective of this section is rather to establish certain basic definitions about topological objects, and to provide some qualitative sense of how the mathematical object and the magnetic ones are related. "Skyrmions in Condensed Matter" by Jung[1] covers this topic in greater depth, and was the source for many of the ideas in this section.

A topological defect, or soliton, is a solution to a set of nonlinear partial differential equations and boundary conditions that is homotopically distinct from the uniform solution[1, 5]. This means that the field in question cannot be 'untangled', or continuously deformed to a trivial or uniform state without introducing a singularity somewhere.

The set of all the values that the field can take on is called the **target space**. The set of points on which this field takes on values is called the **base space**. The configuration of the field can be viewed as a map from the base space to the target space[1, 5]. A main idea which this section seeks to emphasize is the connection or analogy between topological solitons in nonlinear sigma models with different dimensional target and base spaces.

2.1 'Kinks' (Domain walls) in one-dimension

The classic one-dimensional topological defect is a domain wall. Take as an example a line of semiclassical spins $\{\mathbf{s}_i = s\mathbf{n_i}\}$ where each spin can rotate in one direction: $\mathbf{n_i}(\mathbf{x}) = (\cos\phi_i, \sin\phi_i)$. We will further consider the coupling between spins to be ferromagnetic, so if we impose boundary conditions at the ends of the chain, the spins will want to turn continuously from the orientation on one end to the other.

If we impose periodic boundary conditions on the chain, the trivial, lowest energy configuration is for the spins to all point in the same direction. However, the spins can still satisfy the boundary conditions undergoing a full 2π rotation from one end to the other. This configuration is clearly not the lowest energy state, because the spins are all slightly misaligned. However, any attempt at deforming it back to the trivial state will result in a discontinous change in the magnetization somewhere along the length of the chain. These two configurations are therefore topologically, or **homotopically** distinct.

To see this, imagine that the spin-chain is in fact a solid ribbon that has been twisted by 2π and held fixed at either end (see figure 1). Keeping the boundaries fixed, there is no way to untwist the ribbon without

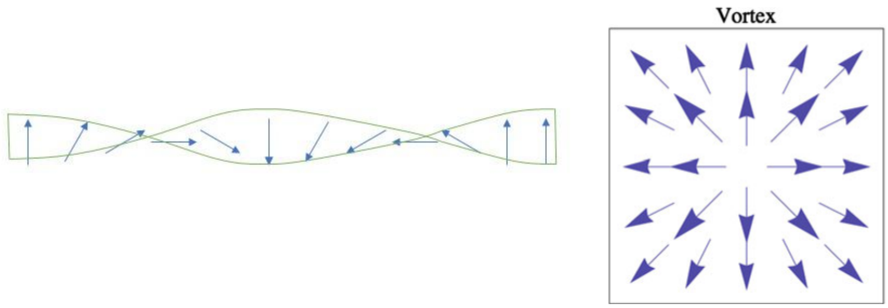

Figure 1: (Left) The domain wall and (green) ribbon analogy. (Right) Vortex configuration[1]

breaking it, and thus introducing a discontinuity.

The **target space** of the spins is the set of all of the directions in which they can point, or a circle. The **base space** is the chain, which can also be thought of as a circle thanks to the periodic boundary conditions. The trivial configuration can be thought of as a mapping from the base space to the target space where every point on the base circle is mapped to the same point on the target circle. If the spins undergo a full 2π rotation, every point on the target circle has at least one base point associated with it. We can therefore define an integer N, called the **winding number**, that gives us the number of times the target space is traversed as we go through the chain once. If we wanted to obtain N for an arbitrary configuration, we could do so using the integral:

$$\oint dx \frac{\partial}{\partial x} \phi = 2\pi N \tag{1}$$

The Lagrangian of the system will further determine what, if any, topological states are energetically stable. It might actually be favorable for the system to create a singularity to go to a lower winding number state. Moreover, if the size of the base space is taken to infinity, a topological soliton has to stay local. This can only be true if there is some energy cost associated with an increase in size in the domain wall[1].

