



EMERGENT STATES OF MATTER

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**Active Flocks that Squawk  
Geese on 2D Sheets**

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## 1 Abstract

This essay explores agent based modeling and the Vicsek model to help explain emergent behaviors of migrating snow geese on a 2D lake surface. We will also review other 2D biological flocks such as surf scooters on the ocean. In addition, we will cover 3D swarms of krill in a more recent paper that takes into account transfer entropy and mutual information to explain swarm behavior due to more complicated social interactions.

## 2 Introduction: Observations of an Amateur Birdwatcher

*“Wild geese that fly with the moon on their wings, these are a few of my favorite things” – The Sound of Music (Film, 1965)*

For about nine days in early March, giant flocks of 3000 snow geese (*Chen caerulescens*) stop by the lake behind my family’s yard in Sinking Spring, Pennsylvania during their migration to Canada. (No really it is about 3,000. I used TrackMate with ImageJ to count the number of white blobs on a small segment of lake to get a density and then multiplied by the approximate lake area after taking time lapse videos with my cellphone camera taped to the window) [11]. My mother and I both wondered why sometimes the birds swim about randomly and other times there’s a flow line of geese promenading through the crowd in one direction. Geese form life long pairs during their second winter migration [2]. I have noticed that paired geese will often swim in similar directions even if there are other geese between them. Once a flow line forms a V shape through the water, all other geese eventually join the unified swim. Joining the flow line seems to be a result of crowd avoidance since geese in the way of the moving front would have to move the least distance if they join. Any other direction would result in further need to avoid clashing with the moving front.

On particularly cold days, I have wondered why the geese choose to sleep on the lake and if the geese activity is actually enough stirring and heating to keep the lake from freezing so that they could access the algae growing underneath. The answer is yes. According to Ronald Ryder’s 1970’s master’s thesis, geese actively break ice and keep sections of water unfrozen even in subzero temperatures given sufficient bird density [9]. Heat production by the feet increases in freezing temperature waters as opposed to freezing temperature air [9]. In addition, goose feces functions as fertilizer increasing the local algae [9]).

I have also wondered why sometimes the geese spread out evenly through out the lake leaving plenty of personal space between themselves instead and just float stationarily. Even geese spread usually happens at night time but will also occur occasionally during the day. During migration season, the largest visiting population is the snow geese, but mallards and Canadian geese also visit in smaller numbers. Sometimes small amounts of mallards and Canadian geese will share the water with the snow geese, but more often, one species will take turns with the lake. Finally, all the geese would take off at once, circle around in a large group, and take off northwards. The effect is so dramatic you can feel the wind from their wing beats! Its astounding that a population of 3000 can make one unanimous decision to move collectively while humans can never reach such consensus (except perhaps subconsciously synchronizing our footsteps).

One morning, I spotted two bald eagles eyeing the lake. Every now and then, they would swoop in

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attempting to capture a goose or a mallard. The water birds were in the spread out formation. A targeted bird would simply dive and reappear elsewhere. After a few tries, the eagle would give up and leave the lake. Bird separation allows for easy ducking into the water to avoid an eagle without running into other ducks. This might seem counter intuitive given that schools of fish and herds of zebra never separate since it is easier to separate and target an individual. However, in this case the eagle is not a talented swimmer while the waterfowl is an expert diver. (Alternatively if I were in my home town in Louisiana the alligators cant fly but the waterfowl can). Since predators are restricted to the air or water, geese sleeping on the water makes sense. Even in their half awake and half asleep state, they can flea from a predator quickly. Spread formation explained by predator avoidance is my own hypothesis and I make the disclaimer that I am a mere bird watcher and not an ornithologist, so I could be wrong. However, this would not be the first time that birds split in response to a predator. In the Great Emu Wars of 1932, the Australian army gave up after multiple attempts to comb down the emu population at the request of farmers who lost their crops. In response to attacks, emus would split and scatter. This was an excellent survival strategy as the emu army suffered very few casualties. (Turns out fences are a much brighter idea at deterring flightless birds than guns).



