Particles may not be employed, if they don't pull themselves up by their boostraps

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Abstract

Our current understanding of physics lies in a reductionist approach. An attempt to put physics under a non-hierarchical footing more in line with Emergence was proposed by Chew in 1968. In this essay, I will explore this approach based on self-consistency, talk about its success, its decline, and its renewed life in critical phenomenon. I will describe how time could possibly emerge from the bootstrap approach. I will elaborate on possible future connections with different fields of physics that could benefit from the Bootstrap approach and vice versa. Finally, I will end with a brief remark on shedding the idea of a particle.

1 Introduction

One of the lessons that I took from Emergent States of Matter is that symmetries are more fundamental than particles. Breaking symmetries in interesting ways give rise to interesting particle/quasi-particle behaviours. When translational symmetry gets broken in solids it gives rise to emergent "particles" like phonons. This class changes the way one thinks about physics. It shows us ways to obtain the same outcomes as the traditional microscopic methods but with a lot fewer assumptions.

Traditionally doing physics meant that every phenomenon or law at some scale l has its origin at some microscopic level $a << l$. "Understanding" in physics usually corresponds to answering what is the underlying smaller scale constituents that give rise to larger scale complex phenomenon. We usually employ particles in our definitions to try and explain larger scale concepts. But this corpuscular/particle notion has some drawbacks. The finer we go in detail, the more assumptions we need to add. But as we saw in class, simpler, broader generalized theories could help provide the same level of description, without incorporating all of the small scale assumptions. Another issue with the traditional method is that this reductionist approach does stop somewhere, but its not rigourous as to why it stops where it stops. Why do we stop at the Standard model of particle physics? Even when most of the particles are unstable and very short lived.

G.F Chew thought of a radically different idea in describing reality that is in the spirit of emergence which he called the Bootstrap Theory [3] (Interestingly, G.F Chew was a faculty here at U of I in the 50s). The Bootstrap theory looks at general constraints bought to us by experiments or symmetries or underlying mathematics, and then uses self-consistency to obtain quantifiable results. Only a few assumptions go into it and it gives powerful results. Self consistency arguments may seem strange but self-consistent calculations are found plenty in computational physics and quantum chemistry where Hamiltonians are iteratively solved and recursively fed back into their machinery.

2 Bootstrap Theory

In traditional particle physics, we treat particle interactions perturbatively. We first draw out the Feynman diagrams and then add more and more terms in decreasing order of their contribution to the total sum. The bootstrap picture does not treat interactions in this hierarchical fashion. It is more akin to how we treat all the infinite prime numbers. Prime numbers are fundamentally equal, and are also elementary constituents, since all numbers can be written as the product of primes. This sort of non-heirarchy is what Chew calls 'nuclear democracy' [3]. Nuclear democracy as implied by Chew means no particle is more fundamental than any other in the realm of strong interactions. There are no fundamental particles, each particle is a composite of all other particles. To make more sense of this argument, consider an example of a 'classical' bootstrap of the ρ meson.[5] The ρ meson decays into two pions. The bootstrap theory for the ρ meson considers it to be composed of two pions bound together and also the force generator of ρ *simultaneously*! The meson and its properties exist because it can bootstrap itself into existence. Going further, Chew mentions that 'each particle helps generate other particles which in turn generate it' [3]. All the hadrons generate each other and basically bootstrap themselves into existence. The bootstrap theory is built out of the notion of self-consistency, where self-consistency is in some sense a sufficient condition for determining nature.

To achieve particle democracy, the bootstrap employs the S-matrix approach. Around the 1950s, quantum field theory (QFT) had run into some difficulties as it was unable to make meaningful numerical predictions for strong interactions. The S-matrix program was an alternative that could avoid the infinite renormalization effects that were plaguing QFT. It garnered quite a bit of success, and after reaching its peak, evolved into string theories. The S-matrix refers to the scattering matrix, which is a unitary matrix that describes probabilities before and after scattering without describing the collision itself. Each element of the S matrix describes a particular nuclear reaction. The bootstrap claims that through experiments, we can provide few general constraints that will be sufficient to define a unique S-matrix. Chew showed us that unitarity, Lorentz invariance, together with first and second degree analyticity were sufficient to determine one and only one S-matrix for hadrons [3].

