The Impact of Superfluids and Superconductors on Neutron Star Cooling

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Abstract

Neutron stars, remnants of stellar collapse whose densities exceed those of atomic nuclei, consist of a strongly interacting liquid of unbounded protons and neutrons (to first approximation) whose temperature is greatly below the Fermi temperature. They are subject to attractive nuclear interactions, so it is commonly believed that Cooper pairs will form in the degenerate matter. This would give rise to superfluid neutrons and superconducting protons. However, modelling degenerate nuclear matter remains an elusive problem, so little is known about the physics of the condensates. This essay will explore perhaps the most significant observational consequence of super-phases in neutron stars: their impact on the star's rate of cooling from a thermal initial configuration into a degenerate ground state. Specifically, we will discuss Cassiopeia A, the youngest and most rapidly cooling known neutron star, observations of which have allowed constraints to be placed on the super-phase phenomena.

1 Introduction and Historical Notes

The existence of neutron stars was first proposed in 1934, just two years after the neutron was discovered[1]. Seeking an explanation for supernovae, the astronomers Baade and Zwicky proposed that a main sequence star, at the end of its life, would most of its gravitational binding energy[2] in a violent explosion and a remnant consisting of tightly packed neutrons would form. Oppenheimer and Volkoff performed the first calculation based on this idea in 1939 and they found that relativistic gravity enforced a maximum mass of a stable neutron core around 0.72 $M_\odot$. However, all observed neutron stars are significantly more massive than this limit: a typical neutron star mass is 1.4 $M_\odot$ and the largest known neutron star exceeds 2 $M_\odot$. The reason for this discrepancy is the equation of state Oppenheimer and Volkoff used: they took the neutrons to be ideal fermions when they really interact via repulsive nuclear forces at high densities, leading to a much higher pressure at a given density than the ideal EOS predicts. Even in the most elementary treatments, the interactions between constituent particles play a central role [1].

The modern view of neutron stars asserts that, to first approximation, they are strongly interacting fluids of neutrons. Their densities exceed those of atomic nuclei found on Earth. So, in a stellar implosion, electrons are captured by protons in inverse beta decay and an electrically neutral fluid of free neutrons remains. This fermionic nuclear fluid is believed to be degenerate; that is, it is believed to have a temperature much less than its characteristic Fermi temperature. This makes the neutrons condense into the lowest energy state allowed by the Pauli principle. Hence, the microphysics and thermodynamics of this matter in this "ground state" are largely governed by the density, with finite temperature and heat conduction having perterbative effects [2].

The possibility of superfluidity in atomic nuclei resulting from Cooper-esque pairing between nucleons was explored very shortly after the development of BCS theory for electronic superconductors. In 1959, while calculating the moment of inertia for a nucleus with superfluidity, Migdal speculated that such effects could manifest in macroscopic neutron cores of stars[3] [3]. After the first pulsar (rotating neutron star with a strong magnetic field) was discovered in 1967, the problem of degenerate nuclear matter, including the effects of superfluidity, received intensified theoretical attention. Ginzburg presented an overview of the possibility in 1969, describing the pairing mechanism in degenerate neutron cores and identifying the effect this phase would have on the heat capacity [2].

Although an interesting theoretical topic, it has been difficult to identify a macroscopic property of neutron stars that would be directly impacted by superfluidity. Traditionally, pulsar glitches have been seen as such a manifestation. The rotation frequencies of pulsars are observed to gradually speed-down. This is expected as the star emits electromagnetic and gravitational radiation. However, the frequency is also observed to rapidly increase at periodic intervals. The mechanism behind this phenomenon, termed "glitching," remains

[1]However, there is a legend that Landau hypothesized such objects and worked out the existence of a maximum mass the same day he learned of Chadwick's announcement.

[2]At the time, it was thought that a star's radiance was fueled by its gravitational binding energy. This was before the details of thermonuclear processes inside stars were understood.

[3]He was referring to the outdated idea that main sequence stars could develop degenerate neutron cores as they aged.

elusive. The first attempt to explain it by the elastic properties of the star's crust (the "starquake" model) was quickly discredited. One theory that has gained popularity is the idea that glitches result from friction between the lattice nuclei and a neutron superfluid in the inner crust (see Sec. 2 and Fig. 1). In this picture, the quantized vorticies of the superfluid would cause the fluid to rotate faster than the lattice until the friction would eventually force the fluid to match the lattice's angular speed. Angular momentum would then be transferred to the crust, causing it to spin-up. However, it was shown that the required superfluid for this mechanism exceeds the amount of superfluid which can exist in the crust [4]. So, the glitch mechanism remains an open problem.

This essay will explore another macroscopic manifestation of superfluid neutrons (and superconducting protons): its impact on the star's cooling rate. Although neutron stars are expected to eventually become degenerate, they are created in supernova explosions, so they begin with a finite temperature and cool to a ground state. The mechanism responsible for Cooper pairing between nucleons is expected to create a significant channel for neutrino emission and for the star to lose energy. Moreover, observations of young neutron stars cooling can constrain properties of the super-phases, in principle.