2.2 Vortices

Consider again a smoothly varying field of two-dimensional spins $\mathbf{n}$, but now embedded in a two-dimensional target plane rather than on chain. The mapping from the base space to the target space is characterized by the same homotopy[1]. This can be seen if you consider a closed loop in the base plane, which has the same topology as a 1-D line with periodic boundary conditions. Rotations of $\mathbf{n}$ must clearly still have an integer winding number N. Slightly moving a piece of the loop through a region where $\mathbf{n}$ varies smoothly corresponds to a small continous change of our spin chain, and therefore cannot change the winding number[1]. If we now decrease the size of our loop to 0, we can see that there has to be a singularity at that point for there to N to be nonzero. This is called a **vortex** configuration. An easy example with $N = 1$ is the configuration with all spins pointing radially outward (see figure 1):

$$\mathbf{n} = \hat{\mathbf{r}} \tag{2}$$

2.3 Skyrmions in 2D

The skyrmions of the kind observed in chiral magnets have 3D spins arranged in a 2D plane. However, since the target space of the spins is the surface of a sphere, it is easier to start by visualizing 2D skyrmions as a configuration of spins also on the surface of a sphere (see figure 2). This spherical surface will later be mapped to the surface of a plane.

Since the base space and the target space have the same dimension again, it would make sense to again define a quantity that compares the number of times the target space is covered if we scan across the base sphere once. We should be able to extract it by integrating the differential piece of the target solid angle $d\Omega_t$ per differential solid angle from the base space $d\Omega_b$ [1]:

$$N_{sk} = \frac{1}{4\pi} \oiint d\Omega_b \frac{d\Omega_t}{d\Omega_b} \tag{3}$$

The quantity being integrated here is the Jacobian of the mapping between the base and target spaces. Topological configurations with a nonzero $N_{sk}$are called **2D skyrmions**. A simple configuration with $N_{sk} = 1$ again consists of all the spins pointing radially outwards:

$$\mathbf{n} = \hat{\mathbf{r}} \tag{4}$$

It is easy to see $N_{sk} = 1$, because there is a one-to-one mapping between the base and target space. It is also possible to see that this is topologically distinct from the trivial state with every spin pointed along, say, $\hat{\mathbf{z}}$.

In order to employ this concept in the context of magnetic skyrmions, we have to map the base sphere to an infinite Euclidean plane. This can be done using the process of **stereographic projection**[1]. This is a one-to-one mapping that can be visualized as the 'unfurling' of the sphere so that a single point on the bottom is mapped to the origin of our plane, while a point on the top of the sphere corresponds to a point at infinity on the plane. This process is described in the figure (see figure 2) where a raw emanating from the top of the sphere (infinity) connects the pair of points from the sphere and plane to be mapped to each other.

Taking the example of our $N_{sk} = 1$ skyrmion from before, our mapping to the plane would yield a spin configuration of the form[1]:

$$\mathbf{n_s}(x, y) = \frac{1}{r^2 + R^2}(2Rx, 2Ry, r^2 - R^2) \tag{5}$$

Here R is the radius of the spherical surface we are projecting, x and y are Cartesian coordinates on the plane, and $r = x^2 + y^2$. The winding number in terms of these new coordinates is[1, 2]:

$$N_{sk} = \frac{1}{4\pi} \oiint d^2 r \mathbf{n} \cdot (\partial_x \mathbf{n}(x, y) \times \partial_{\mathbf{y}} \mathbf{n}(x, y)) \tag{6}$$

An extra parameter γ called the helicity is also often defined, which represents an azimuthal rotation of all the spins but does not change the winding number. In the limit $r \to \infty$, all of the spins point the same direction ($\hat{\mathbf{z}}$), since they represent a single point on the sphere. This is also our boundary condition for the skyrmion. Meanwhile, the spins at the origin point in $-\hat{\mathbf{z}}$ (see figure 2). The sphere radius R controls how fast the rotation of the spins on the plane approach their value at infinity. As with the domain wall, there needs to be an energy cost to increasing the size of the skyrmion, otherwise it will expand to infinity in order to decrease the energy of its curvature. This can sometimes be provided by Zeeman coupling to a

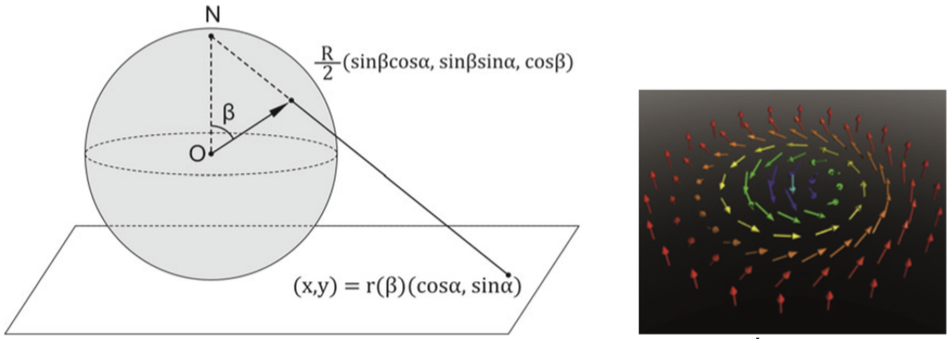

Figure 2: (Left) "Stereographic projection from a spherical surface to the Euclidean plane from a sphere of diameter R." A ray emanating from the north pole of the sphere connects the point on the sphere to be projected and its destination on the plane. Adapted from ref. [1] (Right) Skyrmion configuration with a nonzero helicity. Adapted from ref.[2]

magnetic field, as the number of spins not pointing parallel to the field can be thought of as approximately proportional to the area of the skyrmion[1].