For an example of a bootstrap calculation consider two particles A and B. A interacts with itself and produces B, and to keep things simple we shall assume the force between A and B don't produce any new particles.

$$A + A \to B \tag{1}$$

Next let the B particles interact with each other as above and form another particle C. But let's consider the bound state C to have the same mass as A, so it would look like

$$\begin{aligned} B + B &\to C \\ B + B &\to A \\ A + A &\to B \end{aligned} \tag{2}$$

The processes described in equation (2) will be described by the scattering amplitudes that will go in the S-matrix. Now noting that the S-matrix is unitary and is analytic, you can calculate parameters like mass of B in terms of A. As the masses enter the equation both at input and output, you can obtain unique masses for A and B. This type of calculation was done for the ρ meson and the numbers agreed with experiment. [13] A big win for Chew's bootstrap approach.

2.1 Decline of traditional bootstrap

Chew's theory unfortunately relied heavily on the S-matrix approach, and the S-matrix approach had started falling out of fashion as gauge field theories such as Quantum Chromo-Dynamics (QCD) started having much greater success. The S-matrix wasn't able to deal with weak and electromagnetic phenomenon that involved massless particles. Under the bootstrap program, the self-consistency equations were often hard to work with and were very complicated. The successful description of particles from quarks and gluons and the discovery of both the quark and the gluon placed the final nail on bootstrap theory's coffin.

The second coming of Bootstrap came in the realm of statistical physics, where it found good use in calculating properties of second-order phase transitions and critical phenomenon.

2.2 Conformal Bootstrap Theory

At the critical point, there are fluctuations at all scales and we see an absence of any characteristic scale. This fact led Alexander Polyakov in the 70's to show that critical points possess "conformal symmetry"[9]. To understand conformal symmetry consider this, start with rotations in space. If you add Lorentz boosts you will obtain the Lorentz group, then if you add space-time translations to the mix you will get the Poincaré group. Finally if you extend this group by adding scale transformations and a special transformation called the conformal transformation (that first inverts the space, then translates and then inverts again, most importantly it preserves all angles) you get the conformal group. So if your system is invariant under all these transformations then we say that it has conformal symmetry. This symmetry especially in 2D was used to rediscover the exact solution of the 2D Ising model. And thus began the field of Conformal Field Theories (CFTs).[2].

The bootstrap method is super useful when there is scale invariance [8]. This scale invariance is seen when fluctuations dominate near the critical point. These fluctuations can be physically seen in a lab through the phenomenon of critical opalascence.

Let's consider a liquid/gas near the critical point and calculate how the fluctuations are correlated

$$\langle \delta\rho(\vec{r}_1)\delta\rho(\vec{r}_2)\rangle \sim \frac{\text{const}}{|\vec{r}_1 - \vec{r}_2|^{2\Delta}} \tag{3}$$

where, $\rho(\vec{r}_i)$ is the density of liquid at $\vec{r}_i$, $\delta\rho(\vec{r})$ is the deviation of $\rho(\vec{r})$ away from average, $|\vec{r}_1 - \vec{r}_2| >> a$ the inter-molecular distance and Δ is the critical exponent. This critical exponent is universal for all liquids, and also for magnets.

In CFT, measurements of the system at some point x is characterized by an infinite set of local operators $\mathcal{O}_1(x), \mathcal{O}_2(x), ...,$. When we constrain the system to obey conformal symmetry then the two-point correlation function of scalar operators takes the form

$$\langle \mathcal{O}(x)\mathcal{O}(y)\rangle = \frac{1}{|x-y|^{2\Delta_{\mathcal{O}}}} \tag{4}$$

Here $\Delta_{\mathcal{O}}$ is called the scaling dimension for the operator $\mathcal{O}$. Clearly, from the equations above we can see that these scaling dimensions are directly related to the critical exponents measured in experiments. In fact it goes even further, as the conformal symmetry reduces all