Section 2 will provide an overview of the matter found inside neutron stars and identify the important Cooper pairing mechanisms. Section 3 will discuss the physics of the possible pairing mechanisms found inside neutron stars and describe the theoretical uncertainties. After describing the physics of the neutrino cooling mechanism in Section 4, Section 5 will discuss what observations of Cassiopeia A, the youngest known neutron star, can tell us about the super-phases.

2 Neutron Star Matter

Neutron star matter is degenerate and fermionic. It is significantly more complicated than the simple picture of a "free neutron fluid" presented in Sec. 1. The microphysics and thermodynamics are governed largely by the density. At very low densities, in the *crust*, finite nuclei still exist and free electrons form a degenerate Fermi liquid, very similar to terrestrial solids. As the distance below the surface increases, so does the density. The spacing between nuclei decreases and the electrons and nucleons interact in nontrivial ways until a neutron liquid of the kind described in Sec. 1 emerges in the *core*.

The transition from "ordinary matter" to neutron liquid occurs in three rough "phases" divided by two characteristic densities: $\rho_{ND} = 4 \times 10^{11}$ g/cm^3, the neutron drip density, and $\rho_{NS} = 2.8 \times 10^{14}$ g/cm^3, the nuclear saturation density. The current understanding of the microphysics in each phase, obtained from a combination of direct experimental experience, extrapolation from these experiments, and *ab initio* modelling, is now presented.

1. **Nuclear Lattice** ($\rho \lesssim \rho_{ND}$) Finite atomic nuclei (presumably Iron-56) form a lattice with degenerate electrons forming a band structure. This is the same kind of matter found in white dwarf stars and it is very similar to crystalline solids found on Earth. When $\rho \sim 10^7$ g/cm^3, the electron Fermi energy exceeds the nuclear barrier. Electrons are captured in the inverse of beta decay and more neutron-rich nuclei are created.

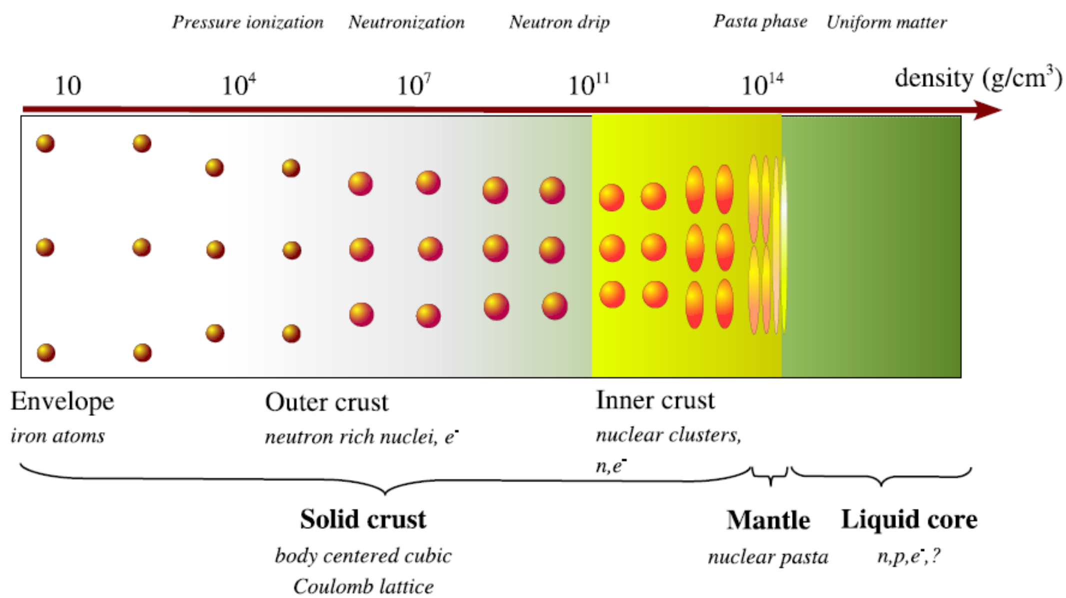

Figure 1: A representation of the different stages of degenerate fermionic matter encountered inside a neutron star. Free neutrons capable of condensing into a superfluid first appear at the neutron drip phase, and both neutrons and protons are expected to Cooper pair in the nuclear liquid phase. (Taken from Ref [2])

2. **Neutron Drip** ($\rho_{ND} \lesssim \rho \lesssim \rho_{NS}$) When the density reaches ρ_{ND}, the chemical potential of the neutrons in nuclei becomes zero. This causes them to "drip" out of the nuclei and create a "neutron gas" in chemical equilibrium with the finite nuclei. The presence of the gas affects the proton-neutron balance in the nuclei (or *clusters*): instead of favoring an equal balance between protons and neutrons, neutron-rich clusters are favored. It is expected that some of the free neutrons will condense into a superfluid. As $\rho \to \rho_{NS}$, some models predict that the clusters become unstable to deformations and form rod-like and slab-like objects called "pastas."