Now we are ready to examine a magnetic system where the skyrmion is stabilized in a ferromagnet by a combination of the Zeeman term, and chiral magnetic interactions.

3 Ginzburg-Landau functional for chiral magnets

3.1 DM interaction and Helical magnetism

As mentioned above, the skyrmions dealt with in this paper arise in magnetic systems without inversion symmetry, which results in chiral interactions. These materials often host what is called helimagnetism, where spins on nearby atoms point roughly in the same direction, but precess around a common axis when viewed on a longer length-scale. The most well known examples are the cubic B20 materials such as MnSi, which this theory was originally developed for[1, 3]. Because the features we are interested in are these long-wavelength modulations, we can coarse-grain our lattice to treat the magnetization as a continuous field $\mathbf{M}$. To explain the inhomogeneity, Bak and Jensen were the first ones to write down a Ginzburg-Landau theory for the free energy density in terms of powers of the magnetization and its derivatives[3]:

$$F(\mathbf{r}) = J(\nabla\mathbf{M})^2 + D\mathbf{M}\cdot(\nabla\times\mathbf{M}) + \mathbf{B}\cdot\mathbf{M} + a(\mathbf{M})^2 + b(\mathbf{M})^4 + \sum_i cM_i^2 + dM_i^4 \tag{7}$$

The positive term J describes a ferromagnetic coupling between spins that wants to align neighboring spins, thus reduce the gradient of the magnetization. The term with the coefficient D, called the Dzyaloshinskii-Moriya (DM) interaction, is an exchange term which models a spin-spin interaction of the form $S_i \times S_{i+1}$ [3, 1, 2]. Finally, the term $\mathbf{B}\cdot\mathbf{M}$ represents a Zeeman coupling to an external magnetic field that wants to align everything with $\mathbf{B}$.

Terms a and b are important when examining the stability of the skyrmion lattice, but are often treated as constants when deriving the helimagnetic state. When the system is locally ferromagnetic, the magnetization will be close to uniform, and the heterogeneities we are looking are manifested through modulations of the spin direction [2, 3]. Meanwhile, in terms c and d the summation over subscript i represents summation over all vector components. The physical interpretation is that these are spin-orbit coupling terms that take on the symmetry of the cubic lattice, and are thus often called **anisotropy energy** (F_A). Because they represent the by far the smallest energy scale in the problem, they are usually ignored, however they will be important when setting the ordering direction of the spin spirals[3, 1]. Dropping the anisotropy and absorbing the constant magnitude of the local magnetization into the coefficients:

$$F(\mathbf{r}) \approx J(\nabla\mathbf{n})^{\mathbf{2}} + \mathbf{D}\mathbf{n}\cdot(\nabla\times\mathbf{n}) + \mathbf{B}\cdot\mathbf{n} \tag{8}$$

Let us consider the system first at zero field. Because is D/J is a small number, locally the ferromagnetic interaction wins again, and neighboring spins point in approximately the same direction. However, because of the DM interaction, the free energy is minimized when

$$\mathbf{n}(\mathbf{r}) = \frac{1}{\sqrt{2}}(\mathbf{n_1}\cos(\mathbf{q}\cdot\mathbf{r}) + \mathbf{n_2}\sin(\mathbf{q}\cdot\mathbf{r})) \tag{9}$$

This shows that over long lengthscales, spins will rotate around a common axis (ordering direction $\mathbf{q}$, which is implicitly determined by F_A). The energy of this state is minimized when $|\mathbf{q}| = D/J$, which shows that this feature is a result of competition between the two energy scales. This is the low-temperature **spin-spiral** state. If the field strength is increased above a critical value, it is favorable for $\mathbf{q}$ to align parallel to so that no spins are pointing antiparallel to it. The spins will then be canted into the field direction, so that they are no longer perpendicular to $\mathbf{q}$. This is called the **conical state**.

