The development of the inflation models

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Chunyu Lu

Abstract

This article is about different inflation models and phase transitions related to the models. We would focus on what kind of observational effects they can predict and explore different cosmological surveys to see the constrains they could give to the inflation model.

I. INTRODUCTION

The standard cosmology model shows that our universe starts with a big-bang. It's widely accepted now that shortly after the big-bang, there is an inflation period. The inflation means that the universe expanding rapidly(exponentially) through a short time.[1]. The reason why the concept of inflation is bring up at is to solve several puzzles about the universe at first[2].

First is the horizon problems. The largest distances that is causally connected is called comoving horizon η[1]. The measurement of the cosmic microwave background shows that the initial universe is homogeneous and isotropic in a large scale. This scale is even much larger than the horizon, which means the modes are not causally connected. Then it is strange that the photons separated so far way have similar temperature[1].

Then the flatness of the universe. The energy density ρ of the universe is very close to the critical energy density ρ_{cr} from measurements.

$$\Omega \equiv \rho/\rho_{cr} \tag{1.1}$$

$\Omega > 1$ means that the universe has positive curvature and it is closed, $\Omega < 1$ means that the universe has negative curvature and is open. The measurement shows that $\mid \Omega - 1$lmid is very close to zero. So the universe is close to flat, that not close or expanding too quickly to form the structure today. That means the initial condition should be fine tuned to satisfy the requirement[2, 3].

Third, the relic density problems. The prediction from the theory of the early univiese about monopoles, cosmic strings or other defects is not consist with the experiment measurement.

The inflation model could explain these puzzles quite well, and we can see that in the following. As there are so many models of inflation. The first model of inflation bring up by Alan Guth[2], and then the "new inflation model" to solve the exit problem[4] by fine tuning, later on, consider more than one scalr field like "extended inflation" which also ends with first order transition but in modified gravity (Brans-Dicke theory)[5] and so on. We will introduce several popular models through the history of the development of the inflation theory. And Show the reason why they are interesting. This paper is organized as below.

II. DIFFERENT KINDS OF INFLATION MODELS

The inflation has developed for about 40 years, there is a large amount of different inflation models. There are several different ways to separate these models. First is based on the initial condition, second by different regimes possible in inflation, third by the how the inflation end[5].

The "old inflation" and the "new inflation" assume that the universe is in the thermal equilibrium at the beginning.

A. First Inflationary Model of Alan Guth

1. *Standard Model at That Time*

First we need the metric of the universe. This model is under the Robertson-Walker metric:

$$d\tau^2 = dt^2 - R^2(t)[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta d\phi^2)] \tag{2.1}$$

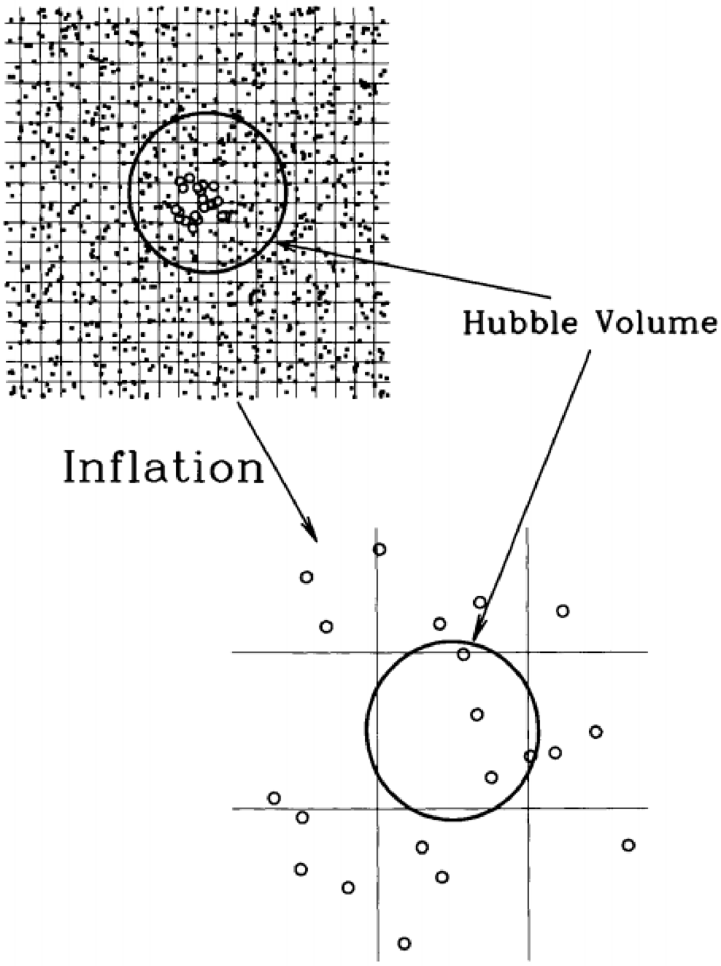

FIG. 1: inflation [1]