Figure 1: Left: Zoomed in photo of snow geese (*Chen caerulescens*) in my back yard. The center bird is a blue morph snow goose. Right: Each white speck a snow goose covering most of the lake. Both photos are taken in Sinking Spring, Pennsylvania in March 2021.

### 3 Boids: Computational Flock Simulations

The first successful attempts to computationally animate flock behavior was in 1987 by C. W. Reynolds [5]. The agents affectionately called “boids” were governed by three constraints: 1. Collision avoidance 2. Velocity mating with other boids in a neighborhood of a fixed radius 3. Attraction towards boids within a fixed short radius. These simulations would quickly reach an ordered phase where all boids would move in unison. However the model does not swap back over to a disordered state like real birds. Boids are relatively computationally simple to simulate and can be parallelized easily for large simulated flocks. The motion of a boid is determined by its 8-neighbors once all radii

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are scaled to unity [6]. In 2018, Tang et. Al. performed a clever study simulating classical boids along with boids with more complicated communication strategies mimicking a social network [10]. A neural net would train on both types of boids and try to predict the next move. The adversarial learning algorithm knew the boid cohesion, alignment, repulsion, and update mechanism but did not know specific parameters or weights, the underlying interaction mechanisms, or the vision range and angles of boids. The learning algorithm was less successful in figuring out boid parameters and predicting the next movements for boids who had an underlying network-based proximity [10]. In addition, the networked boids enter the ordered state faster than their classical boid counterparts [10]. Complex social dynamics makes predators less likely to predict next movements. This is extremely interesting because it demonstrates the difficulty of extracting interaction rule parameters computationally – implying it is also difficult to extract interaction parameters in live biological samples.

## 4 Vicsek Model

There is an excellent set of notes on the Vicsek model written by Francesco Ginelli that I enjoyed reading. This brief section on the Vicsek model are my own condensed notes on Ginelli’s notes [6]. The Vicsek model describes nonequilibrium active matter that are self propelled and follow local alignment interactions with neighbors. Neighbors can be defined by nearest physical distance (metric neighbors) or by topology and neighbors may change as a consequence of motion. Momentum is not conserved in the model due to dissipative forces or drag but number of particles is generally conserved.

$$\begin{aligned}
 r_i^{t+\delta t} &= r_i^t + \delta t v_o s_i^{t+\delta t} \\
 2d : \\
 s &= (\cos(\theta), \sin(\theta)) \\
 \theta_i^{t+\delta t} &= \text{Arg}[\sum_j n_{ij}^t s_j^t] + \eta \xi_i^t
 \end{aligned}$$

where  $\eta$  is the noise strength and  $\xi$  is a delta correlated scalar noise. In general the noise term can also be a vector.  $n$  is the connectivity matrix which is a 1 if  $i$  and  $j$  are neighbors and a 0 if  $i$  and  $j$  are not neighbors. (This can change with neighbors).  $v_o$  is particle speed.  $s$  is orientation. And  $r$  is the current position of a particle. This model is not sensitive to the exact type of noise, if direction is determined by  $t$  or  $t + \delta t$  or the existence of a short range repulsion force. In 2D, the state can be characterized by a polar order parameter or center of mass velocity. A small center of mass velocity near zero occurs in the disordered state when particles move randomly. A larger velocity indicates the ordered state where all particles move in a unified direction. Direction of motion is not predetermined and instead arises from fluctuations resulting in a spontaneously broken continuous symmetry. If one considers a model with metric neighbors and low density and rescales so that the relevant radius within which neighbors are found is unity, the mean free path of particles becomes  $l \approx 1/\rho_o$ . Particles then align after effective “collisions.” The persistence length  $l_p$  is defined as how far the particle can travel before losing its previous head direction and accounts for dissipative terms. Intuitively the persistence length is inversely proportional to the variance of the noise  $l_p \approx v_o/\eta^2$ . At the transition point to ordered motion,  $l = l_p$ . Setting the two expressions for the persistence length equal, we have a critical noise threshold  $\eta_c \approx \sqrt{\rho_o}$ . Similarly one can fix noise and velocity to arrive at a critical density or fix density and noise to arrive at a critical velocity. Locally high density areas