In certain B20 helimagnets, such as MnSi, a **multispiral** state with three spirals superimposed on another becomes stable right at the boundary between conical order and paramagnetism [2, 6]:

$$\mathbf{n}(\mathbf{r}) \approx \mathbf{n_0} + \sum_{i=1}^{3} \mathbf{n_i}(\mathbf{q_i}\cdot\mathbf{r} + \mathbf{\Delta r}) \tag{10}$$

As is described in the following section, small-angle neutron scattering determined that, at least in MnSi, there are three propagation vectors of equal magnitude, with an angular separation of $\frac{2\pi}{3}$ coexisting. This describes a spin texture with hexagonal symmetry. Now we will see how this hexagonal state was originally discovered using neutrons scattering.

4 Experimental observation of skyrmion lattices

4.1 Small-Angle Neutron Scattering

Scattering probes are some of the most powerful techniques in the experimental arsernal in the analysis of ordered phases of matter, built upon the well-known principle that scattering off of a potential will be proportional to the fourier-transform of the two-point correlation function of the scattering density in terms of the transfered momentum. For elastic scattering, the incoming and outgoing momenta are related by $\hbar(\mathbf{k_i} - \mathbf{k_o}) = \hbar\mathbf{q}$, where $\hbar\mathbf{q}$ is the momentum transfer, and $|\mathbf{k_i}| = |\mathbf{k_o}|$. In the elementary case of scattering off a set of parallel, equally spaced planes, this is manifested as Bragg's Law:

$$m\lambda = 2d sin\theta \rightarrow \frac{|\mathbf{q}|}{4\pi} = \frac{sin\theta}{\lambda} \tag{11}$$

Unlike X-rays, neutrons do not have a Coulomb interaction with the electron clouds inside a material, and are scattered exclusively by atomic nuclei and magnetic moments inside a sample. This makes elastic

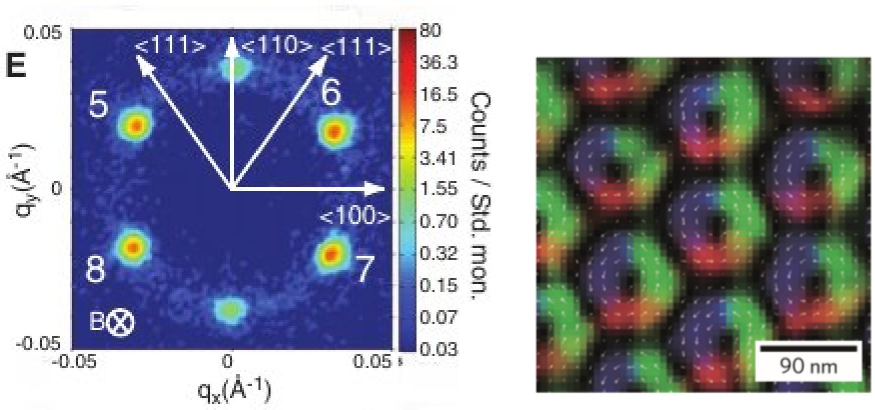

Figure 3: (left) SANS pattern showing hexagonally symmetric satellite peaks. adapted from ref. [6](right) illustration of the hexagonal skyrmion lattice in the SkX phase. adapted from ref. [2]

neutron scattering an excellent tool for the study of magnetic structures. Because the measured scattering intensity yields the fourier-transform of the system, looking at larger structures means looking at lower $|\mathbf{q}|$, or smaller scattering angles θ.

Small-angle neutron scattering (SANS) was successfully employed by Muhlbauer et al. in 2009 to probe the magnetic structure of chiral helimagnet MnSi in order to prove the existence of the skyrmion lattice (SkX) in the so-called A-phase of this class of magnets[6, 2]. Besides neutron scattering, the A-phase also has its signatures in hall-conductivity, heat capacity, and magnetization, so it was well known at the time that there existed some unexplained phenomenon in that region. By this time, there were many predictions of some form of topological crystal formation in the A-phase.

It was also known that applying a strong enough magnetic field to the sample in the spiral state would allign the spiral ordering vectors to be parallel with the field ($\mathbf{q_s} \parallel \mathbf{H}$). Pursuant to this, previous neutron studies had aligned the neutron beam perpendicular to the field ($\mathbf{k_i} \perp \mathbf{H}$), because this would probe the ordering direction of the conical state.