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As we can adjust R, k needs only take 0, +1, -1 to present all kinds of shapes of the universe, where k = 0 for flat universe, k = +1 for closed universe, k = -1 for open universe.

The general relativity give us the equation of motion:

$$\ddot{R} = -\frac{4\pi}{3}G(\rho + 3p)R, H^2 + \frac{k}{R^2} = \frac{8\pi}{3}G\rho \tag{2.2}$$

Assume the total entropy is conserved and the expansion is adiabatic, then:

$$\frac{d}{dt}(\rho R^3) = -p\frac{d}{dt}(R^3) \tag{2.3}$$

And,

$$\frac{d}{dt}(sR^3) = 0 \tag{2.4}$$

The thermodynamic functions will lead to the evolution of universe on T:

$$(\frac{\dot{T}}{T})^2 + \epsilon(T)T^2 = \frac{4\pi^2}{45}Gn(T)T^4 \tag{2.5}$$

where n, ϵ is defined by:

$$n(T) = N_b(T) = +\frac{7}{8}N_f(T)$$
$$\epsilon(T) = \frac{k}{R^2T^2} = k^2[\frac{2\pi^2}{45}\frac{n(T)}{S}]^{2/3} \quad (2.6)$$

With the measurement at that time, the S needs to satisfy:

$$S > 10^{86}, \mid \epsilon \mid < 10^{-58} n^{2/3} \quad (2.7)$$

This lead to:

$$\mid \frac{\rho - \rho_{cr}}{\rho} \mid = \frac{45}{4\pi^3}\frac{M_p^2}{nT^2}\epsilon < 3 * 10^{-59} n^{-1} \quad (2.8)$$

If taking T 10^{17} Gev and n 10^2 according to knowledge at that time, the last equation gives:

$$\mid \frac{\rho - \rho_{cr}}{\rho} \mid < 10^{-55} \quad (2.9)$$

which is just the flatness problem.

Furthermore, the distence light can travel from t = - to time t is:

$$l(t) = R(t)\int_0^t dt' R^{-1}(t') = 2t \quad (2.10)$$

And the distance evolves as:

$$L(t) = [s_p/s(t)]^{1/3} L_p, \quad (2.11)$$

where p means present value. Compared with last two equations we have:

$$\frac{l^3}{L^3} \; 4 * 10^{-89} n^{-1/2} (Mp/T)^3 \quad (2.12)$$

This is the horizon problem.

2. *How Inflation Solve the Puzzles*

Give up the adiabaticity in expansion, and assume the initial and current total entropy(sR^3) entropy satisfy:

$$S_p = Z^3 S_0 \quad (2.13)$$

Then the equation (2.8) need to multiply Z^2 on the RHS, and (2.10) needs to multiply Z^{-1} on RHS. Thus, take Z large enouth to 10^{28} can solve both the flatness and the horizon problems, which is totally practiable.

When the universe is expanding, it is also cooling down at the same time, as it reach the critical temperature T_c, the the nucleation of bubbles of low temperature phase starts. But the temperature still goes down, when the temperature goes down to T_s, the phase transition take place, and released the amount of energy comparable to T_c, which heat the universe to T_r. Assume the nucleation rate is λ(t), the factor Z has expression:

$$Z_\tau = exp(\chi_\tau) = exp(\frac{3\chi^4}{4\pi\lambda_0}) \quad (2.14)$$

With the expression of λ_0 to be Aρexp(-B), the Z could be easily to reach 10^{28}.