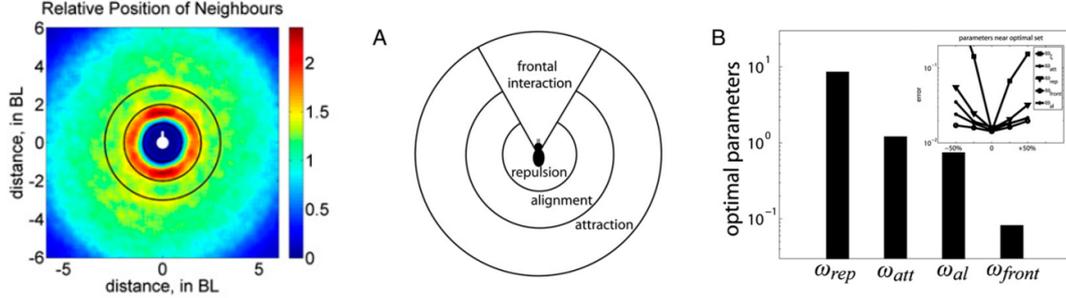
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have positive feedback for local alignment. This creates a long wavelength instability and leads to spontaneous phase separation of ordered bands from the disordered bulk. Individual particles join and leave the bands at a steady rate leading to long range band stability. The bands spread transverse to their direction of motion and have a well defined width. A second transition allows for transition between an ordered state and the disordered state with a single ordered band. This coexistence of phases indicates a first order transition. However, the bands can only exist if the system is larger than the instability wavelength. Therefore, people thought the transition was second order initially. However, if the neighbors are instead topologically defined, long wavelength instability is destroyed and the transition is indeed second order and has critical exponents. Toner and Tu showed that the Vicsek model with a convection term bypasses the Mermin and Wagner theorem that states that long range order cannot exist in  $d = 2$  or lower. This theorem is specific to equilibrium systems while the Vicsek model is inherently out of equilibrium since the flock has continuously changing dynamics. The Mermin Wagner theorem only accounts for stationary particles where information transfer of errors and perturbations diffuses in all directions simultaneously. In the Vicsek model however, the particles are moving so there is diffusive and movement based information transfer. Due to bunching in the direction of motion, information travels faster perpendicular to motion via diffusion than it does in the parallel direction. Orientation fluctuations are scale free in the Vicsek class.

As a review, the Vicsek Class is defined by:

1. Existence of a polar or ferromagnetic alignment term despite the fact the equations are isotropic. If this term is stronger than the noise term, the system can develop collective motion.
2. Particles are self propelled.
3. Three control parameters determine the modulo of the order parameter  $\phi(t) = |\phi(t)|$ : density  $\rho_o$ , noise amplitude  $\eta$ , and velocity shared by all particles  $v_o$ . This means velocity is same for all particles but this constraint is loose as long as  $v$  is within a certain min and max range (true for most animals).
4. Ordered phase direction not picked but emerges from fluctuations.
5. Number of birds conserved but momentum is not. Thus Galileian invariance also broken. The Vicsek Model is calculated in a reference frame where dissipative substrate is at rest.

## 5 Experimental Studies on Sea Ducks, Guppies, and Krill



From Ryan Lukeman et. Al. PNAS 2010

Figure 2: From Lukeman et. Al. PNAS 2010. Left: density map of neighbors showing preference for neighbors in front of the surf scooter duck, a zone of attraction in red, and a zone of repulsion in dark blue closer to the bird. (actual data) Center: A schematic of repulsion, alignment, and attraction zones along with the zone of frontal interactions. Note that this particular bird frequently moves its head so it does not have blind spots. Right:  $\omega$  represents weights of forces. Repulsion strength (rep) is an order of magnitude stronger than attractive (att) and alignment (al) which are both an order of magnitude stronger than front interactions (front) (matched from simulations) [8].