The key difference in the study by Muhlbauer is their orientation of the neutron beam along the magnetic field ($\mathbf{k_i} \parallel \mathbf{H}$), which allows them to probe order in the plane of what we known now to be the skyrmion lattice. Their observation of a hexagonally symmetric set of Bragg peaks (see figure 3), led to their explanation of a multi-spiral phase with three ordering vectors $\mathbf{q_{s(i)}}$ oriented in an equilateral triangle as described above. Using the Ginzburg-Landau theory above as a starting point, they show in their paper that long-range thermal fluctuations near T_C are enough to stabilize the multi-spiral lattice over the conical spiral phase. By minimizing the free energy using a variational calculation, they obtained a result for the magnetic structure, from which they then calculated the topological winding number N_{sk} (as defined above) per unit cell to obtain the result

$$N_{sk}/u.cell = \frac{1}{4\pi} \int d^2r\mathbf{n} \cdot (\partial_x \mathbf{n} \times \partial_y \mathbf{n}) = -1 \tag{12}$$

Muhlbauer et al.'s results suggested that the A-phase of MnSi is a triangular lattice of spiral skyrmions, each with a winding number of $N_{sk} = -1$, and a helicity which has since been calculated to be $\gamma = \frac{\pi}{2}$, which yields a spiraling magnetization (see figure 3) [6, 2].

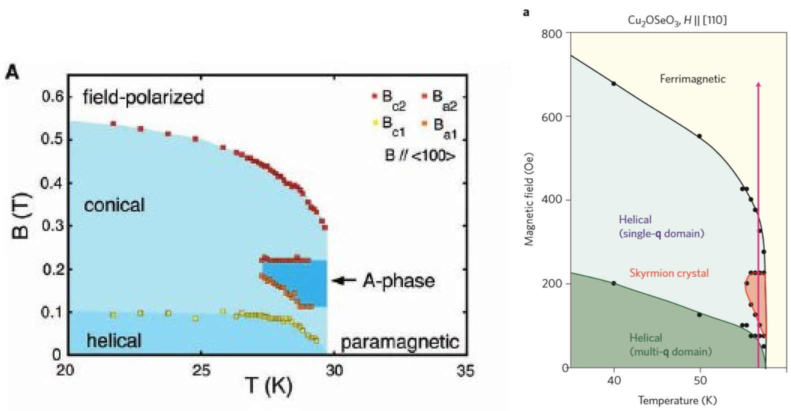

Figure 4: (left) Phase diagram for MnSi showing the A-phase at the boundary between the conical and paramagnetic phase[6]. (right) The phase diagram for CuO_2SeO_3looks remarkably similar, despite a number of differences in other material properties[2].

4.2 What kind of lattices host skyrmions?

As mentioned before, the two-dimensional magnetic skyrmions of the type we have been discussing can have been seen in a variety of systems where there exists some kind of chiral interaction to stabilize the skyrmion crystal. The classic example is the Dzyaloshinskii-Moriya interaction that appears in certain crystals without inversion symmetry. Examples of such materials include crystals with the cubic B20 structure, including MnSi or FeGe, both of which have a helimagnetic state in the bulk. [2]

However, chiral magnetic interactions can also arise through other mechanims, such as competing interactions between nearest neighbors, frustrated interactions, or dipolar interactions in heterostructures[2]. Chiral interactions that stabilize skyrmions are often labelled DM interactions, even when their origins are completely different from the asymmetric exchange mechanism which this label was originally afixed to.[1]

As an example, the material CuO_2SeO_3 studied below has the same spacegroup as the B20 materials, but has a different atomic structure. Furthermore, it has a different kind of magnetism; the B20 materials are **ferromagnetic**, with local moments all aligned in the same direction. In contrast, CuO_2SeO_3 has a kind of magnetic order called **ferrimagnetism**, which means that it has two magnetic sublattices, one with all of its moments oriented in one direction, and the other antiparallel to the first. (Macroscopically this looks a lot like ferromagnetism, and it is sometimes called that in the literature). Yet another difference is that MnSi and the other B20 materials in question are good metals, meaning there are conduction electrons interacting with the static spins, while $CuO_2SeO_3$is an insulator.

Despite these differences, the phase diagrams of MnSi and CuO_2SeO_3 look remarkably similar, with the skyrmion lattice phase occurring in a small pocket near transition[2, 7] (see figure 4). In the next section we will see that the stability of the SkX region in CuO_2SeO_3increases markedly under hydrostatic pressure[7]. A similar effect was also observed in MnSi in high pressure hall conductance measurements[2].

4.3 Stability of the skyrmion lattice

As shown in figure 1, the SkX phase in the bulk is typically only 'stable' in a small region of field and temperature at the boundary of the conical state and paramagnetism. In a number of cases, however, an increase in the size of this stable region is seemingly tied to the anisotropy term H_a which we neglected in our mean-field calculation.

Nagaosa and Tokura [2] point out that SkX in certain thin films, particularly in MnSi and $Fe_{1-x}Co_xSi$, have a much larger region of stability than in the bulk, and that the size is maximized when the film thickness is on the order of, or smaller than the helimagnetic spiral wavelength λ. They mention two competing explanations for this effect. The first has to do with destabilization of the helimagnetic state due to confinement effects on its spirals, while the second has to do with an increased magnetic anisotropy effect due to confinement in the thin film.