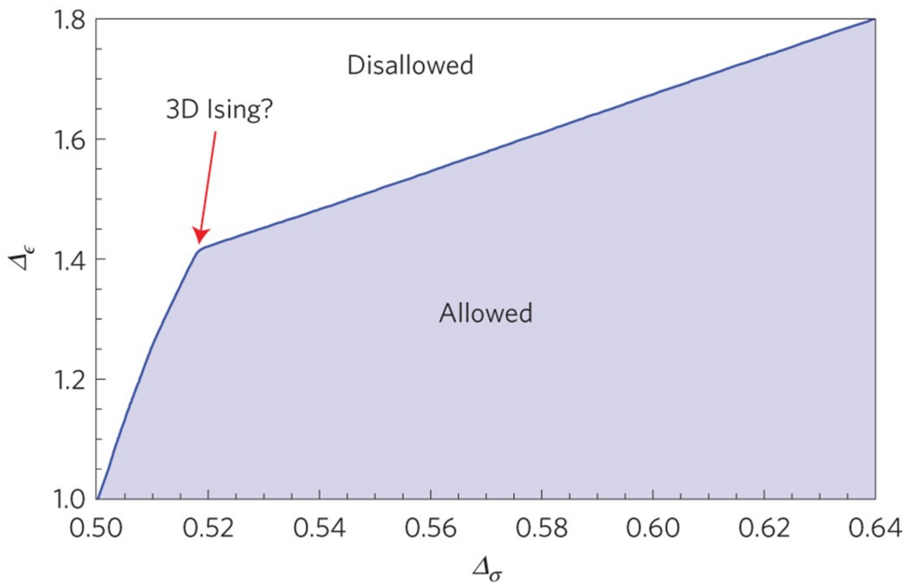

Figure 1: Upper bound on even (y-axis) and odd (x-axis) scalar dimensions in 3D CFT[11]. The only assumptions that went into making this graph are conformal invariance and unitarity. The kink is believed to correspond to the critical dimensions of the 3D Ising model

the CFT observables to a few set of numbers for each operator, out of which one of them is $\Delta_{\mathcal{O}}$. Since there are infinite of these local operators there are infinite of these critical exponents.

Now let's consider the bootstrap method of self-consistency which will be employed to calculate critical exponents. One of the self consistent conditions that places a simple lower bound on $\Delta_{\mathcal{O}}$ is unitarity. Another self-consistency condition arises from the 4-point operator correlation functions which I will not discuss in this essay, although I urge the reader to watch the cited presentation where it is described very well[10].

Using these constraints scientists were able to place universal bounds on these critical exponents. Using these bounds, they were able to map out what critical exponents are possible and what are not. Strikingly these bounds exhibited a kink as seen in Figure (1) near the values of the scaling parameters for 3D Ising model. They were able to further refine their approach and calculate the scaling parameters to really good precision. Their results showed to be consistent with the best Monte Carlo results. A big win for conformal bootstrap.

3 Recent Developments

Current work through the Simon's collaboration has been on using the bootstrap approach to map and understand the whole space of QFTs and CFTs especially the strongly coupled models while making minimal assumptions. Perimeter Institute is regularly having conferences on the Bootstrap approach and it is currently an exciting time to be bootstrapping in High Energy

Physics.

But one interesting avenue that I want to shed light on is how bootstrap is being used in Cosmology. In a 2019 paper by Arkani-Hamed et. al [1], the authors studied spatial correlations in the cosmos which were caused by inflation. Quantum fluctations were constantly producing particle-antiparticle pairs and annihilating these pairs in the early universe. Some of these pairs after being produced were quickly stretched away due to the quickly expanding universe caused by inflation. This caused these pairs to end up in different regions of the cosmos and cause correlations to appear in seemingly distant places in the night sky (Those pairs could decay into more pairs which would give rise to higher order correlations, although those get rarer the further we go up in order). Arkani-Hamed and Maldecena found out that when you restrict the universe to have conformal symmetry it tightly constrains what sort of these cosmological correlations could be produced by inflation. Baumann and Lee the two other co-authors took the results of Hamed and Maldecena's calculations and used the self-consistent bootstrap methods to extend it to three and four point correlation functions for a range of primordial fields. Their equations simplified many equations including Maldecena's dozen page three-point correlation function calculation that collapsed to just a few lines [12]. This shows the power of a general bootstrap approach over a traditional approach. So in short, scientists start with the conformal symmetries, throw in the ingredients of inflation, and what comes out are differential equations. Upon solving these differential equations scientists get the three and four point correlations. To solve these equations one has to consider the singularities.