While computational simulations are important to confirm predictions given by theory and allow for the clean extraction of critical parameters, we have no idea how close simulated flocks are to biological ones unless comparisons are made to field data. In 2010, Ryan Lukeman et. Al. took videos of the sea duck *Melanitta perspicillata* (surf scooter) from a dock under which the birds forage on muscles. They wrote equations of motion taking into account attractive, repulsive, and alignment forces of the birds (see below). And then the authors found the best fit for the force weights by tuning simulation behavior to match actual data. The result shows a zone of repulsion close to each bird. At intermediate radii, there is a zone of alignment influences from neighbors [8]. Immediately outside the zone of alignment is a cohesive attraction to the flock. Although the force from frontal interactions is weaker by an order of magnitude compared to the attractive and alignment forces, it is still enough to create preference for neighbors directly in front of the bird [8]. (See Fig. 2). This study focuses on the ordered phase when all birds are swimming in the same direction with only slight left to right deviations. The supplementary videos show a high level of alignment for flocks up to 100 birds on the ocean surface. While the results most likely apply to snow geese in the ordered phase due to similarities between waterfowl, it does not speak to the disordered phase.

$$\frac{d\vec{x}_i}{dt} = \vec{v}_i$$

$$\frac{d\vec{v}_i}{dt} = \vec{f}_{i,autonomous} + \vec{f}_{i,interaction} + \vec{\xi}_{i,(noise)}$$

$$\vec{f}_{i,interaction} = \omega_{repulsion} \vec{f}_{i,repulsion} + \omega_{attraction} \vec{f}_{i,attraction} + \omega_{alignment} \vec{f}_{i,alignment} + \omega_{front} \vec{f}_{i,front}$$

In 2014 Lukeman made a follow up study taking into account the disorderd phase in surf scooters [7]. To simplify behavior effects from differences between foraging, predator avoidance, and resting

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behavior, only foraging data was taken. In surf scooters, the disordered phase is short lived during foraging behavior. More importantly, front neighbor bias is not observed during the disordered phase. All other force weights remain about the same as the 2010 paper, however this time force weights were derived by optimizing the group's efficiency to organize back into the polarized (ordered) phase [7]. Lukeman notes that it is unclear whether the disordered phase is caused by lower linear velocities (higher turning) or vice versa. However, the conal nature of the attraction and alignment zones only exist if the repulsion zone is empty. Thus collision avoidance plays a roll in reduced linear velocity during the disordered phase. The author suggests perturbations cause the transition from order to disorder for surf scooter attraction force strength  $\omega_{attraction} \approx 5$ . A lower threshold of attraction strength ( $\omega_{attraction} < 4$ ) causes frequent group splitting. Perhaps snow geese have a smaller attraction strength than surf scooters to explain the observation that they both swim in disordered and ordered states for long periods. Of course, my back yard data does not separate foraging, predator avoidance, and resting behaviors. Perhaps one factor in the ordered to disordered transition is the perturbations of tired individuals who no longer want to swim quickly in a group?