Meanwhile, hydrostatic (isotropic) pressure has also been observed to increase the stability of bulk skyrmion lattices. Levatic et al. studied the magnetic phase diagram of Cu_2OSeO_3 at high pressure using AC magnetic susceptibility measurements[7]. This kind of measurement applies a superposition of a static (DC) magnetic field and a small oscillating (AC) component to a sample, which induces a similarly oscillating magnetization in the sample. The induced moment is measured and then compared with the driving field to obtain a component that is in phase with the excitation, χ_{re} and a $\frac{\pi}{2}$ phase-shifted component χ_{im}. At low frequencies, the response of the sample is quasistatic, which means it can be used to obtain the slope of the magnetization vs applied field curve, while higher frequencies can be used to probe dynamics of the system. This tool is especially useful for mapping out phase transitions in magnetic systems.

The skyrmionic region of the phase diagram was observed to grow dramatically with the application of pressure as low as 0.17 GPa, by 2.3 GPa expanding to almost cover the phase diagram as measured by χ_{im}. (see figure 5) To account for this crazy result, they write down a modified version of our coarse-grained free energy:

$$F[M] = A(T)\mathbf{M}^2 + J(\nabla\mathbf{M})^{\mathbf{2}} + D\mathbf{M}\cdot(\nabla\times\mathbf{M}) + (U+K)\mathbf{M}^{\mathbf{4}} - \mathbf{B}\cdot\mathbf{M} \tag{13}$$

This energy is similar to the Landau free energy we have seen close to T_C for the ferromagnetic transition, except for the inclusion of the extra quartic coefficient K as their model for the often-neglected anisotropy Hamiltonian. Again, because we are now examining the instability of the conical state to the skyrmion lattice due to thermal fluctuations, our free energy has to depend on the magnitude of the magnetization, not just the spin direction. The $A(T)$ in front of the quadratic term is a temperature dependent parameter that was empirically adjusted to match the known ferromagnetic transition. U, meanwhile, is a mode-mode interaction term that was present in Bak and Jensen's original model[3]. By tuning the J, D, and K parameters to reflect the change in lattice constant expected under high pressure, they were able to largely reproduce the changes in the empirical phase diagram, but only with the inclusion of the anisotropy term.

4.4 Lorentz Transmission Electron Microscopy

Real-space imaging of the skyrmion lattice is possible using Lorentz Transmission Electron Microscopy (LTEM) [2, 10]. This technique takes advantage of the Lorentz deflection of electrons in a magnetic field to create image local magnetic moments in a material. It is perfect for imaging long-period structures like spin spirals and skyrmions, and is used in a few of the examples covered in the following sections.

5 Isolated skyrmions

5.1 Nucleating a skyrmion

Although the first confirmation of skyrmions in chiral magnets involved a ordered structure of many skyrmions described by a superposition of spiral states, skyrmions can also occur as lone, metastable knots in the local

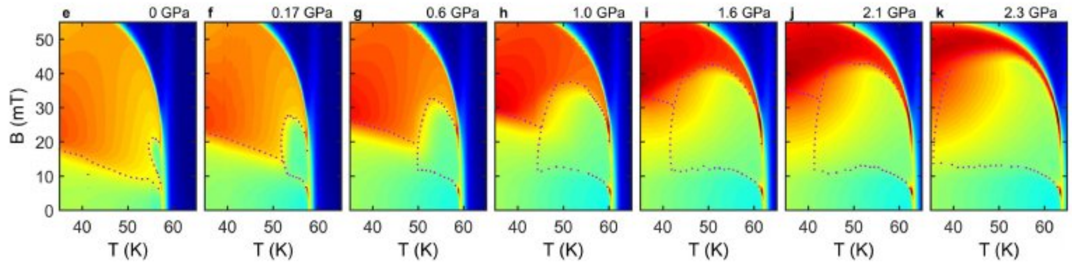

Figure 5: Pressure series of the phase diagram of CuO_2SeO_3, obtained through AC magnetization. The dotted line marks the phase boundaries as determined by the imaginary (out of phase) part of the susceptibility, while the color map represents the real part. [7]

magnetization [2]. This is truly what is meant when skyrmions are called topological solitons, and is also what lead to interest in their potential applications in memory devices, and it is worth investigating the formation of such a state.

Creating isolated topological structures in a chiral lattice requires a nucleation process that locally perturbs the underlying helimagnetic state. An interesting example is the 2015 paper by Jiang et al.[8], which describes the nucleation of a large number of skyrmions in various chiral thin films using a process similar to “bubble blowing”.