3. **Nuclear Liquid** ($\rho_{NS} \lesssim \rho$) When the mean separation between clusters reaches the characteristic size of a cluster at ρ_{NS}, the nucleons cease to exist in finite nuclei. A strongly interacting liquid of neutrons (with small amounts of protons, electrons, and other particles) forms that is still not understood. However, it is expected that some of the neutrons will condense into a superfluid and some of the protons will condense into a superconductor. It is hypothesized that at an even higher density, a first order phase transition will occur to free quark matter.

Degenerate fermionic matter has only been experimentally studied to densities of $\sim 6\times10^{10}$ g/cm^3, through both condensed matter physics and heavy ion collisions. For densities slightly above this point, it is possible to extrapolate from existing data. However, for densities much higher than this point, no terrestrial experiment has yet achieved the densities needed to begin to understand this matter [2]. And, since there are strong interactions governed by the strong force, making theoretical predictions has proven to be exceptionally difficult (there are currently over fifty candidates recognized by CompOSE [5]). It is hoped that neutron star observations, electromagnetic, particle, and gravitational, of which there are still very few, will guide our understanding of this exotic matter.

3 Nucleon Pairing in Neutron Stars

This section discusses the different superphases which could possibly exist in neutron star matter. First, the tempting idea that crust electrons could form a superconductor is dispelled. Next, the mechanism by which free neutrons and protons may pair to form superfluid and superconducting phases, respectively. Although qualitatively similar to the Cooper pairing mechanism in the degenerate electron fluids found in solids, the physics governing the pairing is significantly more complicated and the results are model-dependent.

3.1 Superconducting Electrons in the Crust?

One might think that a superconductor phase emerges in the degenerate electrons of the outer crust. Indeed, terrestrial iron has been found to have a superconducting phase at pressures $\sim 10^5$ atm (a modest value in the context of neutron stars) [2]. However, the critical temperature was found to be $\lesssim 2$K, which is much less than observed surface temperatures. Moreover, the electronic Fermi kinetic energy at neutron star densities greatly exceeds the Coulomb interaction energy, so the critical temperature for this phase is expected to be much lower than temperatures in the star [6].

We can verify this expectation using the results of BCS theory. In the weak-coupling limit, the critical temperature T_c is given by

$$k_B T_c = A E_D \; e^{-2/UN(0)} \tag{1}$$

where E_D is the energy associated with the Debye frequency in the lattice, U is the effective interaction energy between the electrons, $N(0)$ is the electronic density of states on the Fermi shell, and A is a constant of order unity. We also have

$$N(0) = \frac{k_F^2}{\pi^2 \hbar v_F} \tag{2}$$

where k_F is the Fermi wavevector and v_F is the Fermi velocity associated with the electrons. If the electrons are non-relativistic (NR) (true for $\rho \lesssim 10^6$ g/cm^3), then

$$v_F^{(NR)} = \frac{\hbar k_F}{m_e} \tag{3}$$

and it can be shown that

$$T_c^{(NR)} \approx (3.6 \times 10^3 \text{ K}) \left(\frac{\rho}{1 \text{ g/cm}^3} \right)^{1/2} \exp\left(-2.7 \left(\frac{\rho}{1 \text{ g/cm}^3} \right)^{1/3} \right) \tag{4}$$

So, the critical temperature begins at a small fraction of the surface temperature then exponentially decreases as the density increases below the surface. Moreover, when the electrons become highly relativistic (R), $v_F^{(R)} \approx c$ and it can be shown that

$$T_c^{(R)} \approx \frac{E_D}{k_B} \exp(-8/\pi\alpha) \tag{5}$$

(α is the fine structure constant), which is effectively zero for all practical purposes [2].

3.2 Neutron and Proton Pairing

Protons and neutrons, as much more massive particles, have kinetic energies comparable to Coulomb and nuclear interaction energies, so it is highly probable that sizeable superconducting and superfluid condensates form in the inner crust and core. Superconducting protons are expected to pair via the standard Cooper mechanism, but this picture is invalid for the superfluid neutrons. Unlike the Cooper pairing mechanism based on weak correlations between pairs of electrons over long distances in an ionic lattice, the attractive nuclear interaction between neutrons is quite strong over relatively short distances. So, the weak coupling approximation is not valid for this system, meaning there is no natural cutoff energy restricting the interactions to a narrow band.