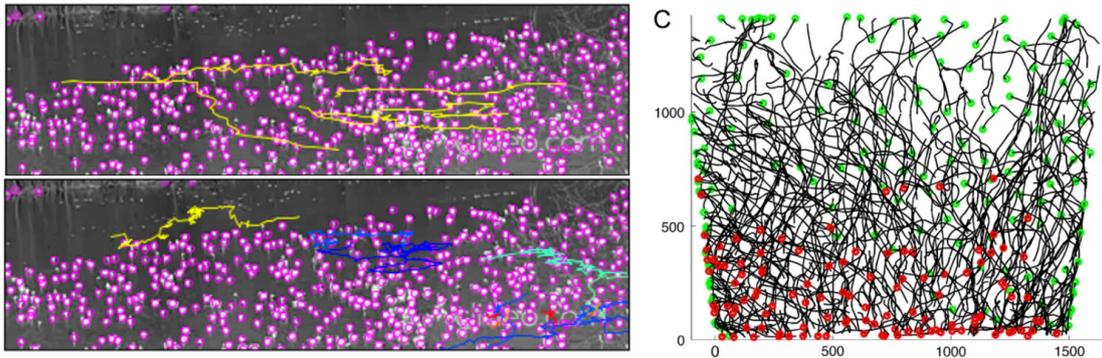


Figure 3: Left Top: Labeled snow geese from my own data showing a selection of the longest length tracks. Left Bottom: Snow geese with a selection of the longest duration tracks. Right: From Lukeman 2014, surf scooter tracks with starting positions in green and end positions in red [7]. All tracks show noisy variability in direction even if there is an overall start to end movement in the flock.

In 2017, Scarlet Davis et. Al. hypothesized that nearest neighbor (metric) interactions and topological interactions are not sufficient to explain guppy shoal dynamics because guppies will swim differently depending on their familiarity with their neighbors [3]. (I promise this is the last time I will mention this same author – our friend Lukeman was also an author in this study). Guppies were taken from the wild, labeled with elastomer, and kept in different tanks to form familiarity bonds with their tank mates which takes 12 days [3]. Guppies can become familiar with up to 40 fish [3]. Four guppies would then be taken from random groups and mixed in a tank for different levels of familiarity and unfamiliarity [3]. The result is that familiar guppies will swim in the ordered state for longer time periods than unfamiliar guppies. Familiar guppies also swam at faster speeds than they did with strangers [3].

I suspect snow geese have similar favored interactions with birds they are more familiar with in their flock and closer family units than birds they are only recently acquainted with for migration

purposes. Snow geese form pair bonds in the second winter of their lives and are visibly correlated in motion with their pair. Unpaired geese are generally young and still have a few streaks in their feathers and make up a smaller portion of the population. I am not sure if young geese still interact mostly with their parents or if they interact equally with flock members to socialize in attempt to form pair bonds. Studying familiarity effects on the snow geese would require individual goose labeling and long term monitoring of the geese over different seasons.

Lukeman [7], mentions that information transfer between individual birds is faster in the disordered phase since the bird will have different neighbors over time. In the ordered phase, the neighbors generally stay the same. Therefore the disordered phase can be advantageous to share information about food and predator locations. In 2021, Alicia L. Burns, et. Al. studied Antarctic krill in search of information transfer behavior effects on the swarm. Transfer entropy is defined as a conditional mutual information in the following way: For a given  $z$ , the conditional mutual information from  $x$  to  $y$  is  $I(x; y|z) = \langle \ln \frac{p(y|x,z)}{p(y|z)} \rangle$ . This tells us how much  $x$  determines  $y$  for a given  $z$  [12] [4]. The information transfer and alignment of krill was strongest in the vertical direction [1]. This makes sense because they have photospheres on their bellies that make them more visible to predators from below. Attacks usually come from above or below and rarely come from the sides[1].