Helimagnetic domains, which show up as stripes in LTEM images of the magnetization, are forced through a constriction, which serves as the local perturbation and leads to numerous stable skyrmions on the other side. The motion of domain walls is achieved by the mechanism of spin-transfer torque, which will be described shortly.

5.2 Skyrmion Hall Effect

When a spin-texture such as a skyrmion moves, it is really the magnetic moments in the material rearranging in a coherent way. In his book[1], Jung derives the equation of motion for the center of mass rigid skyrmion through several different methods. These will not be reproduced here, but the results will be explained. An expression is obtained for the Lagrangian of the center of mass motion of the skyrmion by rewriting the action for a continuum of spins in the XY plane in terms of the topological charge:

$$L_s = \frac{1}{2}M_s\dot{\mathbf{R}}^{\mathbf{2}} - \frac{1}{2}G\hat{z}\cdot(\mathbf{R}\times\dot{\mathbf{R}}) - V(\mathbf{R}) \tag{14}$$

Here M_s is a phenomenological parameter that represents the effective mass of the skyrmion, and is related to magnons that are generated when the spin-structure is set in motion. Meanwhile $G = 2hSQ_s/a^2$ where S is the spin and a is the lattice constant, which appears when we coarsegrain the action and Q_s is the topological charge of the skyrmion. $V(R)$ is a general potential term from the skyrmion's environment. If we find the Euler-Lagrange equations we obtain:

$$M_S\ddot{\mathbf{R}} = -\frac{\partial V}{\partial \mathbf{R}} + G\hat{z}\times\dot{\mathbf{R}} \tag{15}$$

This is equivalent to the equation of a charged particle moving in a magnetic field, and is often called the Skyrmion Hall Effect (SHE). The potential term is due to spin-transfer torque, the net effect of the

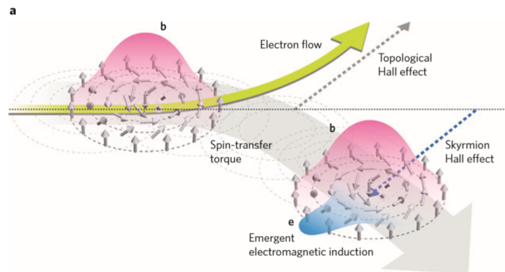

Figure 6: Illustration representing the topological and Skyrmion Hall effects. Notice that transverse components of the skyrmion motion and electron motion are oppositely oriented. [2]

interaction of conduction electrons interacting with the localized magnetic moments [1, 2].

5.3 Topological Hall effect due to skyrmions

Conversely, electrons moving through a magnetic field will feel a Lorentz-force, or equivalently, build up a geometric phase due to the Aharonov-Bohm effect [1]. In chiral magnets that are also good metals, such as MnSi[2, 10], the conduction electrons will also feel an emergent gauge field due to their motion through the local spin-texture.

The usual assumption is that conductions electrons will align with the local magnetization as they move through the lattice[2, 1, 10]. This leads to the conduction electrons also feeling an emergent magnetic field $\mathbf{b}$ equal to the local skyrmion density[2]:

$$\mathbf{b} = \hat{z}\frac{1}{2}\mathbf{n} \cdot (\frac{\partial \mathbf{n}(x,y)}{\partial x} \cdot \frac{\partial \mathbf{n}(x,y)}{\partial y}) \tag{16}$$

as well as to the existence of an emergent electric field $\mathbf{e_i}$:

$$\mathbf{e_i} = \mathbf{n} \cdot (\partial_i \mathbf{n} \times \dot{\mathbf{n}}) \tag{17}$$

The coupling of the conduction electrons to the emergent electromagnetic field is also responsible for the skyrmion hall effect, as was mentioned earlier. The net effect is that if an electric current is flowing through the skyrmion lattice, the skyrmions will start to move parallel to the current, but will be deflected in a transverse direction. Meanwhile, the charge current is deflected in the opposite transverse direction. Because real system has dissipative effects, both skyrmions and electrons will end up with terminal drift velocities in straight paths that are diagonal to the applied voltage[1, 2] (see figure 6). Having examined the dynamics of individual skyrmions, we should talk about the dynamics of a lattice of them.