3. Properties of this inflation model

When T is close to 0, the universe is cooling to true vacuum, the density ρ is now:

$$\rho(T) = \frac{\pi^2}{30}n(T)T^4 + \rho_0 \tag{2.15}$$

As a result, the equation of T changes to:

$$(\frac{\dot{T}}{T})^2 = \frac{4\pi^2}{45}Gn(T)T^4 - \epsilon(T)T^2 + \frac{8\pi}{3}G\rho_0 \tag{2.16}$$

For small enoughϵ, we have:

$$T(t) \approx Const * e^{-\chi t} \tag{2.17}$$

where χ= $\sqrt{8\pi G\rho_0/3}$ And R(t):

$$R(t) \approx Const * e^{\chi t} \tag{2.18}$$

This show that the universe is expending exponentially. But it then leaves a problem about how the inflation is end, This could be solved by later models of inflation.

B. New Inflation

With the Grand Unified Theories, the inflation could driven by scalar field. In Albrecht and Steinhard's paper, they assume a quite complicated form of potential, see the FIG 2. For T > T_{GUT}, the the ϕ equals to zeros phase is the global minimum, so there is no symmetry-breaking. For T < T_{GUT}, the $\phi = 0$ phase is no longer the global minimum, and therefore a phase transition would happen, which could be the classical nucleation.

The rate of bubble creation rate per unit time per unit volume defined by Γ(t) can be approximated at special conditions T arond T_{SD}:

$$\Gamma(t) \approx Sexp(-F_f(T)/kT) \tag{2.19}$$

See the FIG 3 for the the F_f(T) / kT. The curve ends at special temperature T_{TERM}, when the fraction of stable phase is in the order of unity[6]. The sate with Δ_H ¿ 7*10^{-6}, the $F_f(T_{term})$ / kT_{term} is large, but not enough to solve the cosmology puzzles, for Δ_H= 0, the approximation is failed.

And we can see that the exponential expansion would happen when $\phi \ll$T$_{GUT}$ for a time τ which is close to t_{exp}. As a result, the fluctuation region go through large e foldings in spatial expansion even the $< \phi >$ doesn't change much.

We can get fluctuation radius with relation to T see FIG FIG 3. And because the observed universe needs inside the fluctuation region, the value of Δ_H needs to be tuned.

C. Extended inflation

Another solution to end the inflation is still keep the first-order phase transition but use modified gravity. Take the "Extended inflation" as an example[5].

This model assume the Brans-Dicke theory of gravity with action:

$$A = \int d^4x\sqrt{g}[-\Phi R + \omega(\frac{\partial_\mu\Phi\partial^\mu\Phi}{\Phi}) + 16\Phi L_{matter}] \tag{2.20}$$

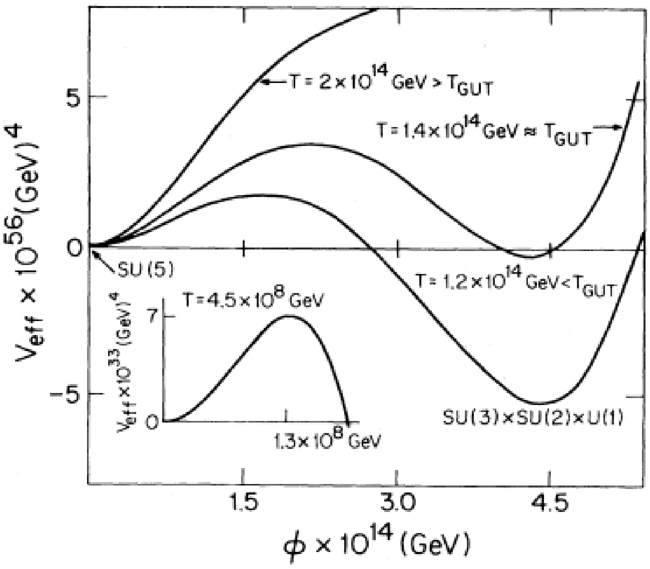

FIG. 2: the effective potential from reference [6]

The parameter ω goes to infinity means the action goes to Einstein theory. This difference could end the inflation reasonable. With this action, the equation of motion changes. For p = 1 = -ρ = -ρ_f and k=0. we have:

$$\Phi = m_P^2(1 + \chi t/\alpha), \tag{2.21}$$

$$R(t) = ((1 + \chi t/\alpha)^{\omega+1/2} \tag{2.22}$$

Where $\alpha^2 = (3{+}2\omega)(5{+}6\omega)/12$. When χt < a, the $\omega > 90$, we have 60e folding of inflation. When χt > a, $\Phi \approx m_P^2(\chi t/\ \alpha)$, R(t) $\approx(\chi t/\ \alpha)^{\omega+1/2}$. This is a power law, and don't need finely tuned. When it comes to power law, the bubble-nucleation is still exponentialy, but the expansion is power law. Define the probability of a point in false-vacuum is p(t), as χt /ω larger than $(3/\pi\omega\epsilon)^{1/4}$, p(t) $\ll 1$.