Instead, the BCS equation for the gap energy Δ,

$$\Delta(\mathbf{k}) = -\frac{1}{2}\int \frac{d^3\mathbf{k}'}{(2\pi)^3}\tilde{V}_{\mathbf{k},\mathbf{k}'}\frac{\Delta(\mathbf{k}')}{\sqrt{(\epsilon(\mathbf{k}')-\mu)^2+\Delta(\mathbf{k}')^2}} \tag{6}$$

subject to the number constraint

$$n = \frac{k_F^3}{3\pi} = \int \frac{d^3\mathbf{k}'}{(2\pi)^3}\left(1 - \frac{\epsilon(\mathbf{k}')-\mu}{\sqrt{(\epsilon(\mathbf{k}')-\mu)^2+\Delta(\mathbf{k}')^2}}\right) \tag{7}$$

must be solved in full given a matrix element $\tilde{V}_{\mathbf{k},\mathbf{k}'}$ for the pairing interaction. When solved for a bare, two-body nucleon-nucleon interaction potential, assuming a free particle energy spectrum ($\epsilon = \hbar^2 k^2/2m$) and the Fermi surface remains well defined ($\Delta(k_F) << \mu$), the gap function is peaked at a neutron density ~ 0.02 fm^{-3} (corresponding to $\sim 0.1\rho_{NS}$) as shown in Fig. 2.

However, BCS theory neglects the many-body effects due to the remaining free neutrons on the pairing. In effect, a many-body calculation must be performed (alla Hartree-Fock) to determine the single particle energy spectrum to be used in the gap equations (6) and (7). The BCS picture is the mean field theory of this more complete picture of superfluid Fermions. Moreover, Eqns. (6) and (7) still neglect important many-body effects. The many-body calculation of the gap function must invoke the Nambu-Gorkov equations of superfluidic fermions. They are nonlinear and highly sensitive to the form of the interaction potential. As shown in Fig. 3, different interaction potentials can yield quite different functional forms for the energy gap. Kaminker and coworkers proposed an analytic formula to describe the different possible forms of the gap function [2]:

$$\Delta(k_F) = \Delta_0\ \frac{k_F^2}{k_1^2+k_F^2}\frac{(k_F-k_2)^2}{(k_F-k_2)^2+k_3^2}. \tag{8}$$

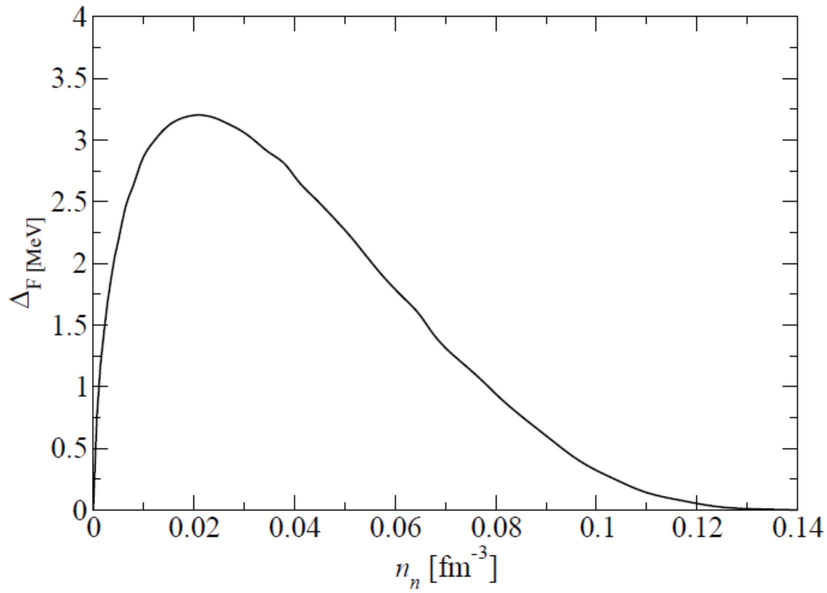

Figure 2: The neutron superfluid gap energy on the Fermi surface $\Delta(k_F) = \Delta_F$ as a function of the nucleon number density predicted by BCS theory. Note that it is peaked near $n_n = 0.02\ \text{fm}^{-3}$ (Taken from Ref. [2]).

where k_F is related to the nucleon density by the first part of Eq. (7) and Δ_0, k_1, k_2, and k_3 are adjustable parameters determined by the interaction model.

The nucleon-nucleon interaction depends strongly on the spin state of the particles. Below ρ_{NS}, in the crust, it is attractive, so neutrons are expected to pair into a singlet $^1S_0$ state. Above ρ_{NS}, the singlet interaction becomes repulsive while the triplet $^3P_2$ state becomes attractive. Thus, the superfluid neutrons in the crust and core are intrinsically different [7].