6 Dynamics of the skyrmion lattice

6.1 Phonon modes

Due to their topological stability, single magnetic skyrmions can often be thought of as localized, rigid objects moving in an emergent electromagnetic field[2, 1, 8]. In this picture, the SkX phase is a periodic array of these particle-like objects, held together by some skyrmion-skyrmion interaction, similar to atoms in a crystal lattice. This suggests that if the lattice is perturbed by displacing one of the skyrmions slightly from its equilbrium position, there would be a restoring force trying to bring it back to equilbrium. In the absence of damping, it is reasonable to expect a low-energy phonon mode that arose from breaking of continuous translation symmetry. An effective Lagrangian for the skyrmion lattice of the following form can be constructed[1]:

$$L[\{R_i\}] = \frac{M_{sk}}{2}\sum_i(\dot{R}_i^2 - \frac{G}{2}\hat{z}\cdot(R_i \times \dot{R}_i) - \frac{\dot{K}}{2}R_i^2) - \sum_{i<j} V(R_i - R_j) \tag{18}$$

Here $R_i$is the center of mass of each skyrmion, and as before the cross-product term is a result of the skyrmion motion in an emergent gauge field, which introduces a transverse component to its velocity. Assuming the potential term is harmonic to lowest order and considering only small perturbations:

$$V(R_i - R_j) \approx \frac{V_0}{2}(R_i - R_j)^2 \tag{19}$$

The lowest-energy acoustic phonon mode is a Goldstone boson, as its energy goes to zero as $|\mathbf{k}|$does, and since it arose from continuous symmetry breaking[2, 1]. Because of the G term, the associated motion is actually a uniform gyration of each skyrmion around their equilibrium position.

Because the magnetization also underwent a kind of continuous symmetry breaking in the formation of the skyrmion lattice, it is reasonable to ask if there is also a gapless spin-wave excitation of the SkX. The answer, as it turns out, is no [2]. Because the skyrmion lattice requires a magnetic field to stabilize it, a spin-wave excitation corresponding to rotations of the magnetic moment the $|\mathbf{k}| = 0$ limit will always have a nonzero energy.

Experimental observation of some dynamical excitations of the skyrmion lattice in Cu_2OSeO_3 were performed by Onose et al. using magnetic resonance spectroscopy in the microwave frequency regime[9]. Such a measurement puts the sample in a uniform magnetic field, then probes the absorption spectrum of gigahertz-frequency electromagnetic waves directed onto the sample. This allows study of the characteristic timescales of magnetic excitations as a function of field and temperature. Two resonance peaks were found in this way only in the SkX region, one for each polarization of the oscillating field. By studying the absorption spectrum, first with the AC field oriented in the skyrmion lattice plane ($H_{AC} \perp H_{DC}$), then perpendicular, ($H_{AC} \parallel H_{DC}$).

The first excitation with the AC field in the plane of the skyrmion lattice ($H_{AC} \perp H_{DC}$). This was interpreted as a gyration of the skyrmions similar to what is predicted for the gapless acoustic mode above. However, it is clear that this mode is not the gapless mode predicted above, as it requires high frequencies to be excited[9, 5, 2].

The second resonance peak occurs only with the driving field in the out of plane direction ($H_{DC} \perp H_{AC}$). This was associated with a 'breathing' excitation of the skyrmions in the lattice. This can be understood as a consequence of the Zeeman term in the skyrmion energy. Because the skyrmion radius is defined by the area where spins are not parallel to the field, an oscillation of the magnetic field causes an oscillation of the skyrmion radius.

6.2 Rotation of the skyrmion lattice

As we know, skyrmions have dynamic properties both in the lattice phase, as well as individual solitons. No real world lattice is free of defects, and the skyrmion lattice is no exception. Various LTEM studies have revealed the existence of structural domains of different orientation in the skyrmion lattice.

A peculiar phenomenon has been observed if a thermal gradient is put across the lattice, along with a parallel electric current. In this case, structural domains of the skyrmion lattice have been observed to undergo rigid rotations [10]. For small gradients, the rotation stops at a finite angle, but for larger ones the domains can rotate continuously with a finite angular velocity. This effect has been seen using a wide variety of methods, including LTEM and SANS.

The explanation put forward by Everschor et al.[10] begins with individual skyrmion motion due to spin-transfer torque induced by the moving conduction electrons, as described in the previous section. By symmetry of the lattice, a single-domain skyrmion crystal under a uniform current with no thermal gradient could experience no net torque. The addition of the thermal gradient creates inhomogeneous dissipative forces across the domain, which yields a net torque. The rigidity of the skyrmion lattice then allows the whole structure to turn in one piece.

7 Open research questions

As was mentioned in the introduction, this is an extremely active subfield of condensed matter physics, largely driven by its potential technological applications.

A class of more fundamental problems lies in the understanding of what actually goes on in the SkX phase for the B20 materials and CuO_2SeO_3 with regards to thermodynamic stability. For example, pressure studies have shown that high pressure can broaden the SkX region, but it is unknown if the skyrmion lattice undergoes any major changes as a result[2, 7].

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