The universe is dominated by true vacuum, and exit the false vacuum[5].

D. Another Hybrid Inflation

Apart from end the inflation with slow rolling or first order phase transition. It could also end with rapid rolling of two combined field. There are two scalar fields and ϕ[7]. And the end of this inflation is dominated by the potential $V(\sigma)$.

Combine chaotic inflation ϕ with spontaneous symmetry breaking filed with potential:

$$V(\phi) = \frac{m^2\phi^2}{2}, V(\sigma) = \frac{1}{4\lambda}(M^2 - \lambda^2) \tag{2.23}$$

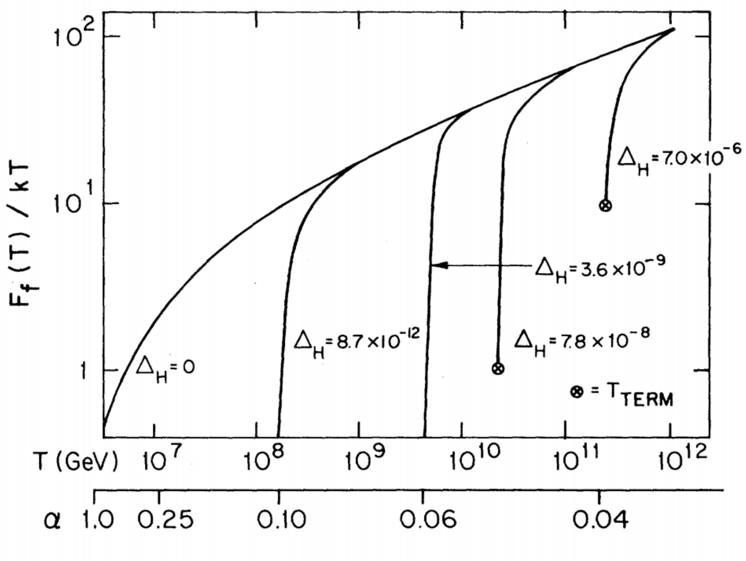

FIG. 3: the effective potential from reference [6]

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T (GeV)	R_U (flat) (cm)	R_U (exp) (cm)	R_U ($\Delta_H = 3.6\times10^{-11}$) (cm)
4.5×10^6	10^{81}	$\gg 10^{470}$	...
4.5×10^7	$10^{-4.6}$	10^{470}	...
1×10^8	10^{-13}	10^{98}	...
3×10^8	10^{-19}	$10^{-4.4}$	...
4.5×10^8	10^{-20}	10^{-13}	$10^{-7.2}$
4.5×10^9	10^{-23}	10^{-23}	10^{-23}

FIG. 4: the effective potential from reference [6]

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The effective potential is:

$$V(\sigma,\phi) = \frac{1}{4\lambda}(M^2-\lambda^2)^2 + \frac{m^2}{2}\phi^2 + \frac{g^2}{2}\phi^{22} \tag{2.24}$$

The σ field is a Higgs field, and effectice mass squared is -M^2 + g^2 ϕ^2. When $\phi > \phi_c$=M/g, the $\sigma = 0$ state is the global minima, at this stage, the Universe is expand as the field σ rolled to 0, ϕ remain large.

When ϕ reaches ϕ_c, the global minimum changes, there is a symmetry breaking happens. For m^2 $\phi_c^2 = m^2$ M^2/g^2 $\ll$M^4/ λ, we have:

$$H^2 = \frac{2\pi M^4}{3\lambda M_P^2} \quad (2.25)$$

For ϕ ¿ ϕ_c, the universe go through a inflation, at the end of the inflation, it is driven by the vacuum energy density V(0,0) = $\frac{M^2}{4\lambda}$ as new inflation. After $\Delta_t = H^{-1}$ from $\phi = \phi_c$, we have:

$$3H\dot{\phi} = m^2\phi \quad (2.26)$$

So we have,

$$\Delta\phi = \frac{m^2\phi_c}{3H^2} \quad (2.27)$$

And,

$$M^2(\phi) = \frac{\lambda m^2 M_p^2}{\pi M^2} \quad (2.28)$$

For $M^3 \ll\lambda$ m M_P^2, within Δt, the field rolls down to the minimum at M(ϕ)/ $\sqrt{}\lambda$, and oscillates around this point. But it is not the end, as there are two field, and it is not minimum of the field ϕ.