4 Neutron Star Cooling

Neutron stars are the remnants of supernovae, where a main sequence star collapses then violently explodes. Thermal processes play a crucial role in this process, so newly formed neutron stars are expected to have a temperature of order 10^{11} K, much greater than the Fermi temperature of the nuclear matter. Assuming it is isolated, it then gradually cools until the matter reaches the degenerate state described in Sec. 2. The cooling is presumed to be driven by particle reactions resulting in neutrino emission. The principal reactions are the *modified Urca process*,

$$n + N \rightarrow p + e + N + \bar{\nu}_e \tag{9}$$

$$p + e + N \rightarrow n + N + \nu_e \tag{10}$$

and nucleon-nucleon bremsstrahlung,

$$N + N \rightarrow N + N + \nu + \bar{\nu} \tag{11}$$

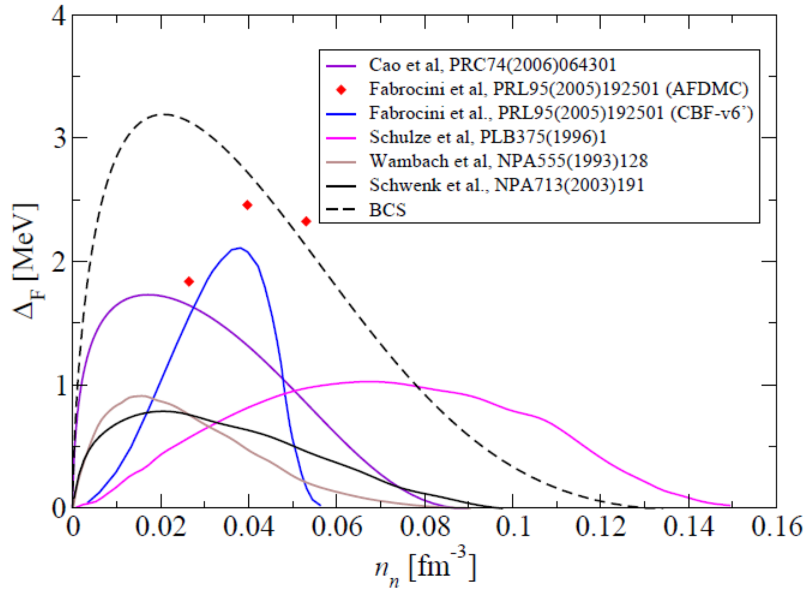

Figure 3: The neutron superfluid gap energy on the Fermi surface $\Delta(k_F) = \Delta_F$ as a function of the nucleon number density predicted by several many-body calculations. The BCS prediction is given by the dotted line. Observe that different calculations, based on different nucleon interaction potentials can yield quite different predictions for $\Delta_F(n_n)$ (Taken from Ref. [2]).

where N denotes either a proton p or a neutron n. The loss of energy via neutrino production in this process leads to the surface cooling to $\sim 10^6$ K on a timescale of 10^4 years. In addition, in the core regions of particularly massive neutron stars where the density is quite large, it is possible to have a *direct Urca process*,

$$n \to p + e + \bar{\nu}_e \tag{12}$$

$$p + e \to n + \nu_e \tag{13}$$

produce even larger neutrino emissivities. The surface would cool to $\sim 10^5$ K on a timescale of 10^4 years in this case.

If superfluid neutrons or superconducting protons are present, then there is another mechanism for neutrino production: the transition between bands to form a Cooper pair. When a nucleon pairs, it loses energy to enter the condensate, resulting in neutrino pair production:

$$N \to N + \nu + \bar{\nu} \tag{14}$$

This process was first recognized in 1976 by Flowers and coworkers and a cooling rate was calculated for the singlet neutron pairing state. Their work was extended to the triplet state by Yakovlev and coworkers [8]. The main points of the analysis will now be summarized.

The reaction in Eq. (14) may be recast using quasi-particles like those found in standard Bogoliubov theory. One instead works with "quasi-nucleons" (denoted by $\tilde{N}$) describing excitations above the super-condensate. In this view, neutrino production is described by

$$\tilde{N} + \tilde{N} \to \nu + \bar{\nu} \tag{15}$$

where two quasi-nucleons annihilate (removing the excitation) and the energy is used to produce two neutrinos. The quasi-nucleon operators are defined using Bogoliubov operators as

$$\hat{\Psi} = \sum_{\mathbf{k},\sigma,\eta} \chi_\sigma \big(e^{i\omega t - i\mathbf{k}\cdot\mathbf{r}} U_{\sigma\eta}(\mathbf{k})\ \hat{\alpha}_{\mathbf{k}\eta} + e^{-i\omega t + i\mathbf{k}\cdot\mathbf{r}} V_{\sigma\eta}(-\mathbf{k})\ \hat{\alpha}^\dagger_{\mathbf{p}\eta} \big) \tag{16}$$

where $U_{\sigma\eta}(\mathbf{k})$ and $V_{\sigma\eta}(-\mathbf{k})$ are matrix elements of the Bogoliubov coherence factors, η labels spin states, and χ_σ is a basis spinor. The Hamiltonian governing the interaction is

