Tsekenis, Goldenfeld, and Dahmen Reply:

Main Point of our Letter.—The main point of our Letter "Dislocations Jam at Any Density" [1] is to discuss dislocations as an example of systems that have no jamming point, in contrast to granular materials. Particles with long-range interactions feel each other regardless of how far apart they are. Therefore a finite stress $\tau_c(\rho)$ is necessary to unjam at all finite densities $\rho > 0$,

$$\tau_c(\rho > 0) > 0. \tag{1}$$

Here $\tau_c(\rho) \to 0$ only for $\rho \to 0$. This behavior is distinctly different from granular materials with short-range interactions where $\tau_c(\rho) = 0$ for $\rho < \rho_J$, where at ρ_J is the jamming point (e.g., see [2]).

Numerical Results.—We performed discrete dislocation dynamics simulations. We found that the critical (yield) stress in a two-dimensional dislocation system under shear scales with the square root of the dislocation density,

$$\tau_c \sim \sqrt{\rho}. \tag{2}$$

Our result (2) proves the main point (1) of our Letter [1]. It also agrees with the Taylor relation [3] and is analogous to the effective velocity of a point vortex in two-dimensional hydrodynamics [4] as we explicitly stated in our Letter.

Theoretical Calculation of Critical Stress.—The main point of this section of our Letter [1] is that even if the mean stress of a collection of uniformly distributed dislocations vanishes ($\langle\tau\rangle = 0$ from Eq. (4) in [1]) the effect of the dislocation density (or number) fluctuations is profound ($\tau_{\rm rms} \sim \sqrt{\rho}$) going beyond energy arguments that Groma *et al.* give in their Comment [5]. In critical phenomena, it is the fluctuations ($\tau_{\rm rms}$) that set the scale for the quantity (τ) that characterizes the phase transition near the critical point (τ_c). In order to show that, we employ analytical calculations that are in general agreement with our numerical findings. Most of the comments on the derivation were matters of preference although we found (ii) and (v) helpful. We can rewrite our original Eq. (5) as (any symbol not defined in this Reply was defined in [1])

$$\langle(\tau_{X,\Delta X})^2\rangle \sim \frac{\langle(N^+_{X,\Delta X} - N^-_{X,\Delta X})^2\rangle}{l^{2\alpha}X^{2\alpha}} \sim \frac{1}{l^{2\alpha}}\frac{\langle N_{X,\Delta X}\rangle}{X^{2\alpha}}. \tag{3}$$

The mean number of dislocations in the ring (first equation after Eq. (5) in [1]) can be expressed as $\langle N_{X,\Delta X}\rangle \sim N\frac{X^{d-1}\Delta X}{X_L^d} \sim X^{d-1}\Delta X$. Substituting into (3) and integrating over the entire region (similar to Eq. (6) in [1]),

$$\tau^2_{\rm rms} \equiv \int\langle(\tau_{X,\Delta X})^2\rangle \sim \frac{1}{l^{2\alpha}}\int_{X_{\rm min}}^{X_L}\frac{dX}{X^{2\alpha-d+1}}, \tag{4}$$

with $X_{\rm min} = b/l$, b the closest two dislocations can be. For $2\alpha > d$ it gives $\tau_{\rm rms} \sim \frac{1}{l^\alpha}\frac{1}{\sqrt{2\alpha-d}}\sqrt{\frac{1}{X_{\rm min}^{2\alpha-d}} - \frac{1}{X_L^{2\alpha-d}}} \sim \frac{1}{l^{d/2}} \sim \sqrt{\rho}$ in the thermodynamic limit, $X_L = L/l \to \infty$. For $2\alpha < d$, $\tau_{\rm rms} \sim \sqrt{\rho}\frac{1}{\sqrt{d-2\alpha}}\sqrt{L^{d-2\alpha} - b^{d-2\alpha}}$ and the thermodynamic limit does not exist. For parallel straight edge dislocations in two dimensions with $2\alpha = 2 = d$ and in general for $2\alpha = d$,

$$\tau_{\rm rms} \sim \frac{1}{l^{d/2}}\sqrt{\ln(L/b)} \sim \sqrt{\rho}\sqrt{\ln(L/b)}. \tag{5}$$

This agrees with our scaling collapse in the bottom Fig. 3 in [1], Eq. (2) is equivalent to (5) for fixed L, and consequently $\tau_c \sim \tau_{\rm rms}$. In addition, we performed the scaling collapse for fixed N and found that (5) works exactly, further verifying that $\tau_c \sim \tau_{\rm rms}$ [6]. We would like to thank the authors of the Comment for pointing out a better choice for the lower limit of the integral. In fact, our analytical calculation above provides a stronger scaling argument for the Taylor relation than Groma *et al.* give in their Comment [5] that is valid for any power law α and dimension d.

Screening.—Screening for a driven nonequilibrium dislocation system (like ours) is not yet resolved in the literature. Our collapse works well with Eqs. (2) and (5), indicating that screening effects were not significantly present in our numerical results. The screened interaction $\tau_{\rm int} \sim e^{-r/r_0}/r$ with $r_0 \sim 1/\sqrt{\rho}$ proposed by the authors of the Comment [5] was extracted from equilibrium systems, which are different from our driven nonequilibrium dynamics and the two may not be comparable. In the equilibrium case, as $\rho \to 0$, $r_0 \to \infty$ and the interaction becomes again the unscreened power law interaction $\tau_{\rm int} \sim 1/r$. As a result we still need to take the limit of $\rho \to 0$ to find the analogue of a jamming point where $\tau_c(\rho) \to 0$. Thus our main point (1) on jamming of dislocations remains valid and as stated in our Letter [1].

Georgios Tsekenis, Nigel Goldenfeld, and
Karin A. Dahmen
Department of Physics
University of Illinois at Urbana-Champaign
Loomis Laboratory of Physics
1110 West Green Street, Urbana, Illinois 61801-3080, USA

Received 16 April 2012; published 28 June 2012
DOI: 10.1103/PhysRevLett.108.269602
PACS numbers: 61.72.Hh, 61.72.Ff, 61.72.Lk, 62.20.fq

[1] G. Tsekenis, N. Goldenfeld, and K. A. Dahmen, Phys. Rev. Lett. **106**, 105501 (2011).
[2] A. Liu and S. Nagel, Annu. Rev. Condens. Matter Phys. **1**, 347 (2010).
[3] J. P. Hirth and J. Lothe, *Theory of Dislocations* (John Wiley and Sons, New York, 1982), 2nd ed.
[4] P. H. Chavanis, Phys. Rev. E **65**, 056302 (2002).
[5] I. Groma, G. Györgyi, and P. D. Ispánovity, preceding Comment, Phys. Rev. Lett. **108**, 269601 (2012).
[6] G. Tsekenis, Ph.D. thesis, University of Illinois at Urbana-Champaign, 2012.