$$\frac{\partial V}{\partial\phi} = m^2\phi + \frac{g^2\phi M^2(\phi)}{\lambda} \quad (2.29)$$

The field rolls to the minimum in a period of time smaller than H^{-1}. The inflation end ends very close to the ϕ reaches ϕ_c. It's behavior is close to:

$$\phi = \phi_c * exp(-\frac{m^2(t - t_c)}{3H}) \quad (2.30)$$

And the scale factor grows as a ∼e^{Ht}

III. EXPERIMENT ABOUT THE INFLATION

From all the models describe above, the simplest one is the single-field slow-roll inflation. This model give some predictions that the precise experiments could verity. Such a the primordial perturbations is adiit's distribution is Gaussian[kamionkowski2016quest], the prediction of the stochastic background of gravitational waves, which would indece "b-mode" in the polarization of CMB.

A lot of CMB surveys like Planck[8], SPT[9], ACT[10]BICEP[11] and so on. Usually the cosmology surveys would give constrains(upper limit) to tensor-to-scalar ratio r, which is an important parameter defind in Inflationary Gravitational Waves. But we haven't measured it yet. See FIG for the results of Planck2018[8]

IV. CONCLUSION

The inflation models can solve the cosmology puzzles. And with decades of development, there are a large amount of inflation models already. It is already widely accepted that it has happen in the early universe. But the constrain for the models is still not good enough.

Cosmological model ΛCDM+r	Parameter	*Planck* TT,TE,EE +lowEB+lensing	*Planck* TT,TE,EE +lowE+lensing+BK15	*Planck* TT,TE,EE +lowE+lensing+BK15+BAO
	r	<0.11	<0.061	<0.063
	$r_{0.002}$	<0.10	<0.056	<0.058
	n_s	0.9659 ± 0.0041	0.9651 ± 0.0041	0.9668 ± 0.0037
	r	<0.16	<0.067	<0.068
	$r_{0.002}$	<0.16	<0.065	<0.066
$+\mathrm{d}n_s/\mathrm{d}\ln k$	n_s	0.9647 ± 0.0044	0.9639 ± 0.0044	0.9658 ± 0.0040
	$\mathrm{d}n_s/\mathrm{d}\ln k$	-0.0085 ± 0.0073	-0.0069 ± 0.0069	-0.0066 ± 0.0070
	r	<0.092	<0.061	<0.064
	$r_{0.002}$	<0.085	<0.055	<0.059
$+N_{\mathrm{eff}}$	n_s	$0.9607^{+0.0086}_{-0.0084}$	0.9604 ± 0.0085	0.9660 ± 0.0070
	N_{eff}	2.92 ± 0.19	2.93 ± 0.19	3.02 ± 0.17
	r	<0.097	<0.061	<0.061
	$r_{0.002}$	<0.091	<0.056	<0.056
$+m_\nu$	n_s	0.9654 ± 0.0044	0.9649 ± 0.0044	0.9668 ± 0.0036
	$\sum m_\nu$ [eV]	<0.24	<0.23	<0.11
	r	<0.12	<0.066	<0.062
	$r_{0.002}$	<0.12	<0.062	<0.057
$+\Omega_K$	n_s	$0.9703^{+0.0045}_{-0.0046}$	0.9697 ± 0.0046	0.9663 ± 0.0044
	Ω_K	$-0.012^{+0.007}_{-0.006}$	$-0.012^{+0.006}_{-0.007}$	0.0006 ± 0.0019
	r	<0.11	<0.064	<0.062
	$r_{0.002}$	<0.10	<0.059	<0.057
$+w_0$	n_s	0.9675 ± 0.0042	0.9669 ± 0.0042	0.9659 ± 0.0040
	w_0	$-1.58^{+0.14}_{-0.34}$	$-1.58^{+0.14}_{-0.34}$	-1.04 ± 0.05

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