One way to obtain a singlularity by self consistency is by folding the four point correlation function and matching it with the three point correlation function in this folding limit (imagine a square folded into a triangle). This picks out a function for the four-point correlation that is oscillatory. Note everything before this was timeless, and now we are seeing an oscillatory function. As Baumann says, "Since oscillations are synonymous with time evolution, this for me was the clearest instance of the **emergence of time**" [12]. Through just symmetries and singularities, the bootstrap method helped bring out time evolution!

4 Future and Discussion

Bootstrap is a powerful approach of looking at whats possible through self-consistent relations and constraints such as symmetries. There are very few assumptions compared to what traditional approaches use. This truly lies in the spirit of emergence. Currently the bootstrap style of physics is constrained to high energy physics, but I feel it has broader connections. I would like to comment on some of the possible connections with fields of research below

4.1 Constructor Theory

Constructor Theory is a theory proposed initially by David Deutsch[4] that expresses physical laws solely in terms of which physical transformations are possible and which are not. These physical transformations are called tasks. In constructor theory if a task has a constructor — a physical entity that can carry out this task repeatedly, then that task exists otherwise it is impossible. Using constructor theory, Marletto and Vedral have been able to describe an experimental test that would be able to prove whether gravity is quantum or not.[7] The constructor theory idea of using the impossible to constrain the possible could be used to enhance the bootstrap method. Also, the self-consistent ideas of the bootstrap method could be added to constructor theory. For example, consider providing a self-consistent relation between a constructor and its task such that the constructor and the task bootstrap themselves into existence! Both the theories use general concepts (such as symmetries or laws) that constrain the space of solutions to squeeze out important results. These exciting top down approaches have potential to be merged and so far no one has been working on merging these two ideas.

4.2 Quantum Circuits

Quantum circuits and simulators have become new platforms for exciting many body experimentation. The 2020 McMillan Prize awardee Dr. Vedika Khemani has been theorizing said new platforms in her group. She has been working on answering deep questions on the border of Statistical Mechanics and Quantum mechanics through the lens of quantum circuits. In a recent paper[6] her lab has shown that by flipping space and time in unitary quantum circuits, they were able to probe steady-state phases of non-unitary dynamics and discover new steady state phases with fractal scaling of its entanglement entropy.

This fractal scaling reminds me of scale invariance seen near critical points. It has also been shown that the bootstrap approach is quite successful when there is scale invariance. One way to extend the program could be to think about the different constraints and symmetries that restrict the space of quantum circuits that give rise to say non-unitary phenomenon, and then apply self-consistent relations to map out circuits that give interesting many-body phenomenon or to discover new phases. Approaching the problem in the same manner CFT/QFT bootstrappers at Perimeter Institute are using to map out the space of CFT/QFTs might prove to be useful.

5 Conclusion

The recent efforts in studying many-body physics through quantum circuits, the phenomenological approaches we took during class, the constructor theorists approaching physics through constraints on what's possible, the bootstrappers building fundamental physics from self-consistent relations; what we are seeing today is that it is possible to do interesting physics without the use of "particles" and particle like description. Instead, we are identifying the key minimal assumptions from which we can obtain powerful results. Each particle description carries with it a baggage of assumptions. The particle approach has been very fruitful in defining and studying the very small. Maybe now, we can think of ways of letting go of this notion of particle and work with other fundamental constraints like symmetries, experiments etc., especially for problems that involve collective behaviour.

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