$$H_w = -\frac{G_F}{2\sqrt{2}}(c_V l_0 J_0 - c_A \mathbf{l}\cdot\mathbf{J}) \tag{17}$$

where $c_{V/A}$ describes the polar/axial vector contributions,

$$J_0 = \hat{\Psi}^\dagger \hat{\Psi}, \quad \mathbf{J} = \hat{\Psi}^\dagger \sigma \hat{\Psi} \tag{18}$$

is the (non-relativistic) quasi-nucleon 4-current defined using the shorthand σ for the Pauli matrices and

$$l^\mu = \bar{\psi}_\nu \gamma^\mu (1+\gamma^5)\psi_\nu \tag{19}$$

is the (relativistic) neutrino 4-current for a neutrino bispinor ψ_ν. The emissivity Q may be calculated from Fermi's Golden Rule. The relevant matrix elements are,

$$I_{00} = \sum_{\eta,\eta'} \left| \langle \mathbf{p},\eta,\mathbf{p}',\eta' | \hat{\Psi}^\dagger \hat{\Psi} | 0 \rangle \right|^2 \tag{20}$$

$$I = \sum_{i=1}^{3} \sum_{\eta,\eta'} \left| \langle \mathbf{p},\eta,\mathbf{p}',\eta' | \hat{\Psi}^\dagger \sigma_i \hat{\Psi} | 0 \rangle \right|^2 \tag{21}$$

where $|0\rangle$ is the quasi-nucleon vacuum state, and the end result is,

$$Q = \left(1.17 \times 10^{21} \frac{\text{erg}}{\text{s cm}^3}\right) \frac{p_F}{m_N v_F} \frac{p_F}{m_N c} \left(\frac{T}{10^9\ \text{K}}\right)^7 N_\nu R \tag{22}$$

where

$$R = \frac{1}{2} \int \frac{d\Omega}{4\pi} \int_0^\infty \frac{z^6 dx}{(e^z+1)^2} (c_V^2 I_{00} + c_A^2 I), \tag{23}$$

N_ν is the number of neutrino species present, m_N is the nucleon mass (the difference between proton and neutron is negligible in this context), $x = v_F k/k_B T$, and $z = E/k_B T$. In the case of singlet pairing, it can be shown that $I = 0$, while in the case of triplet pairing, it can be shown that $I = 2I_{00}$ [7].

The integral R is quite sensitive to the gap function $\Delta(T)$, the particles involved, the angular momentum of the paired state, and the temperature relative to the critical temperature. Approximate expressions for different regimes are derived in Ref. [7]. So, the dominant mechanism for neutrino emission can change depending on the conditions. This dependence is shown in Figs. 4 and 5. As the star cools, there is a crossover from Urca-dominated cooling to condensate-dominated cooling at core densities. Moreover, the critical temperatures of the condensate phases have an impact on the dominant effect [8].

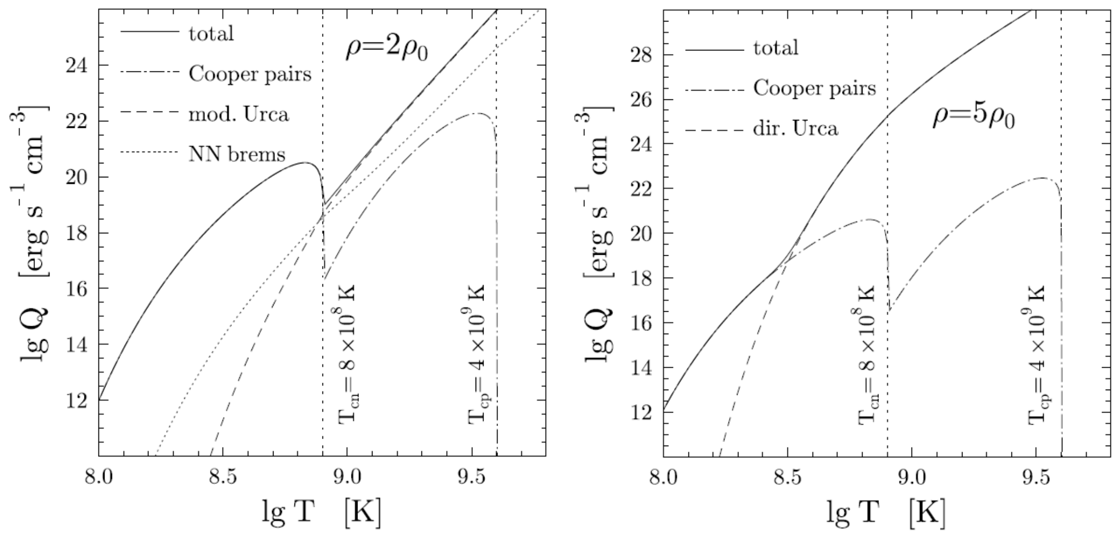

Figure 4: Neutrino emissivities from different mechanisms as a function of temperature and density. Nucleons entering the condensate provide the dominant source of neutrino emission once the temperature drops to some point below the critical temperature for condensation. (Taken from Ref. [7])