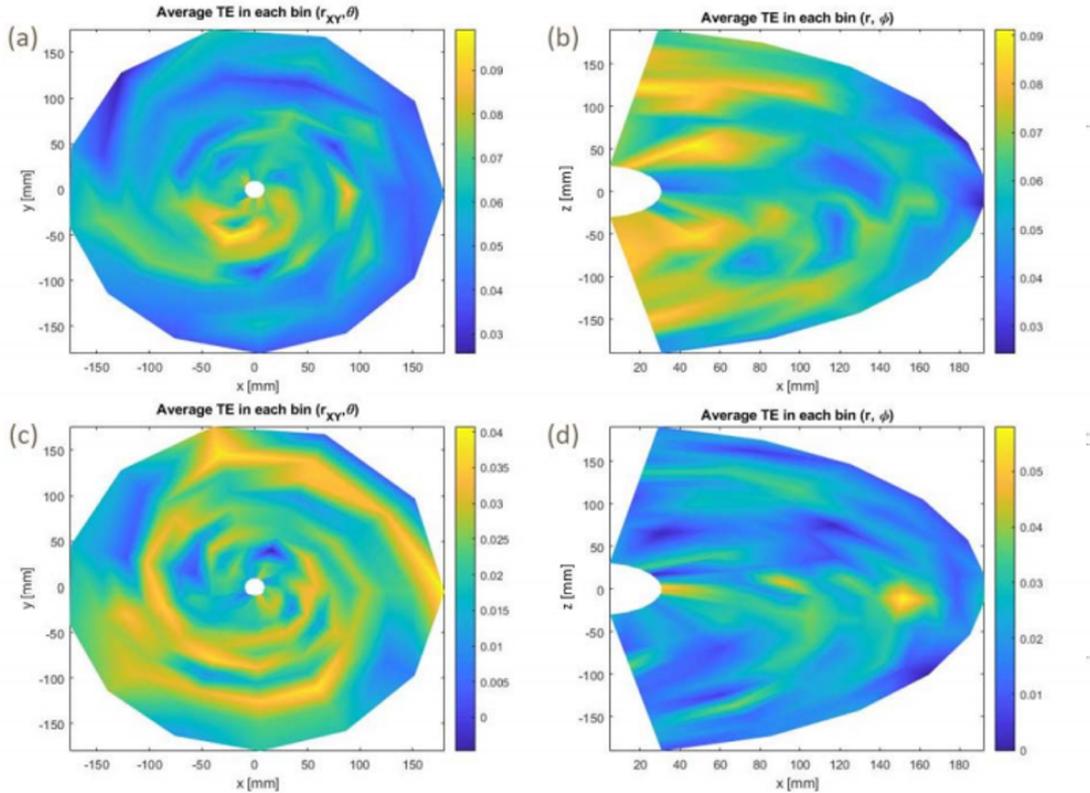


Figure 4: Transfer Entropy on average for a krill swimming from -x to +x with a near neighbor. Directions are given with respect to the animal as opposed to with respect to the environment. Taken from [1]

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## 6 Future Work

While searching for papers, I find a huge number of hits on Arxiv under “boids” but very few hits for actual bird data. This is unsurprising given the relative speed of running simulation studies and the slower process of labeling real data and correcting for bird overlaps for flying birds in photography. It is also much simpler to tell a model what parameters to use and hit go than to extract force interaction parameters from real data [7]. Much of the 2D flocking literature focuses on microscopic molecules or single cell organisms that are easier to grow and control in the lab. I suspect higher order organisms will display more complex social interactions that may deviate them from microorganisms. However, the complex organisms still fall in the Vicsek class since complex social interactions can be considered a type of noise. My original intension was to take my time-lapse of snow geese and write a script to measure mean heading direction as a function of time along with strength of neighbor interactions and perhaps calculate the information transfer entropy as a function of time. FIJI (is just ImageJ) has built in tools to subtract uniform backgrounds. However, my videos have trees and other background objects. I have made an attempt at zooming into a smaller piece of the video with a background of just water and successfully labeled birds using Trackmate [11]. This allowed me to estimate the lake bird density and calculate that my flocks contained about 3000 individuals. Since my videos are taken from the house window, better background subtraction could be achieved with aerial/drone photography to achieve a constant background. To correct for perspective, one can measure the length and width of the lake on google maps and scale the y axis of the videos accordingly to better match the bird spacing in the x direction. In the future, I should trim the videos to separate the disordered from the ordered phases in order to take a meaningful average head direction. It is most likely safe to assume that the birds’ head direction is the same as the direction of its velocities. I found that I need a better way to select for geese who never leave the zoomed in frame of the video or find a way to track the geese over the entire image to better trace individuals from frame to frame. So far, I have located birds and computed the tracks, but I still need to code a way to extract neighbor densities, heading directions, and information transfer entropy. Out of these, I am most excited about applying information theory techniques to the large flock of geese as I am unsure what patterns will emerge. Geese are highly social after all but they are not krill.

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