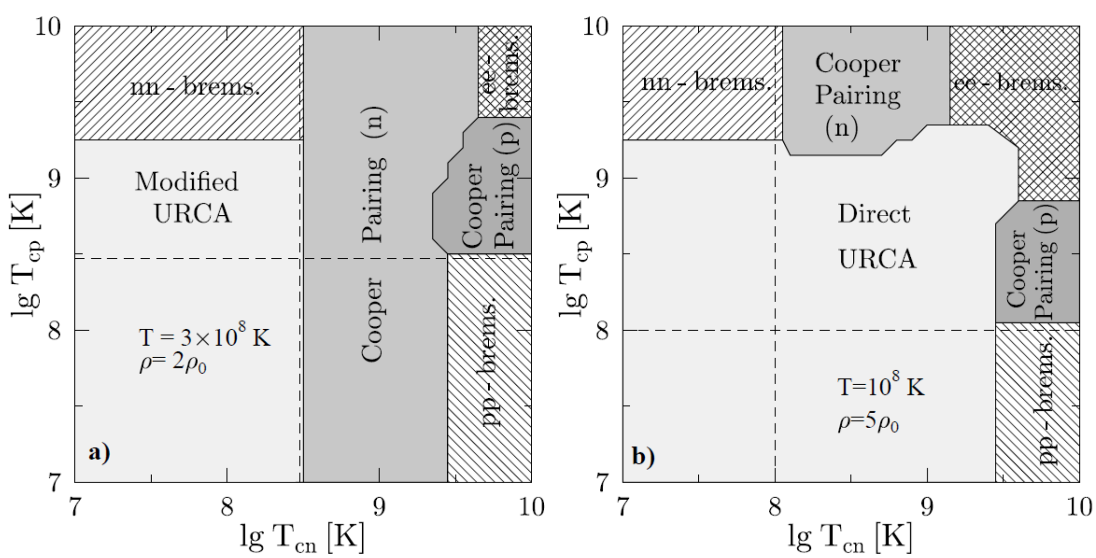

Figure 5: The dominant mechanism for neutrino emission as a function of the critical temperatures for proton and neutron condensation, T_{cp} and T_{cn} respectively, at two different temperatures and densities ($\rho_0 = \rho_{NS}$) (Taken from Ref. [7]).

5 Cassiopeia A

The youngest known neutron star, Cassiopeia A (Cas A), lies at the remnant of the historical supernova SN 1680[4]. It is 11 000 light years away in the constellation Cassiopeia A and about 330 years old. The first modern observation was via radio astronomy in 1947, before the Chandra x-ray telescope identified it as a neutron star in 1999. Since it is so young, Cas A is still in the process of cooling from its original state. This makes it an ideal candidate to test models of neutron star cooling.

Observations from 2000 to 2010 suggest that it is cooling much more rapidly than standard models based on Urca processes predict. One possible explanation is the presence of superfluid neutrons and superconducting protons. As discussed in Sec. 4, there is an increase in neutrino emission once the star reaches the critical temperature for pairing and condensation. Using data from the Chandra telescope allows the cooling of Cas A to be compared to models. Several authors have simulated the cooling of neutron stars, which has allowed them to place modest constraints on the proton and neutron phase condensates.

The singlet-paired superfluid neutrons in the crust are expected to be most emissive when the star is still very young (it is expected to cause an initial phase of relaxation within the first 100 years of the star's life). So, observations of Cas A are not expected to reveal much on this phenomenon. However, the rapider-than-predicted cooling may yield insights into the triplet-paired neutrons and superconducting protons in the core. Since the gap function is density-dependent (as discussed in Sec. 3.2), the maximum density of the star (the central density in stable configurations) governs the neutrino emission mechanism and therefore the cooling rate. Shternin and coworkers discuss this for Cas A. They use the observed mass to constrain the central density and simulate the cooling for different phenomenological models of neutron pairing (As shown in Fig. 6) [9].

Predictions regarding Cooper pairing and condensation into superfluids and superconductors vary greatly with the choice of microscopic model, so it is necessary to consider several when comparing to observation. Ho and coworkers extended the work of Shternin *et al.* by comparing cooling curves predicted my multiple microscopic models. They used a formula for the gap energy similar to Eq. (8) and they considered three possibilities: superfluid neutrons in the crust, superconducting protons in the core, and superfluid neutrons in the core. They found that the processes related to superfluid neutrons in both the crust and core took place too early in the cooling the process to be relevant to current observations of Cas A. However, they found that it was possible to constrain superconducting proton models (as shown in Fig. 7) [10].

6 Summary and Outlook

Cooper pairing is a common feature to most microscopic models of degenerate nuclear matter, leading to superfluid neutrons and superconducting protons. While the process shares some features with the much better-studied pairing mechanism in electron bands, it is significantly more complicated for nucleons. They pair as a result of strong short-ranged nuclear forces

[4]Although there is no record of an observed supernova from this time, it may have been inadvertently recorded as a main sequence star by John Flamsteed in 1680.

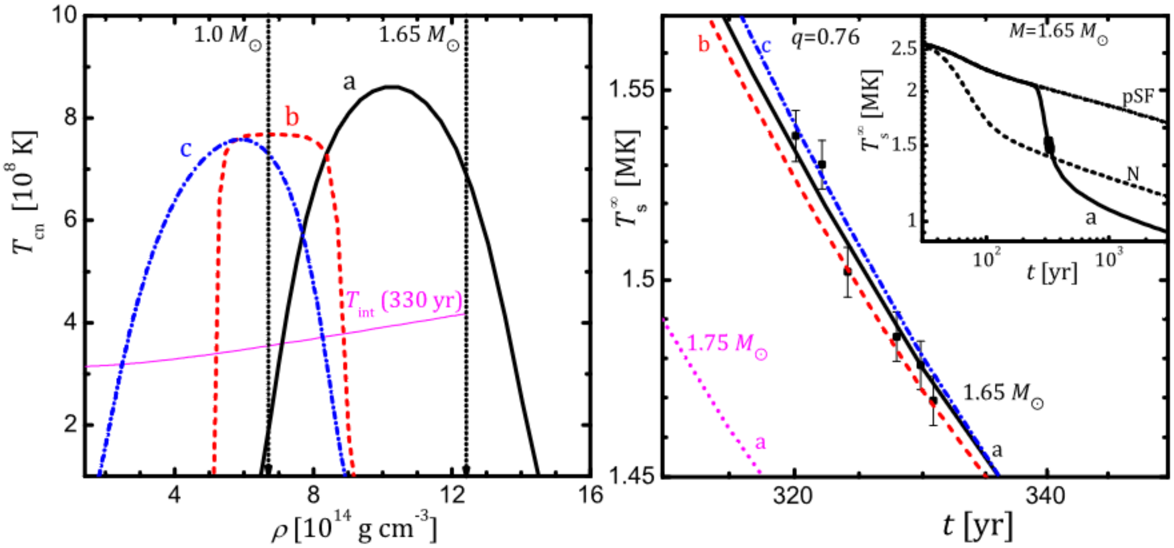

Figure 6: Left: The critical temperature T_{cn} of the superfluid neutron phase as a function of density for different pairing strengths. Right: the predicted cooling curves for the models on the left compared to observed surface temperatures for Cas A. (Taken from Ref. [9])

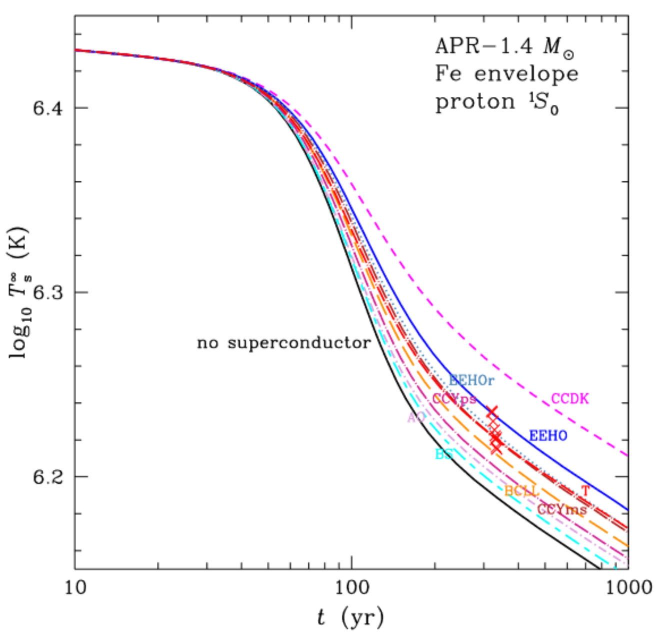

Figure 7: The observed surface temperature as a function of time for different core EOS models taking proton superconductivity into account. Unlike the case of superfluid neutrons, it is possible to constrain microscopic models using the observed data. (Taken from Ref. [10])

while electrons pair by weak long-range coupling to mechanical disturbances in a lattice. As a result, the results for the gap energy and critical temperature are highly sensitive to the particular model.

However, neutron star observations allow models to be constrained, since the Cooper pairing mechanism directly impacts the rate at which a young neutron star cools via neutrino emission. Cas A, as the youngest known neutron star, is perhaps the best candidate for such studies. Although it is already too old for neutron superfluidity to play a significant role, it is still possible to glean information about superconducting protons. Now that it is possible to observe gravitational waves from neutron stars, it will be possible to place constraints on the degenerate nuclear matter EOS, and place even further constraints on the Cooper pairing mechanisms.

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